Performance Analysis of ECG Data Compression Techniques

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Abstract- *ECG can identify the electrical activity of the heart. A muscular organ, the heart rhythmically pumps blood throughout the body. It's necessary to send and store large signal data. The ECG signal data must then be effectively compressed. In this article, we compared the effectiveness of various ECG compression techniques.These techniques are crucial for lowering communicated data size without losing clinical information. Discrete Cosine Transform (DCT), Discrete Sine Transform (DST), Fast Fourier Transform (FFT), the enhanced method Discrete Cosine Transform- II (DCT-II), and Blaschke unwinding AFD are the transformation methods on which these schemes are based.We test records that have been chosen from the MIT-BIH arrhythmia database. Percent Root Mean Square Differences (PRD) and Compression Ratio (CR) are used to evaluate performance.*

Keywords- ECG, DCT, DST, FFT, DCT-II, Blaschke unwinding AFD

I. INTRODUCTION

Electrocardiographic signals may be recorded on a long timescale (i.e., several days) for the purpose of identifying intermittently occurring disturbances in the heart rhythm. As a result, the produced ECG recording amounts to huge data sizes that quickly fill up available storage space. Transmission of signals across public telephone networks is another application in which large amounts of data are involved. For both situations, data compression is an essential operation and, consequently, represents yet another objective of ECG signal processing. Signal processing has contributed significantly to a new understanding of the ECG and its dynamic properties as expressed by changes in rhythm and beat morphology. For example, techniques have been developed that characterize oscillations related to the cardiovascular system and reflected by subtle variations in heart rate. ECG Data Compression is required to reduce the disk space required to store the data, as ECG is a continuous data taken for a very long interval of time. Also by compressing redundant data from the signal can be removed which actually takes considerably large area in memory. The need of signal transmission over telephone lines or antenna for remote analysis makes the compression and data reconstruction of the signal an important issue in signal processing. ECG is a graphic display of the electrical activity of the heart. Due to low cost and noninvasion, ECG signal has been extended for heart disease diagnosis and ambulatory monitoring. For storage and transmission of large signal data, it is necessary to compress the ECG signal data. Data compression has its application in many fields and so as in the field of medical science. ECG is an important parameter that measures patient's health and reports abnormalities if any. This thesis has done a survey of various kinds of ECG data compression techniques. Recently, numerous research and techniques have been developed for compression of the signal. These techniques are essential to a variety of application ranging from diagnostic to ambulatory ECG's. Thus, the need for effective ECG compression techniques is of great importance. The non-invasive extraction of physiological and clinical information hidden in biomedical signals is an important and fascinating field of research. Non-invasive assessment of the physiological parameters of a patient enables to study the physiology and patho-physiology of the investigated system, with minimal interference and inconvenience. Endogenous biomedical signals from physiological systems are acquired for a number of reasons including diagnosis, post surgical intensive care monitoring, neonatal monitoring and guide therapy and for research. The electrocardiogram (ECG) is a non- stationary signal containing information about the physiological condition of the heart. The electrical activity of the heart depicts the morphology and durations of the P-QRS- T intervals (Figure 1). The P, QRS complex and T features of ECG reveal the rhythmic depolarization and re polarization of the myocardium contractions of heart's atria and ventricles [1]. The time intervals between various peaks contain clinical information about the nature of possible disease afflicting a heart [2].

Due to low cost and non-invasion, ECG signal has been extended for heart disease diagnosis and ambulatory monitoring resulting in enormous volume of the data. In course of a 24-h ECG observation or multichannel biological signal acquisition, real-time data compression methods are required for the effective use of communications channels such as wired channel, wireless environment and cloud computing. The ECG data compression is also required for the transmission of ECG signals across intensive care units, emergency tele-medical services, telemedicine, home care, space programs, sports, military, public telephone networks, cellular networks and wireless communication systems [4-5]. ECG is having possibility of redundant information reduction through inter and intra beat correlation, which is the basic cause of its compression [6]. The fundamental goal of data compression is efficient transmission or storage while preserving the significant diagnostic features.

In general, ECG compression can be classified into lossy and lossless techniques [7]. The lossless compression guarantee the integrity of reconstructed data while compromised compression ratio (CR), with nearly 0% reconstruction error, on the other hand lossy compression is having high CR with varying level of reconstruction error [6].

ECG signal compression techniques widely fall into three categories of direct method, transformation method and parameter extraction method [7, 8]. The direct data compression method openly analyzes and reduces data points in the time domain and the example includes turning point (TP) [25], amplitude zone time epoch coding (AZTEC) [3], Improved modified AZTEC technique [9], coordinate reduction time encoding system (CORTES) [48], SLOPE [10], the delta algorithm and the Fan algorithm [11]. The transformed method analyzes energy distribution by converting the time domain to some other domain and example includes Fourier transform, Fourier descriptor [12], the discrete cosine transform (DCT) [13], DCT with modified stages [14, 15] and wavelet transform [16], and the compressed sensing [17]. The parameter extraction method is based upon dominant feature extraction from raw signal; examples include neural based or syntactic methods [18],peak picking and linear prediction method [19]. The other methods for compression includes ASCII based encoding for incorporation of ECG data as ASCII character in existing technology [20-23].

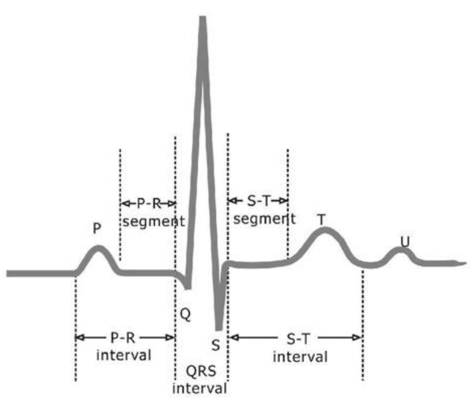

Fig 1 : Time intervals of ECG

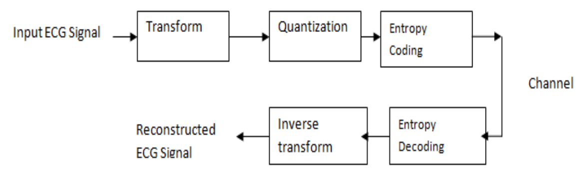

Fig. 2 Block diagram of transform based compression method

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Data compression has its application in many fields and so as in the field of medical science. ECG is an important parameter that measures patient's health and reports abnormalities if any. This thesis has done a survey of various kinds of ECG data compression techniques. Recently, numerous research and techniques have been developed for compression of the signal. These techniques are essential to a variety of application ranging from diagnostic to ambulatory ECG's. Thus, the need for effective ECG compression techniques is of great importance. Many existing compression algorithms have shown some success in electrocardiogram compression; however, algorithms that produce better compression ratios and less loss of data in the reconstructed signal are needed.

II. DISCRETE COSINE TRANSFORM(DCT)

The Discrete Cosine Transform (DCT) was developed to approximate Karhunen-Loeve Transform (KLT) when there is high correlation among the input samples, which is the case in many digital waveforms including speech, music, and biomedical signals. The DCT $D = [d_0\ d_1\ d_2\ d_3 \ldots\ldots\ldots. d_{N1-1}]^T$ Of the vector x is defined as follows

$$d_0 = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x_{n_1} \qquad (1)$$

$$d_k = \sqrt{\frac{2}{N}} \sum_{n1=0}^{N-1} x_{n_1} \cos\frac{(2n_1+1)K\pi}{2N}, \quad k = 1,2,\ldots\ldots\ldots\ldots\ldots N-1 \qquad (2)$$

Where d_k is the k_{th} DCT coefficient. The inverse discrete cosine transform (IDCT) of d is given by

$$x_{n_1} = \frac{1}{\sqrt{N}} d_0 + \sqrt{\frac{2}{N}} \sum_{k=0}^{N-1} d_k \cos \frac{(2n_1+1)K\pi}{2N}$$

n_1=0,1,2………………..N-1 (3) (3.3)

There exist fast algorithms, Order (Nlog$N$), to compute the DCT .Thus, DCT can be implemented in a computationally efficient manner. Two recent image and video coding standards, JPEG and MPEG, use DCT as the main building block. A discrete cosine transform (DCT) expresses a sequence of finitely many data points in terms of a sum of cosine functions oscillating at different frequencies. DCTs are important to numerous applications in science and engineering, from lossy compression of audio (e.g. MP3) and images (e.g. JPEG) (where small high frequency components can be discarded), to spectral methods for the numerical solution of partial differential equations. The use of cosine rather than sine functions is critical in these applications. For compression, it turns out that cosine functions are much more efficient whereas for differential equations the cosines express a particular choice of boundary conditions. In particular, a DCT is a Fourier-related transform similar to the discrete Fourier transform (DFT), but using only real numbers. DCTs are equivalent to DFTs of roughly twice the length, operating on real data with even symmetry (since the Fourier transform of a real and even function is real and even), where in some variants the input and/or output data are shifted by half a sample. Discrete Cosine Transform is a basis for many signal and image compression algorithms due to its high decorrelation and energy compaction property. A discrete Cosine Transform of N sample is defined as

$$F(u) = \sqrt{\frac{2}{N}} C(u) \sum_{x1=0}^{N-1} f(x_1) \cos \frac{(2x_1+1)u\pi}{2N}$$

u = 0,1,2……………..N-1 (4)

Where

$$C(u) = \begin{cases} \frac{1}{\sqrt{2}}, & for\ u = 0 \\ 1, & otherwise \end{cases}$$

The function $f(x)$ represents the value of x_{th} samples of input signals. $F(u)$ represents DCT coefficients. The inverse DCT is defined in similar fashion as

$$f(x_1) = \sqrt{\frac{2}{N}} C(u) \sum_{u=0}^{N-1} C(u)\ F(u) \cos \frac{(2x_1+1)u\pi}{2N}$$

x_1 = 0,1,2…………N-1 (5)

III. DISCRETE SINE TRANSFORM

Discrete sine transform (DST) is a Fourier-related transform similar to the discrete Fourier transform (DFT), but using a purely real matrix. It is equivalent to the imaginary parts of a DFT of roughly twice the length, operating on real data with odd symmetry (since the Fourier transform of a real and odd function is imaginary and odd), where in some variants the input and/or output data are shifted by half a sample. Like any Fourier-related transform, discrete sine transforms (DSTs) express a function or a signal in terms of a sum of sinusoids with different frequencies and amplitudes. Like the discrete Fourier transforms (DFT), a DST operates on a function at a finite number of discrete data points. The obvious distinction between a DST and a DFT is that the former uses only sine functions, while the latter uses both cosines and sines (in the form of complex exponentials). However, this visible difference is merely a consequence of a deeper distinction: a DST implies different boundary conditions than the DFT or other related transforms.

Formally, the discrete sine transform is a linear, invertible function F: R^N -> R^N (where R denotes the set of real numbers), or equivalently an N × N square matrix. There are several variants of the DST with slightly modified definitions. The N real numbers $x_0,\ldots.x_{N-1}$ are transformed into the N real numbers $X_0,\ldots..X_{N-1}$ according to

$$X_k = \sum_{n=0}^{N-1} x_n \sin \frac{\pi}{N+1}(n+1)(k+1)$$

k =0,1,…….. N-1 (6)

IV. FAST FOURIER TRANSFORM (FFT)

A fast Fourier transform (FFT) is an efficient algorithm to compute the discrete Fourier transform (DFT) and it's inverse. There are many distinct FFT algorithms involving a wide range of mathematics, from simple complex-number arithmetic to group theory and number theory. A DFT decomposes a sequence of values into components of different frequencies but computing it directly from the definition is often too slow to be practical. An FFT is a way to compute the same result more quickly. Computing a DFT of N points in the naive way, using the definition, takes O(N2) arithmetical operations , while an FFT can compute the same result in only O(N log N) operations.

Fast Fourier Transform is a fundamental transform in digital signal processing with applications in frequency analysis, signal processing etc. The periodicity and symmetry properties of DFT are useful for compression. The uth FFT coefficient of length N sequence {f(x)} is defined as

$$F(u) = \sum_{x=0}^{N-1} f(x)\, e^{\frac{-j2\pi ux}{N}}$$
u = 0,1,2……………..N-1 (7)

And its inverse transform is calculated from

$$f(x) = \frac{1}{N}\sum_{u=0}^{N-1} F(u)\, e^{\frac{j2\pi ux}{N}}$$
x = 0,1,2………. N-1 (8)

V. DISCRETE COSINE TRANSFORM–II (DCT – II)

The most common variant of discrete cosine transform is the type-II DCT [54]. The DCT-II is typically defined as a real, orthogonal (unitary), linear transformation by the formula

$$C_k^{II} = \sqrt{\frac{2-\delta_{k,0}}{N}} \sum_{n=0}^{N-1} x_n \cos\left[\frac{\pi}{N}\left(n+\frac{1}{2}\right)k\right] \quad (9)$$

for N inputs x_n and N outputs C_k^{II}, where $\delta_{k,0}$ is the Kronecker delta (= 1 for k = 0 and = 0 otherwise). DCT-II can be viewed as special case of the discrete Fourier transform (DFT) with real inputs of certain symmetry. This viewpoint is fruitful because it means that any FFT algorithm for the DFT leads immediately to a corresponding fast algorithm for the DCT-II simply by discarding the redundant operations. The discrete Fourier transform of size N is defined by

$$X_K = \sum_{n=0}^{N-1} x_n \quad (10)$$

where $\omega_N = e^{-2\pi iN}$ is an N_{th} primitive root of unity. In order to relate this to the DCT-II, it is convenient to choose a different normalization for the latter transform as

$$C_K = 2\sum_{n=0}^{N-1} x_n \cos\left[\frac{\pi}{n}\left(n+\frac{1}{2}\right)k\right] \quad (11)$$

$$2\cos\left(\frac{\pi l}{N}\right) = \omega_{4N}^{2l} + \omega_{4N}^{4N-2l} \quad (12)$$

$$C_K = 2\sum_{n=0}^{N-1} x_n \cos\left[\frac{\pi}{N}(n+1)k\right] \quad (13)$$

$$C_K = \sum_{n=0}^{N-1} x_n\, \omega_{4N}^{(2n+1)k} + \sum_{n=1}^{N-1} x_n\, \omega_{4N}^{(4N-2n-1)k} \quad (14)$$

Thus, the DCT-II of size N is precisely a DFT of size 4N, of real-even inputs, where the even-indexed inputs are zero.

VI. BLASCHKE UNWINDING AFD

Algorithm 1 illustrates how the Blaschke unwinding AFD is applied to compress a real-valued signal. First, the input real-valued signal F is projected to H^2 space and we get F^+. In practice, we could safely assume that

$$2\Re F^+ = F + c_0 \quad (15)$$

holds, where $\Re$ means taking the real part and c_0 is the zeroth Fourier coefficient of F. c_0 is the first data point we save for the signal compression, and F^+ is initialized as the first remainder F1.

Algorithm I : Blaschke Unwinding AFD based Compression

Input: Real-valued input signal F, sets of parameters a$\in$ D and the decomposition level N.

Output: $\{c_n\}_{n=1}^N$, $\{a_n\}_{n=1}^N$ and a finite number of zeros $\left\{\{r_{n_j}\}_{j=1}^{M_n}\right\}_{n=1}^N$

l: Get the projection signal $F^+$of F in the Hardy space.
2: Initialize $F_l = F^+$.
3: for n = 1 to N do.
4: Obtain the inner function I_n and outer function O_n of F_n so that $F_n = I_n O_n$;
5: Get zeros $\{r_{n_j}\}_{j=1}^{M_n}$, of I_n by Algorithm 2;
6: Get $a_n = \arg\max\{(1-|a|^2)|O_n(a)|^2 : a \in D\}$;
7: Get $c_n = \langle O_n e_{a_n} \rangle$;
8: Get $F_{n+1} = \frac{F_n - c_n I_n e_{a_n}}{I_n} \frac{1-\overline{a_n} z}{z-a_n}$;
9: return $\{c_n\}_{n=1}^N$, $\{a_n\}_{n=1}^N$, $\left\{\{r_{n_j}\}_{j=1}^{M_n}\right\}_{n=1}^N$.

Second, extract the inner function by calculating zeros of F_1 by the method introduced in [59], where we assume that F_1 has finite roots on $\overline{D}$:= {z ∈ C| $\|z\|$ ≤ 1} [59]. The detailed steps of numerical calculation for calculating zeros of F_1 are performed in Algorithm 2. Then accordingly, get the outer function O_1 by the Nevanlinna factorization. Third, The set of {a_n}, n = 1, 2, . . . , in consisting of discrete points in $\mathbb{D}$ is generated by dividing $\mathbb{D}$ into rectangular grid to get the TM system and evaluators {e_a} . Then, the decomposition of O_1 is based on the TM system. During the decomposition, the maximal selection principle is applied in the selection of a1 with the aid of evaluators. Suppose the decomposition level is N ∈ N. Iterate the above three steps, each on the remainder of the previous step, for N times, and we end up with {e_{a_n} }, the modified Blaschke products {B_n}, and M_n zeros, for n = 1, . . . N. As a result, we obtain 2N + 1 parameters, including $\{c_n\}_{n=0}^{N}$, and $\{a_n\}_{n=1}^{N}$, as well as $\sum_{n=1}^{N} M_n$ zeros. c_n and a_n, where n = 1, . . . N, as well as $\sum_{n=1}^{N} M_n$ zeros, are other data points we save for the data compression.

Algorithm II: Procedure for calculating zeros

Input: F, $\delta > 0$

Output: zeros of F, $\{r_{n_j}\}_{j=1}^{M_n}$

l: Determine for M, $M = \frac{1}{2\pi i}\int_{|z|=1} \frac{F'(z)}{F(z)}\, dz$
2: Initialize $G_1 : F$
3: **for** j = 1 to M_1 **do**.
4: Evaluate arg $\min_{z \in D_{1-\delta}} |G_j(z)|$;
5: Get r_j satisfying $G_j(r_j) = 0$;
6: Get $G_{j+1} := G_j \frac{1 - r_j z}{z - r_j}$;
7: **return** $\{r_j\}_{j=1}^{M_n}$.

VII. RESULT ANALYSIS

We used data in the MIT-BIH database to test the performance of the six coding techniques. The ECG data is sampled at 142Hz and the resolution of each sample is 11bits/samples. The amount of compression is measured by CR and the distortion between the original and reconstructed signal is measured by Percentage Mean Square Difference (PRD). A data compression algorithm must represent the data with acceptable fidelity while achieving high CR.

Figure 3 shows the original ECG signal record 100 which are selected from MIT-BIH arrhythmia database and its reconstructed waveform when compressed by FFT.

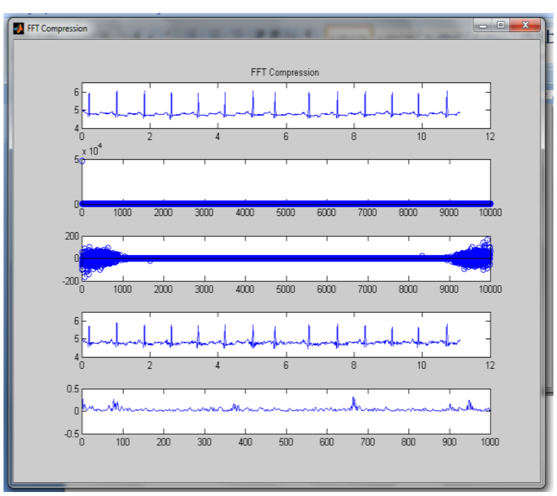

Figure 3 FFT compression of MIT-BIH record 100

Fig.4 shows the original ECG signal record 100 which are selected from MITBIH arrhythmia database and its reconstructed waveform when compressed by DCT.

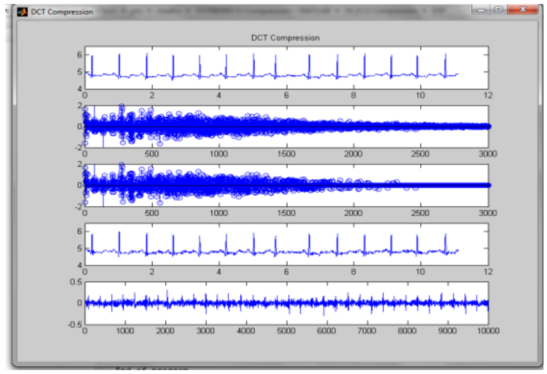

Figure 4 DCT compression of MIT-BIH record 100

Figure 5 shows the original ECG signal record 100 which are selected from MIT-BIH arrhythmia database and its reconstructed waveform when compressed by DST.

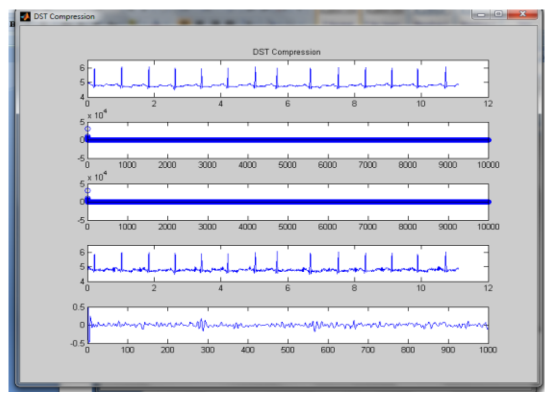

Figure 5. DST compression of MIT-BIH record 100

Figure 6 shows the original ECG signal record 100 which are selected from MIT-BIH arrhythmia database and its reconstructed waveform when compressed by DCT-2.

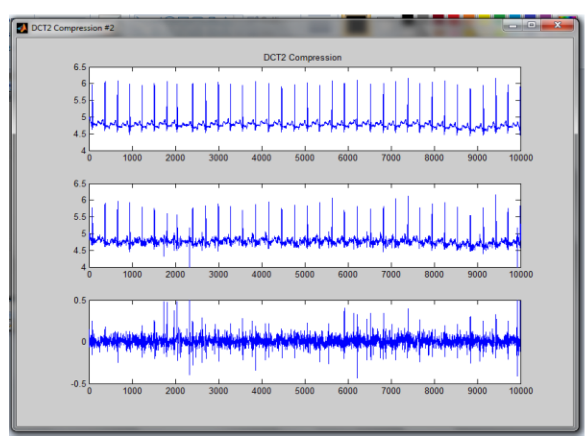

Figure 6 DCT-2 compression of MIT-BIH record 100

In Blaschke unwinding AFD, the compression consists of two steps. The first step carries out the Hardy projection and the Blaschke unwinding AFD compression. The second step is the lossless Huffman encoding. For the decompression, it is the inverse of the compression, including the Huffman decoding and the inverse Blaschke unwinding AFD process.

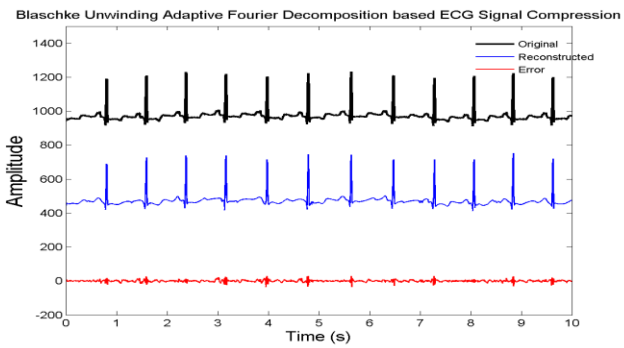

Figure:7 Waveforms of original, reconstructed and error signals with N = 8 taken from record 100

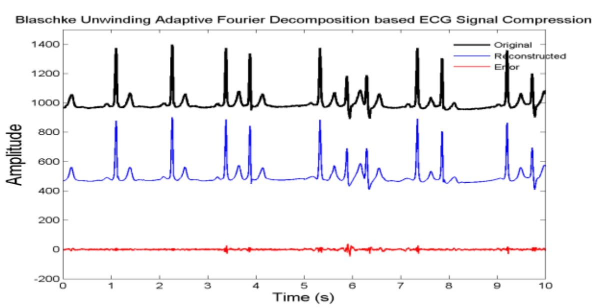

Figure:8 Waveforms of original, reconstructed and error signals with N = 10 taken from record 100

The comparison table shown in Table 1 details the resultant compression techniques. This gives the choice to select the best suitable compression method. A data compression algorithm must represent the data with acceptable fidelity while achieving high CR. As the PRD indicates reconstruction fidelity; the increase in its value is actually undesirable. Blaschke unwinding AFD which leads to a high compression rate with a high fidelity. Compared with existing algorithms, like FFT, DCT, DST and DCT-2.

Table 1 Comparision of resultant compression techniques

Method	**CR**	**PRD**
FFT	16.01	1.10
DCT	16.87	1
DST	11.62	1.19
DCT2	22.21	1.27
Blaschke unwinding AFD(N=8)	39.34	0.71
Blaschke unwinding AFD(N=10)	26.09	0.57

VIII. CONCLUSION

Among the five techniques presented, DST provides lowest CR and distortion is also high. DCT improves CR and lowers PRD. Next is FFT which gives CR 16.01 with PRD as 1.10. But DCT-II provides an improvement in terms of CR of 22.21 but PRD increases up to 1.27. Thus an improvement of a discrete cosine transform (DCT)-based method for electrocardiogram (ECG) compression is presented as DCT-II in terms of amount of compression. From table 1 we can observe that using Blaschke unwinding AFD based compression we are getting higher compression rate in comparison to other compression technique. Hence this technique is better than other ECG compression technique in both point of consideration.

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