Pythagorean Triangle With Area/Perimeter As Krishnamurthy Number

Dr. G. Janaki[2], R. Sarulatha[2]
[1]Associate Professor, Dept of Mathematics
[2]Dept of Mathematics
[1, 2] Cauvery college for women, Tiruchirappalli-620018, Tamil Nadu, India

Abstract- *This article investigates the patterns of Pythagoreantriangles employing Krishnamurthy Number as ratio of Area/Perimeter. Further some fascinating relations among the sides are also presented.*

Keywords- Krishnamurthy Number, Non-Primitive, Primitive, Pythagorean Triangles, Special numbers.

I. INTRODUCTION

Mathematics is the Science of structure, order and relation that has evolved from counting, measuring, and describing the shapes of objects. Mathematics deals with logical reasoning and quantitative calculations. It has been an indispensable adjunct to the physical science and technology. Number Theory explores the fundamental nature of numbers and seeks to understand their intricate pattern and structure. [1-3] is referred for detailed view on subject. In addition to figurate numbers, Special numbers play an integral part of number theory. Some intriguing numbers include Jarasandha number, Harshad number, Canada numbers, Disarium numbers and so on. [4-10] portrays copious problems concerning special numbers and its relation to Pythagorean triangles. The main vision of this article is to seek the patterns of Pythagorean triangle symbolising Krishnamurthy number as Area/Perimeter ratio with intriguing properties.

II. BASIC DEFINITIONS

Definition 2.1: The ternary quadratic Diophantine equation given by $u^2 + v^2 = w^2$ is known as Pythagorean equation where u, v and w are natural numbers. The equation above is also known as Pythagorean triangle and denote it by $\Psi(u, v, w)$.Further, in Pythagorean triangle $\Psi(u, v, w)$: $u^2 + v^2 = w^2$, u, v are called its legs and w its hypotenuse.

Definition: 2.2: Most cited solution of the Pythagorean equation is $u = f^2 - g^2$, $v = 2fg$ and $w = f^2 + g^2$, where $f > g > 0$. If f and g have opposing parities and gcd$(f, g) = 1$, then this solution is referred to as Primitive otherwise non-primitive.

Definition 2.3: A Krishnamurthy number is a number whose sum of the factorial of digits is equal to the number itself. The Krishnamurthy numbers are 2, 145, 40585.

III. ANALYSIS TECHNIQUE

Area and perimeter of the triangle are denoted by $\dot{A}$and $\dot{P}$ respectively with the hypothesis that

$\frac{\dot{A}}{\dot{P}} =$ Krishnamurthynumber

The relationship mentioned above leads to the expression

$\frac{g(f-g)}{2} =$ Krishnamurthy number(1)

Case 1: One digit Krishnamurthy number

$\frac{g(f-g)}{2} = 2$ (2)

$\Rightarrow \quad g(f-g) = 4$(3)

Table1 shows the values of the generators f *and* g,satisfying (3)after scrutiny.

Table 1

g	f	u	v	w	$\dot{A}$	$\dot{P}$	$\frac{\dot{A}}{\dot{P}}$
1	5	24	10	26	120	60	2
2	4	12	16	20	96	48	2
4	5	9	40	41	180	90	2

Case 2:Three digit Krishnamurthy number

$\frac{g(f-g)}{2} = 145$ (4)

$\Rightarrow \quad g(f-g) = 290$(5)

Table 2 displays the values of the generators f *and* g,satisfying (5) after estimation.

Table 2

g	f	u	v	w	$\dot{A}$	$\dot{P}$	$\frac{\dot{A}}{\dot{P}}$
1	291	84680	582	84682	24641880	169944	145
2	147	21605	588	21613	6351870	43806	145
5	63	3944	630	3994	1242360	8568	145
10	39	1421	780	1621	554190	3822	145
29	39	680	2262	2362	769080	5304	145
58	63	605	7308	7333	2210670	15246	145
145	147	584	42630	42634	12447960	85848	145
290	291	581	168780	168781	49030590	338142	145

Case 3:Five digit Krishnamurthy number

$$\frac{g(f-g)}{2} = 40585 \quad (6)$$

$$\Rightarrow \quad g(f-g) = 81170 \quad (7)$$

Table 3 shows the values of the generators f *and* g,satisfying (7) after scrutiny (shown below).

Table 3

g	f	u	v	w	$\dot{A}$	$\dot{P}$	$\frac{\dot{A}}{\dot{P}}$
1	81171	6588731240	162342	6588731242	534813903482040	13177624824	40585
2	40587	1647304565	162348	1647304573	133718300759310	3294771486	40585
5	16239	263705096	162390	263705146	21411535269720	527572632	40585
10	8127	66048029	162540	66048229	5367723316830	132258798	40585
8117	8127	162440	131933718	131933818	10715656575960	264029976	40585
16234	16239	162365	527247852	527247877	42803298744990	1054658094	40585
40585	40587	162344	3294446790	3294446794	267416834837880	6589055928	40585
81170	81171	162341	13177300140	13177300141	1069608041013870	26354762622	40585

IV. INTRIGUING RESULTS

- From aforementioned cases, leg v, area $\dot{A}$and perimeter $\dot{P}$ are even.
- In all cases of Pythagorean triangles, the leg u and hypotenuse w are of same parity.
- If the generators f, g are consecutive, then leg $v$and hypotenuse w are consecutive.
- Generators f, g are of same parity, then u, v ar also of same parity.f, g are of opposite parity, then u, v are also opposite parity.
- In each case $u + v - w$ has same number.
- In Table 1, From the above evaluation it is there are 2 primitive and 1 non-primitive.
- In Table 2, all 8 Pythagorean triangles are primitive.
- In Table 3, all 8 Pythagorean triangles are primitive

REFERENCES

[1] Mordell, L. J. (1969). Diophantine equations. Academic press.

[2] Ore, O. (1988). Number theory and its history. Courier Corporation.

[3] Andrews, G. E. (1994). Number theory. Courier Corporation.

[4] P. S. N. Sastry. Jarasandha numbers. Themathematics teacher, No.9 Vol.37(3,4):502– 504, 2001.

[5] Janaki, G., and C. Saranya. "Pythagorean Triangle with area/perimeter as a Jarasandha Number of orders 2 & 4." International Research Journal of Engineering and Technology 3.7 (2016): 1259-1264.

[6] Janaki, G., and S. Vidhya. "Pythagorean Triangle with Area/Perimeter as a 4-digit consecutive Sphenic numbers."

[7] Janaki, G. and Gowri Shankari, A. 2023. "Integer right triangle with area/perimeter as a Canada numbers", Asian Journal of Science and Technology, 14, (02), 12399-12402.

[8] G.Janaki and R.Radha (2017), Pythagorean Triangle with Area/Perimeter as a Harshad number of digits 4,5 and 6, IJRASET, Vol. 5, No. 12,pp 1754-1762

[9] Janaki, G., and P. Saranya. "Special pairs of Pythagorean triangles and Narcissistic numbers." IJMRD 3.4 (2016): 106-108.

[10] Janaki, G., and S. Vidhya. "Special pairs of Pythagorean triangles and 3-digit consecutive sphenic numbers." International Journal of Academic Research and Development 1.11 (2016): 29-31.