Thermal Stress Analysis of FGM Cylinder With Cut-Outs

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Abstract- *Functionally graded materials (FGMs) are a new generation of engineered materials in which the material properties are continually varied through the thickness direction by mixing two different materials, thus no distinct internal boundaries exist and failures from interfacial stress concentrations developed in conventional structural components can be avoided. FGMs are widely used in many structural applications such as mechanics, civil engineering, optical, electronic, chemical, mechanical, biomedical, energy sources, nuclear, automotive fields, and shipbuilding industries to eliminate stress concentration and relax residual stresses and enhance bond strength In this work, thermal stress behavior of FGM cylinder with cut-outs is studied under normal and cryogenic conditions. Material properties are calculated theoretically using the rule of mixtures Total of three different cut-out models are studied under different two thermal boundary conditions.*

Keywords- Nylon 6, glass fibre reinforced, Epoxy Carbon Woven (395 GPa) Prepreg, PBT, glass fibre reinforced Using ANSYS software.

I. INTRODUCTION

Functionally graded materials were introduced for the first time by the material scientists as heat resistant materials for use in space planes and nuclear reactors. Recently, large amount of researches about application of these materials in wide range of industries such as dental and orthopaedic implants, energy conversion, heat generators and sensors have been carried out. Additional potential applications of FGMs include their use as interfacial zones to improve the bonding strength and to reduce residual stresses in bonded dissimilar materials and as wear resistant layers such as gears, cams ball and roller bearings and machine tools. FGMs are the combination of two different materials, ceramic and metal. The volume fraction of these materials vary uniformly along a certain direction(s). Therefore FGMs have a non uniform microstructure and a continuously variable macrostructure. Materials have always been a crucial part of humans from the time the first man was created. As time progressed, so did the enhancement of technology and knowledge. Man started to engineer their own materials from existing raw materials. These engineered materials go back to 1000 BC in the form of composites of straw and mud. These composite improved over time and in the last four decades, the world was introduced with more sophisticated composites such as fiber reinforced plastics. However due to the limitations of delamination in such composites materials, another breed of composite material has given birth in the form of Functionally Graded Material (FGM). FGM is not new to us. Our nature is surrounded with FGMs. For example, the bone, human skin and the bamboo tree are all different forms of FGM.

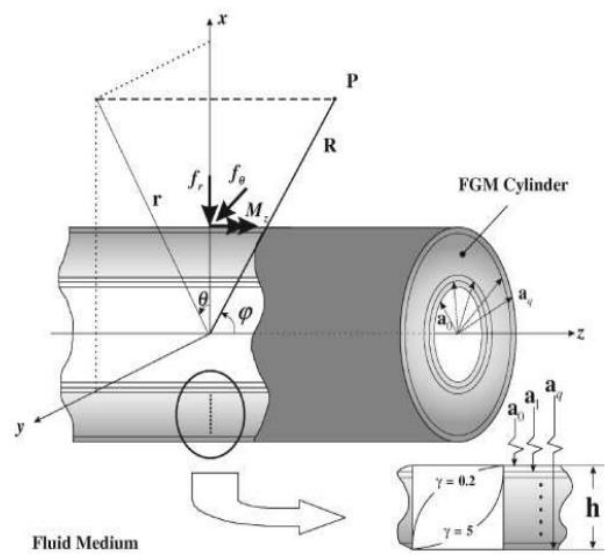


Fig:FGM CYLINDER WITH NOMENCLATURE

PROCESSING TECHNIQUES OF FGM:

Thin FGM are generally in the form of surface coatings, depending on the service requirement from the process there are a vast range of surface deposition processes to pick it from. The process of multi-particle setting in the manufacture of FGMs by co-sedimentation is modeled in. The models can be used to design powder composition and to predict the volume fractions obtained in FGMs. As examples, Tic-Ni system FGMs are designed and manufactured. The predictions fit well the actual results obtained. An experiment

with Mo - Ti system FGM is also used to validate the prediction model.
Other areas of application are:

- automobile engine components
- heat exchanger
- nuclear reactor components
- sensors
- cutting tool insert coating
- turbine blade
- Tribology
- Fire retardant doors, etc.

RECENT RESEARCH EFFORTS IN FUNCTIONALLY GRADED MATERIAL: Many experimentations been regulate on etiquette of FGM, and the literature is very high on this because of the wide areas of application of this novel material. Presentation of functionally graded material under localized transverse loading was examined by Kashtalyanand Woodward, on property evaluate study was conducted by Lu et al. A complete review on performance of functionally graded material was published in the year 2007 by Byrand Birman. An overview on fracture deportment of functionally graded material was conducted by Shanmugavel et al... More reviews on FGM available in the review study on research and development by Tilbrook et al. and Cherradi et al., they also conducted review study on crack propagation in functionally graded materials. A number of researches have also been conducted in the areas of analysis and modelling work on functionally graded material; some of these work can be found in. There are still more to be done in terms of research to improve the performance of manufacturing processes of FGM.

AREAS OF APPLICATION OF FGM

1.FGM in Aerospace
2.FGM in Medicine
3.FGM in Defence
4.FGM in Energy

FUTURE RESEARCH DIRECTION: FGM is an excellent approach material that will revolutionize the produce world in the 21st century. There are a number of roadblocks for perceive this objective. Cost is a major problem with considerable part of the cost expended on fabrication method and powder processing. Solid freeform fabrication technique offers a greater advantage for manufacturing functionally graded material but there are still plenty of issues that need to be resolved with this optimistic technology. More research needs to be supervise on developing the performance of solid freeform processes through extensive characterization of functionally graded material in other to generate a comprehensive database and to develop a predictive model for proper process control. Further work should also be done to improve the process control through development of more powerful feedback control for overall functionally graded material fabrication process improvement. This will improve the overall performance of the process, bring down the cost of functionally graded material and improve reliability of the fabrication process.

MODELING: There are various typify of drawings required in the dissimilar fields of engineering and science. In elder days, various delineation implements like drafting shape, T-equality, scale etc. are a necessity to load drawings gently and carefully. But to keep meliorate satisfaction in modifying the design and from calculations, the process of busk an attractive is made in the computer second-hand certain software's. This utility of computer systems is condition as electronic computer assist design. It replaces manual dragging with a machine driven anapophysis.

CAD:

- Reduced labor.
- Less amount of time for drawing.
- Minimum wastage of materials for drawing.
- Higher accuracy in drawing.
- Evolution of better design.
- Ease in modification.
- Less possibility for errors.
- Repeatability.
- Customization of product by coloring.
- Adjustability of drawing to require to scale.

CATIA:

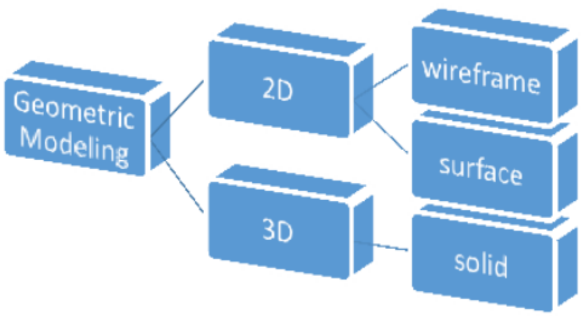
- Modification of model became much easier and accurate.
- It enables life like experience of the model by making the virtual representation to appear as realistic as possible.
- It is much faster than the previously existing design software's.
- CATIA provides easy assembly of the individual parts by automatically recognizing the joints.
- It provides clear 2d models which are easy to visualize and understand.
- It provides a very accurate representation of the product with many user friendly features.

GEOMETRIC MODELLING: The computer compatible mathematical description of geometry of an object is called geometric modelling. The CAD software allows the mathematical description of the object to be displayed as an image on the computer. Various steps for creating a geometric modeling

Creation of basic geometric elements by using commands like points, lines and circles.

Transformation of the basic elements based on requirements by using commands like scaling, rotating and joining.

Creation of geometric model by using various commands that cause the integration of the elements into desired shape.



Geometric modelling

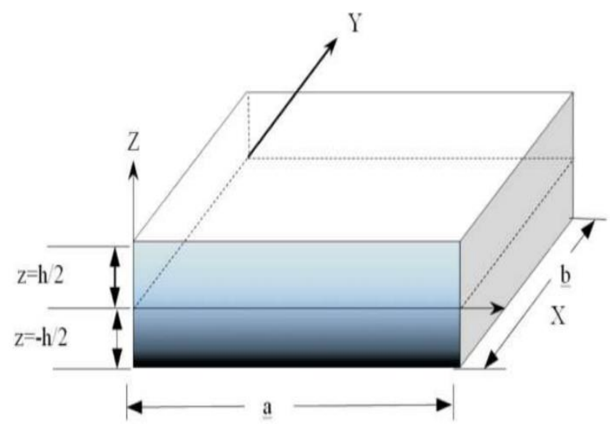
SOLID MODELLING: It is a type of 3d modelling that contains volumetric information about the model to be generated. It gives complete representation about the model. Solid modelling can be created and modified very quickly when compared to other types of modelling. Three approaches that are generally used for creating a solid model are:

1. Constructive Solid Geometry
2. Boundary representation
3. Hybrid scheme

Unlike wireframe and surface modelling, it allocates special address as well as verifies that the model is well framed or not. When compared to wire frame, solid models can be created easily without having to define individual locations. It produces unambiguous and complete models. It is best suitable for calculating mass properties. Solid modelling is very much suitable for automated applications.

PART MODELLING: The Version 5 Part Design application makes it possible to design precise 3D mechanical parts with an intuitive and flexible user interface, from sketching in an assembly context to iterative detailed design. Version 5 Part Design application will enable you to accommodate design requirements for parts of various complexities, from simple to advance. This application, which combines the power of feature-based design with the flexibility of a Boolean approach, offers a highly productive and intuitive design environment with multiple design methodologies, such as post-design and local 3D parameterization Select Start -> Mechanical Design -> Part Design from the menu bar

ANSYS: ANSYS is general-purpose finite element analysis (FEA) software package. Finite Element Analysis is a numerical method of deconstructing a complex system into very small pieces (of user-designated size) called elements. The software implements equations that govern the behaviour of these elements and solves them all; creating a comprehensive explanation of how the system acts as a whole. These results then can be presented in tabulated or graphical forms. This type of analysis is typically used for the design and optimization of a system far too complex to analyse by hand. Systems that may fit into this category are too complex due to their geometry, scale, or governing equations. ANSYS is the standard FEA teaching tool within the Mechanical Engineering Department at many colleges. ANSYS is also used in Civil and Electrical Engineering, as well as the Physics and Chemistry departments.



Set an FGM rectangular plate as an example. The sale is length a, width b and total thickness h. The plate is made from a mixture of two kinds of materials, and the composition is described as follows. The formulations are deduced here with the assumption of the linear elastic material behaviour and small displacements and strains. The material properties varied with plate thickness can be expressed as:

$$X(z) = X_m V_m(z) + X_c V_c(z),$$

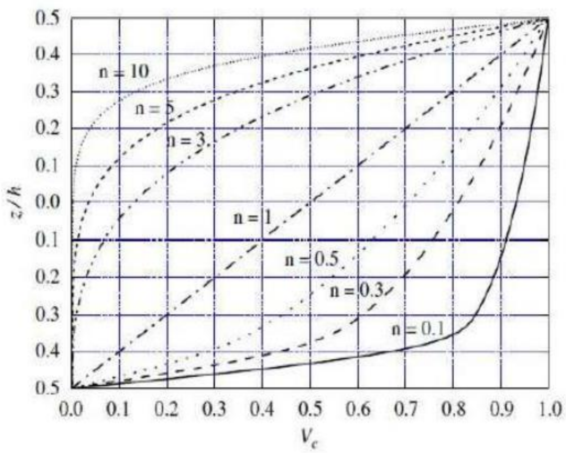
where X denotes the effective material property such as density, Young's modulus etc. X(m) and X(c) is the properties of the two different materials.
(m) And V(c) are the volume fraction of the material according to the conditions of:

$$V_m(z) + V_c(z) = 1,$$
$$V_m = \left(\frac{2z+h}{2h}\right)^k, \quad (k \geq 0).$$

In which k is the volume fraction exponent? Taking Eqs. (3) Into the Eqs. (1), the following Equation can be obtained:

$$X(z) = (X_m - X_c)\left(\frac{2z+h}{2h}\right)^k + X_c.$$

The relation between volume fraction and exponent



Variation of the volume fraction through the thickness

All Case with 10 layers:

Properties	Aluminium (metal)	Titanium carbide (ceramic)
Young's modulus (e)	70Gpa	460Gpa
Poisson's ratio (V)	0.3	0.336
Thermal conductivity (K)	204 W/mk	5.64 W/mk
Co-efficient of thermal expansion (α)	$23*10^{-6}$ /k	$7.4*10^{-6}$ /k
Density (ρ)	$2.7*10^3$ kg/m^3	4930 kg/m^3

$P(z) = P_2 + (P_1 - P_2)\, V_1$

V_1= (0.90, 0.80, 0.70, 0.60, 0.50, 0.40, 0.30, 0.20, 0.10)

p1=aluminium, p2=titanium carbide

By taking the values of v1= 0.90

P (e) = 460+ (70−460)* 0.90=109Gpa

P (V) =0.336+ (0.3−0.336)* 0.90=0.3036

P (k) =5.64+ (204−5.64)* 0.90=184.164W/mk

P (α) = $(7.4*10^{-6}) + (23*10^{-6}-7.4*10^{-6})* 0.90=2.144*10^{-5}$/K

P (ρ) = $(4930) + (2.7*10^3-4930)* 0.90=2923$Kg/m^3

By taking the values of v1= 0.80

P (e) = 460+ (70−460)*0.80 =148Gpa

P (V) =0.336+ (0.3−0.336)* 0.80 =0.3072

P (k) =5.64+ (204−5.64)* 0.80 =164.328W/mk

P (α) = $(7.4*10^{-6}) + (23*10^{-6}-7.4*10^{-6})* 0.80 =1.988*10^{-5}$/K

P (ρ) = $(4930) + (2.7*10^3-4930)*0.8 =3146$Kg/m^3

By taking the values of v1= 0.70

P (e) = 460+ (70−460)* 0.70=187Gpa

P (V) =0.336+ (0.3−0.336)* 0.70=0.3108

P (k) =5.64+ (204−5.64)* 0.70=144.492W/mk

P (α) = $(7.4*10^{-6}) + (23*10^{-6}-7.4*10^{-6})* 0.70=1.832*10^{-5}$/K

P (ρ) = $(4930) + (2.7*10^3-4930)*0.70=3369$Kg/m^3

By taking the values of v1= 0.60

P (e) = 460+ (70−460)* 0.60=226Gpa

P (V) =0.336+ (0.3−0.336)* 0.60=0.3144

P (k) =5.64+ (204−5.64)* 0.60=124.656W/mk

P (α) = $(7.4*10^{-6}) + (23*10^{-6}-7.4*10^{-6})* 0.60=1.676*10^{-5}$/K

P (ρ) = $(4930) + (2.7*10^3-4930)*0.60=3592$Kg/m^3

By taking the values of v1= 0.50

P (e) = 460+ (70−460)* 0.50=265Gpa

P (V) =0.336+ (0.3−0.336)* 0.50=0.318

P (k) =5.64+ (204−5.64)* 0.50=104.82W/mk

P (α) = $(7.4*10^{-6}) + (23*10^{-6}-7.4*10^{-6})* 0.50=1.52*10^{-5}$/K

P (p) = (4930) + $(2.7*10^{3}-4930)*0.50=3815 Kg/m^{3}$

By taking the values of v1= 0.40

P (e) = 460+ (70−460)*0.40=304Gpa

P (V) =0.336+ (0.3−0.336)* 0.40=0.3216

P (k) =5.64+ (204−5.64)* 0.40=84.984W/mk

P (α) = $(7.4*10^{-6}) + (23*10^{-6}-7.4*10^{-6})* 0.40=1.364*10^{-5}$/K

P (p) = (4930) + $(2.7*10^{3}-4930)*0.40=4038 Kg/m^{3}$

By taking the values of v1= 0.30

P (e) = 460+ (70−460)*0.30=343Gpa

P (V) =0.336+ (0.3−0.336)* 0.30=0.3252

P (k) =5.64+ (204−5.64)* 0.30=65.148W/mk

P (α) = $(7.4*10^{-6}) + (23*10^{-6}-7.4*10^{-6})* 0.30=1.208*10^{-5}$/K

P (p) = (4930) + $(2.7*10^{3}-4930)*0.30=4261 Kg/m^{3}$

By taking the values of v1= 0.20

P (e) = 460+ (70−460)*0.20=382Gpa

P (V) =0.336+ (0.3−0.336)* 0.20=0.3288

P (k) =5.64+ (204−5.64)* 0.20= 45.312W/mk

P (α) = $(7.4*10^{-6}) + (23*10^{-6}-7.4*10^{-6})* 0.20=1.052*10^{-5}$/K

P (p) = (4930) + $(2.7*10^{3}-4930)*0.20 =4484 Kg/m^{3}$

By taking the values of v1= 0.10

P (e) = 460+ (70−460)*0.10=421Gpa

P (V) =0.336+ (0.3−0.336)* 0.10=0.3324

P (k) =5.64+ (204−5.64)* 0.10= 25.476W/mk

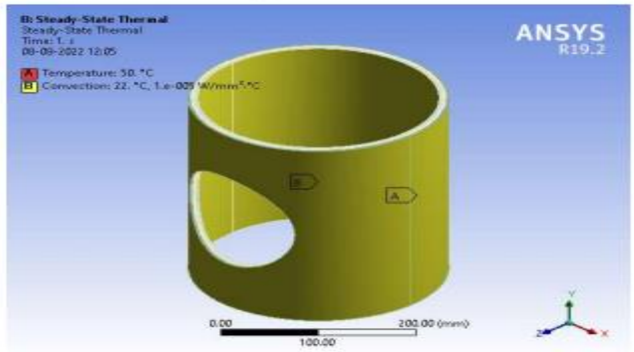
P (α) = $(7.4*10^{-6}) + (23*10^{-6}-7.4*10^{-6})* 0.10=8.96*10^{-5}$/K

P (p) = (4930) + $(2.7*10^{3}-4930)*0.10 =4707 Kg/m^{3}$

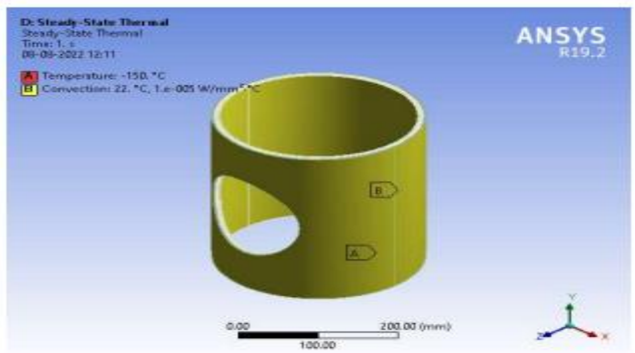
Boundary conditions

Thermal

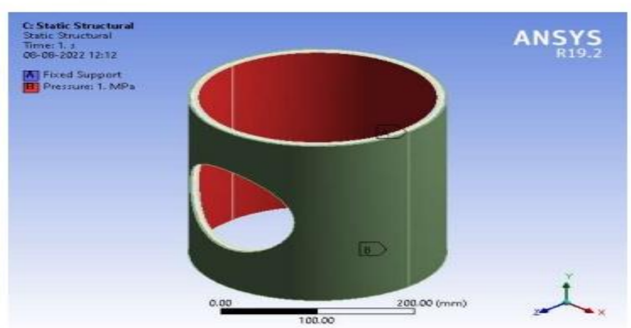
General boundary conditions



Cryogenic boundary conditions

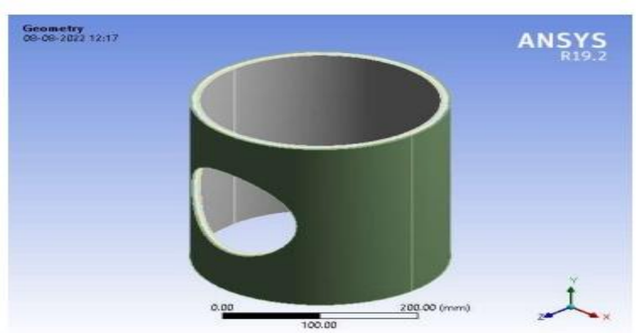


Structural

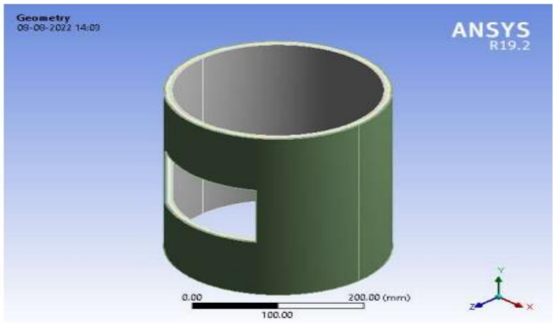


Geometry

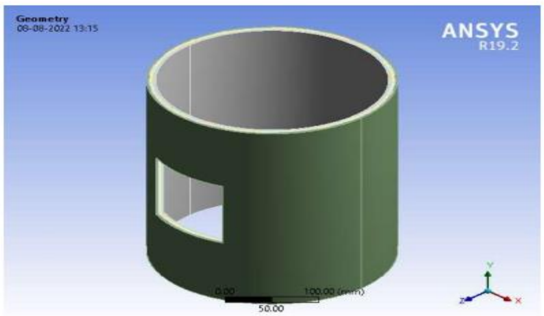
Circular cut-out



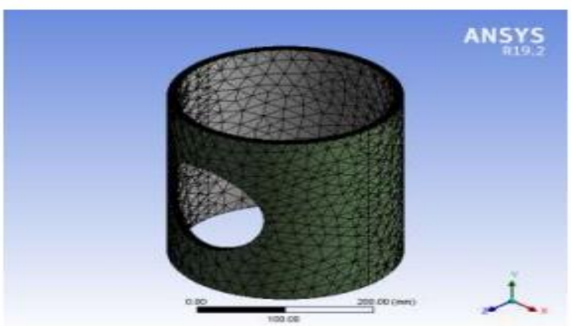
Rectangular cut out



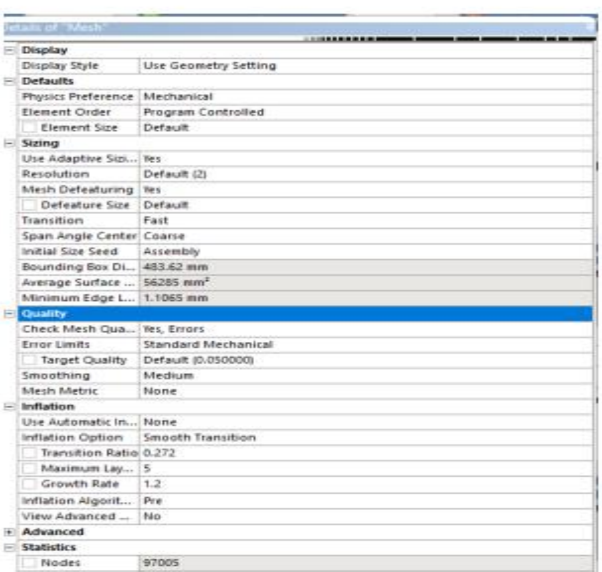
Square cut out



Meshed model



Details

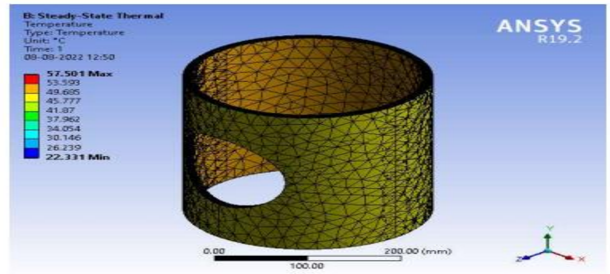


Details of "Mesh"

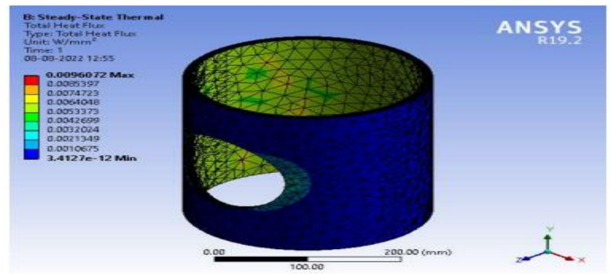
Display	
Display Style	Use Geometry Setting
Defaults	
Physics Preference	Mechanical
Element Order	Program Controlled
Element Size	Default
Sizing	
Use Adaptive Sizi...	Yes
Resolution	Default (2)
Mesh Defeaturing	Yes
Defeature Size	Default
Transition	Fast
Span Angle Center	Coarse
Initial Size Seed	Assembly
Bounding Box Di...	483.62 mm
Average Surface ...	56285 mm²
Minimum Edge L...	1.1065 mm
Quality	
Check Mesh Qua...	Yes, Errors
Error Limits	Standard Mechanical
Target Quality	Default (0.050000)
Smoothing	Medium
Mesh Metric	None
Inflation	
Use Automatic In...	None
Inflation Option	Smooth Transition
Transition Ratio	0.272
Maximum Lay...	5
Growth Rate	1.2
Inflation Algorit...	Pre
View Advanced ...	No
Advanced	
Statistics	
Nodes	97005
Elements	45316

Circular cut-out thermal analysis at 50 °C

Temperature

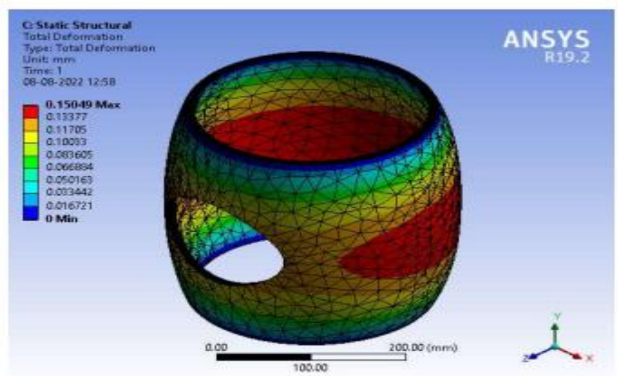


Total heat flux

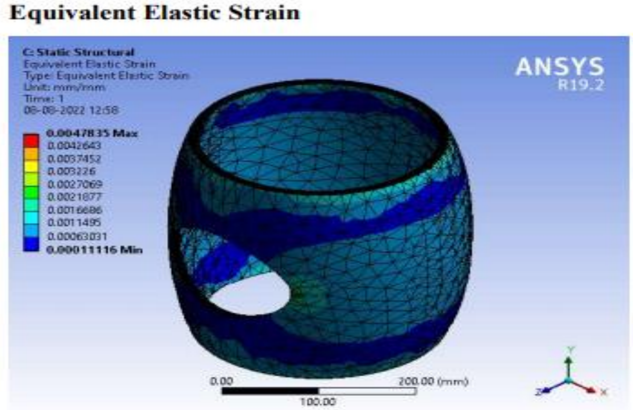


Circular cut-out structural analysis at 50 °C

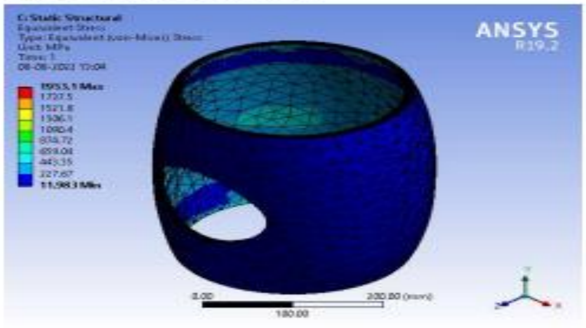
Total Deformation



Equivalent Elastic Strain

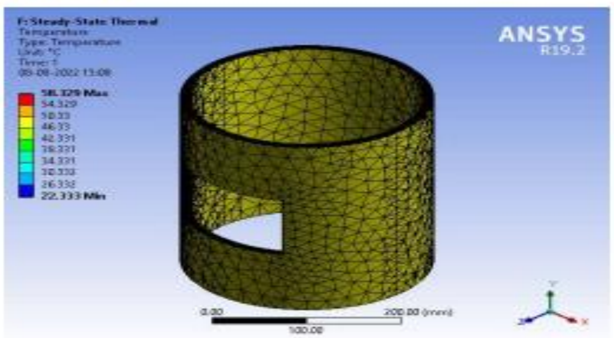


Equivalent (von-Mises) Stress

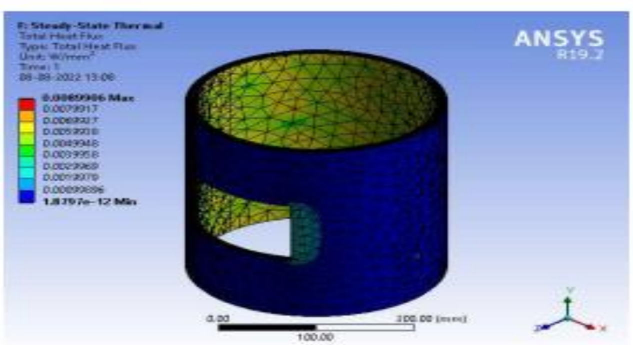


Rectangular cut-out thermal analysis at 50 °C

Temperature

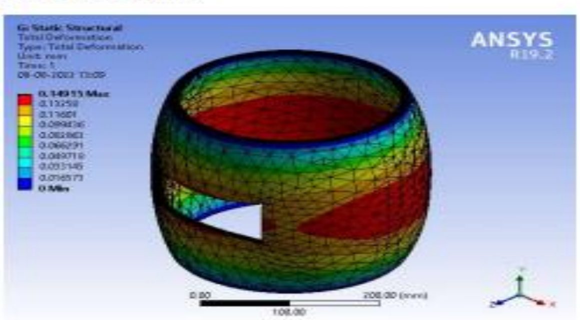


Total heat flux

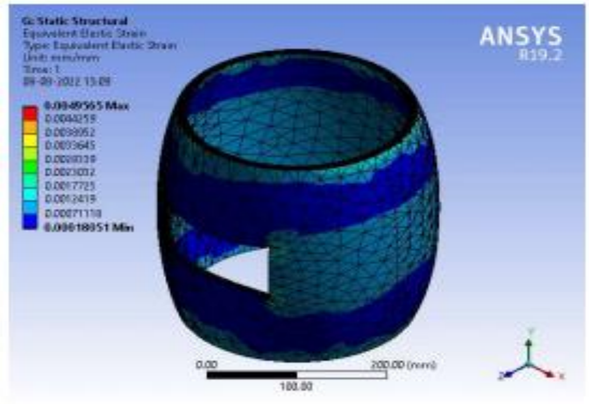


Rectangular cut-out structural analysis at 50 °C

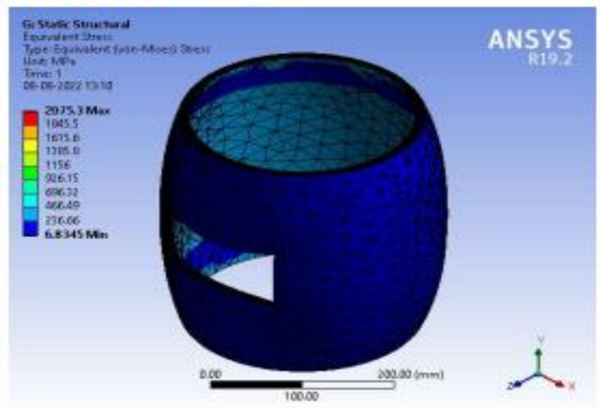
Total Deformation



Equivalent Elastic Strain



Equivalent (von-Mises) Stress



FGM CYLINDER WITH CUT-OUTS AT 50 ºC THERMAL

Thermal analysis at 50 °C	Temperature (°C)			Total Heat Flux (W/mm²)			Directional Heat Flux (W/mm²)		
	Minimum	Maximum	Average	Minimum	Maximum	Average	Minimum	Maximum	Average
circular cut-out	22.331	57.501	48.936	3.41E-12	9.61E-03	7.68E-04	-6.71E-03	6.60E-03	7.46E-06
rectangular cut-out	22.333	58.329	48.953	1.88E-12	8.99E-03	7.66E-04	-7.04E-03	6.97E-03	6.65E-06
square cut-out	22.29	57.655	48.955	2.74E-12	8.20E-03	7.40E-04	-6.74E-03	6.68E-03	8.67E-06

STRUCTURAL

structural analysis at 50 ℃	Total Deformation (mm)			Equivalent Elastic Strain (mm/mm)			Equivalent (von-Mises) Stress (Mpa)		
	Minimum	Maximum	Average	Minimum	Maximum	Average	Minimum	Maximum	Average
circular cut-out	0	0.15049	8.91E-02	1.11E-04	4.78E-03	9.84E-04	11.983	1953.1	248.73
rectangular cut-out	0	0.14915	9.08E-02	1.81E-04	4.96E-03	9.71E-04	6.8345	2075.3	246.61
square cut-out	0	0.14939	9.24E-02	1.23E-04	3.84E-03	9.68E-04	10.337	1613.1	246.88

FGM CYLINDER WITH CUT-OUTS AT -150 ºC
THERMAL

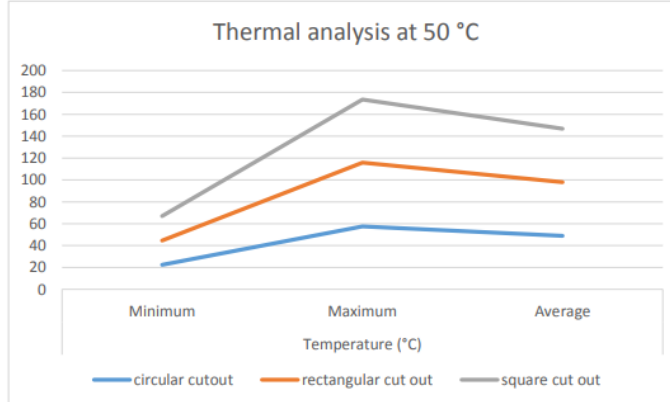
Thermal analysis at -150 ℃	Temperature (℃)			Total Heat Flux (W/mm²)			Directional Heat Flux (W/mm²)		
	Minimum	Maximum	Average	Minimum	Maximum	Average	Minimum	Maximum	Average
circular cut-out	-196.08	19.967	-143.46	2.10E-11	5.90E-02	4.71E-03	-4.05E-02	4.12E-02	-4.58E-05
rectangular cut-out	-201.16	19.955	-143.57	1.15E-11	5.52E-02	4.71E-03	-4.28E-02	4.32E-02	-4.09E-05
square cut-out	-197.02	20.222	-143.58	1.68E-11	5.03E-02	4.54E-03	-4.10E-02	4.14E-02	-5.33E-05

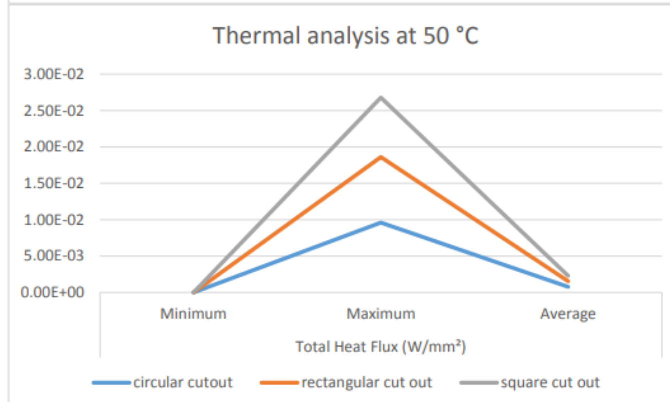
STRUCTURAL

structural analysis at -150 ℃	Total Deformation (mm)			Equivalent Elastic Strain (mm/mm)			Equivalent (von-Mises) Stress (Mpa)		
	Minimum	Maximum	Average	Minimum	Maximum	Average	Minimum	Maximum	Average
circular cut-out	0	0.89455	0.52729	6.51E-04	2.94E-02	5.90E-03	62.08	11990	1488.4
rectangular cut-out	0	0.88717	0.53831	1.08E-03	3.06E-02	5.82E-03	46.27	12797	1476.5
square cut-out	0	0.88976	0.54775	7.44E-04	2.37E-02	5.80E-03	52.012	9955.7	1475.5

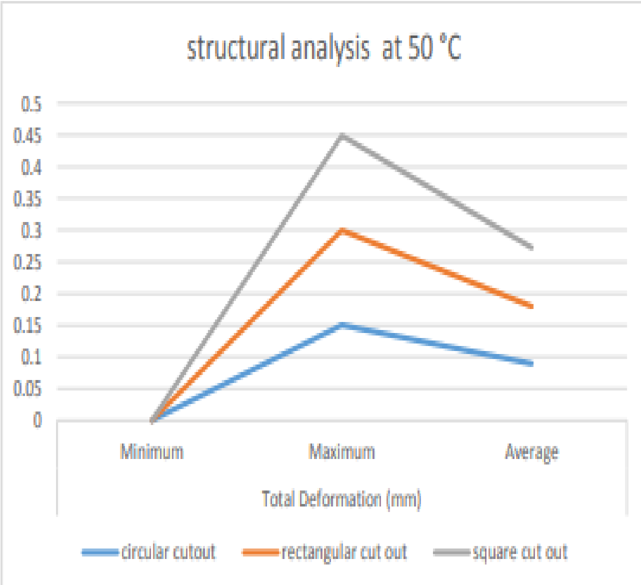
GRAPHS

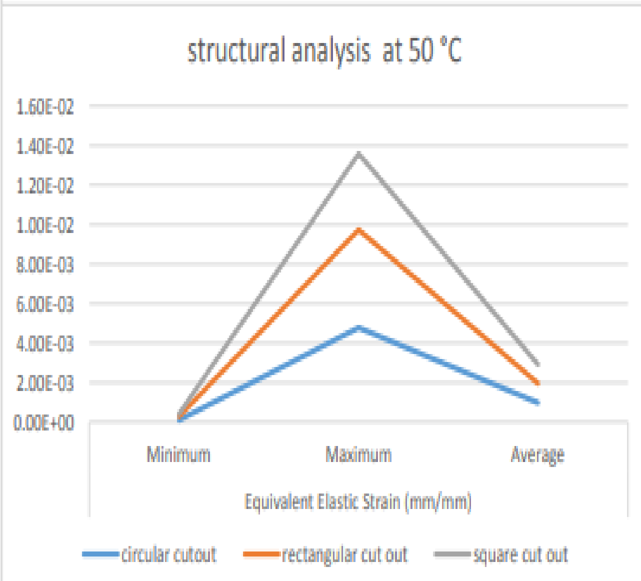
FGM CYLINDER WITH CUT-OUTS AT 50 ºC
THERMAL

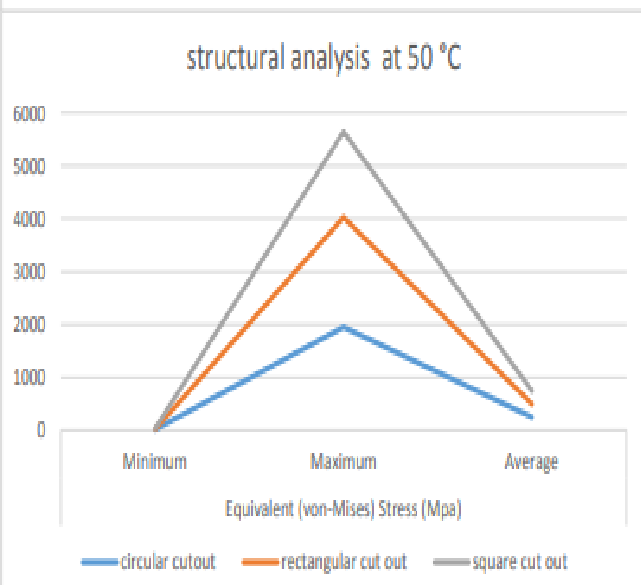




STRUCTURAL

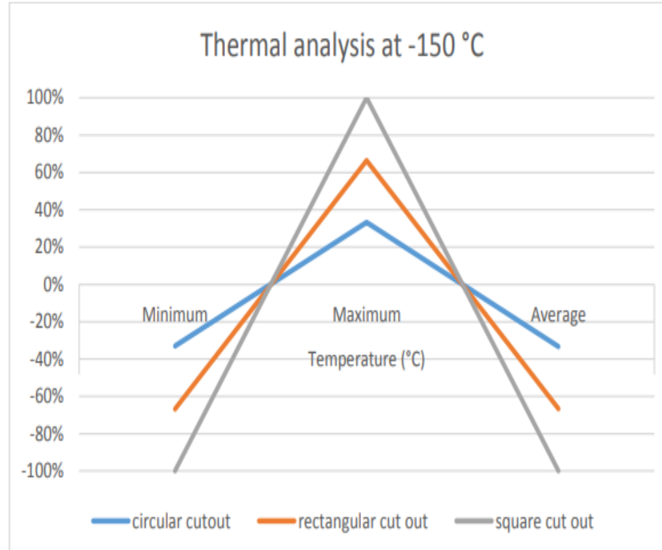


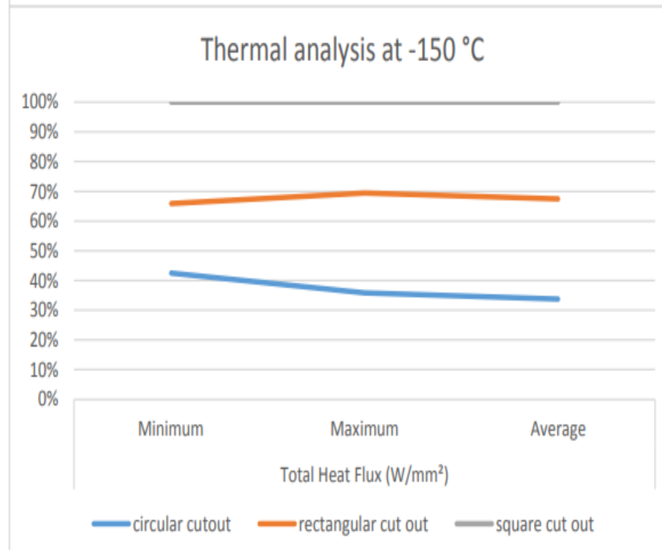




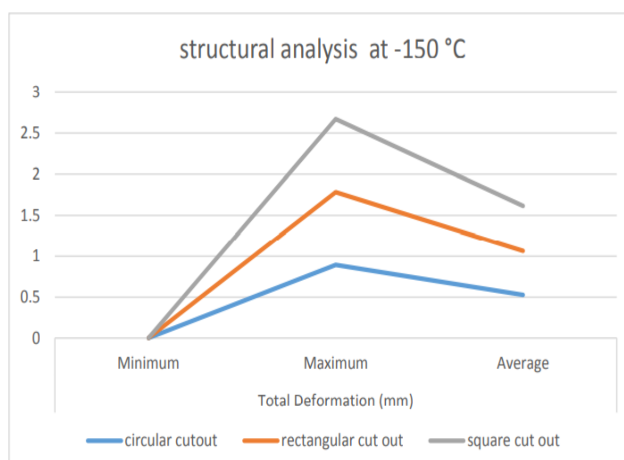
FGM CYLINDER WITH CUT-OUTS AT -150ºC

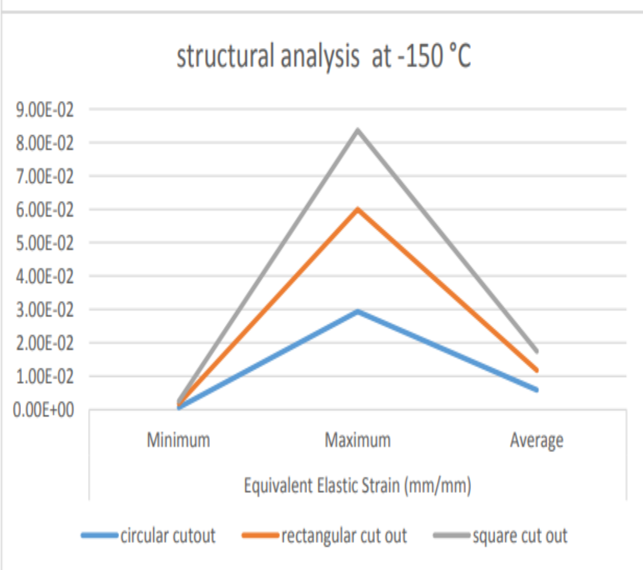
THERMAL

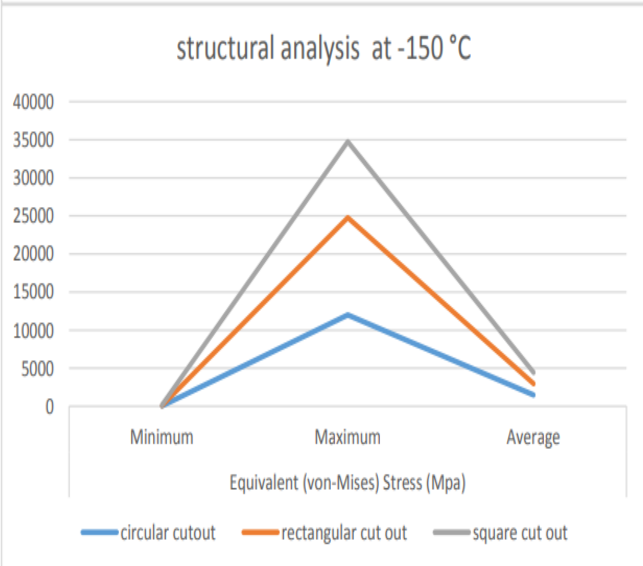




STRUCTURAL







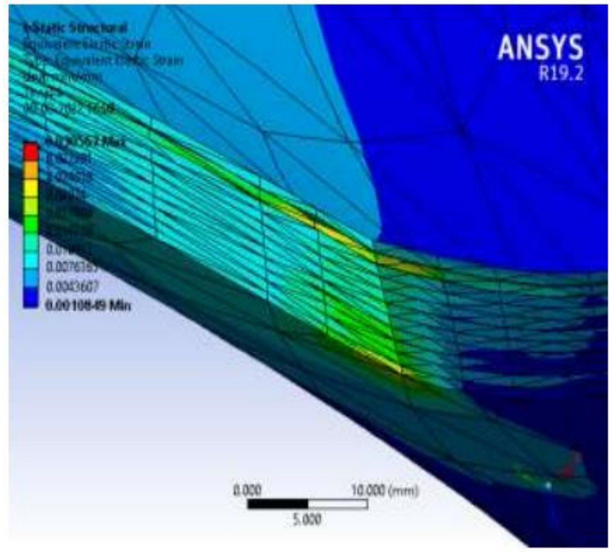
II. CONCLUSION

In this work model behaviour of FGM plates with cut-outs are studied under thermal condition (normal temperature and cryogenic temperature). Total Three different cut-out models are studied under different conditions. Namely

a. Normal temperature 50 oC.
b. Cryogenic temperature -150 oC.

The observations made are as follows

1. There is not much difference between a square cut-out and rectangular cutouts
2. Cryogenic temperatures created compressive stress in the shell
3. Rectangular and square cut-outs models show significant high stress in the internal layers as shown in the picture.



4. Stress is very high at cryogenic temperatures.
5. Models with filleted or round corners could reduce stress.

III. FUTURE SCOPE

Here all the work is carried using simulation, only material property calculations are made theoretically, more theoretical work is suggested. Also, tough manufacturing FGM components is very costly, experimentation is also suggested. We can also apply this study to more complex cut outs and design.

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