Prediction of Flow Separation Over A Flat Plate

Mr Bhushan R Walve[1], Prof Maheshwar M C[2], Prof Akash Bhise[3]
[1]Dept of Mechanical Engineering
[2, 3]Professor, Dept of Mechanical Engineering
[1, 2, 3] DPCOE, Pune.

Abstract- *Flow separation is one of the important factor that affects the performance of an airfoil. The performance and stability of an airplane is often degraded by flow separation. In this study the separation of flow is studied by studying the pressure distribution on the surface of the cylinder by keeping the G/D (gap to diameter ratio) value constant and varying Velocity ratio VR (dimensionless cylinder rotation rate) value and vice versa. Streamlines have been generated around the cylinder for the given conditions. Two methods were used to predict the separation points and the results were compared with the experimental results. It was concluded from the study that both the methods yielded almost similar results. The separation point decreased with increasing value of VR keeping the value of G/D constant.*

Note: Experimental values were taken from the paper "Use of a rotating cylinder to induce laminar and turbulent separation over a flat plate"

Nomenclature

Θ	= angle of attack
ζ	= boundary layer co-ordinate
u_e	= boundary layer edge velocity
r	= circulation
z_0	= centre of rotating cylinder
z_i	= centre of image cylinder
C_p	= coefficient of pressure
ρ	= density
U	= flow velocity
δ_m	= momentum thickness
H	= shape factor
C_f	= skin friction drag
Ψ	= uniform stream
φ	= velocity potential
λ	= flow parameter

I. INTRODUCTION

The ability to manipulate a flow field to effect a desired change is of immense practical importance. Flow control is a topic that is widely pursued by scientists and researchers than any other topic in fluid dynamics (Experimental Study of Flow Separation Control of an Airfoil by Suction and Injection, 2010). Boundary layer separation also referred to as flow separation is the most unwanted effect for many applications and is even dangerous in aviation due to its unstable flow profile combined with drag increase, high energy losses and reduced lift in case of airfoil (Sturm, 2012). A laminar flow is a smooth uninterrupted flow over the surface of the wing or any part of the airplane in flight and there is no energy transfer between the layers of the boundary layer. It is found at the front of a streamlined body. Turbulence is created if the laminar flow is interrupted over a wing section, it causes a increase in drag and loss of lift. An airfoil designed for minimum drag and uninterrupted flow over its surface is called a laminar airfoil. All the boundary layers start off as laminar but there are some factors that destabilize these laminar boundary layers causing it to transition to turbulent boundary layers. The parameters like adverse pressure gradient, surface roughness, heat and acoustic energy are some examples of the destabilizing factors (Laminar flow airfoil, 2015). Pressure gradient is an important factor which influences a flow. A negative pressure gradient also referred to as favourable pressure gradient is a parameter which enables the flow, on the other a positive pressure gradient also referred to as adverse pressure gradient is a parameter which causes the flow to separate.

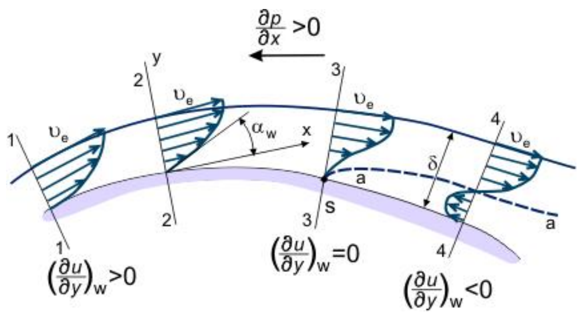

Figure 1: Flow past a curved surface

In figure 1 the geometry of the surface is such that we have a negative pressure gradient from the start ie. Point 1 till Point 2 after which the negative pressure gradient will counteract the retarding effect of the shear stress in the boundary layer. An adverse pressure gradient is obtained for the above geometry downstream of point 2. The adverse pressure gradient starts decreasing further and the velocity at the wall reduces causing an increase in thickness of the boundary layer. At point 3 the wall stress becomes zero. The wall stress starts to decrease from this point and the flow reverses and a region of recirculating flow is generated. The flow no longer follows the contour of the body and we say that the flow is separated (Aerodynamics for students, 2005). In short flow separation occurs when the boundary layer travels far enough against the adverse pressure gradient that the speed of the boundary layer relative to the object falls to zero. The fluid is detached from the surface of the body and takes the form of eddies and vortices (D., 2017). The separated laminar flow is highly sensitive to disturbances which causes it to transition to turbulent state. The transition region is located away from the airfoil at the outer boundary of the separated flow area. The boundary layer of the turbulent flow increases rapidly, forming a turbulent wedge which might reach the airfoil surface again. The point where the turbulent flow reaches the airfoil surface again is called the **reattachment point**. The volume enclosed by the regions of separated laminar flow and turbulent flow is called a **laminar separation bubble (LSB).** The mechanism of LSB was first observed by Jones in 1930 and further explored by Gaster in 1960s. An experimental study was conducted by both of them to analyse the stability behaviour of the separation bubble. Horton in 1968 develpoed a novel semi-empirical bubble model which is widely used in such studies. McGregor, Young and Horton were among the others who contributed in the characterization of LSB (Shah, 2015). The flow may be circulating inside the bubble and the direction of the flow at the surface of the bubble may be opposite to that of the outer flow. The separation bubble is quite stable as there is no energy transfer with the outer flow (Hepperle, 2006). The distance between the separation point and the reattachment point is defined as the size of the separation bubble and is generally expressed in terms of the airfoil chord length. At lower Reynold’s number of the order $1x10^5$ the separation bubble obtained is generally longer and it covers up to 20-30% of the chord length. At higher Reynold’s number the separation bubble is shorter and harmless

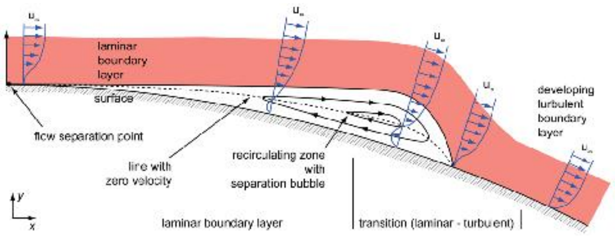

Figure 2: Typical airfoil setup with an occurring flow separation followed by a reattachment (Sturm, 2012).

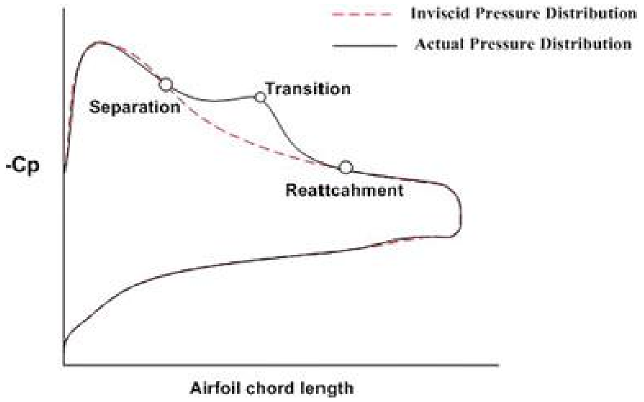

Figure 3: Surface pressure distribution of airfoil with laminar separation bubble.

The hump in the plot of pressure gives the interpretation of the LSB. For inviscid flow (dashed line in plot) no laminar separation bubble is observed. The formation of LSB has a great influence on the skin friction drag (C_f). Von Karman integral boundary layer equation can be used to express the skin friction drag and is given as

$$\frac{1}{\rho u_e^2\theta}*\frac{d(\rho u_e^2\theta)}{d\zeta}=\frac{C_f}{2\theta}-\frac{H}{u_e}*\frac{du_e}{d\zeta} \qquad (1)$$

Where ζ is boundary layer co-ordinate, u_e is the boundary layer edge velocity, H is the shape factor defined as the ratio between the boundary layer momentum and the boundary layer displacement thickness. The skin friction drag inside the bubble is nearly zero and the integral boundary condition becomes

$$\frac{\Delta(\rho u_e^2\theta)}{\rho u_e^2\theta}=-H\frac{\Delta u_e}{u_e} \qquad (2)$$

Where $\rho u_e^2\theta$ represents the total momentum defect. Considering the boundary-layer edge velocity, the shape factor and that the total momentum defect as average quantities, the above equation can be written as

$$\Delta(\rho u_e^2\theta)=-\rho u_e\delta^*\Delta u_e \qquad (3)$$

From the above equation it can be seen that the drag due to the separation bubble is proportional to the average mass defect and edge velocity jump. Hence it can be said that the size and position of the separation bubble are a function of airfoil shape, angle of attack, Reynolds number and environmental interruptions (Shah, 2015). It is very important to know whether the boundary layer will separate from the surface of the airfoil and if it does it is also necessary to know where will the flow separate. The determination of separation points are very important and can be found out by using different methods like Head's method, Entrainment method, Stratford's method, Goldschmied's method (Cebeci, 1972).

II. METHODOLOGY

The complex velocity potential is an analytic function and can be written by combining the velocity potential φ and Lagrange's stream function ψ.

$$\bar{w} = \bar{\varphi} + i\psi \tag{4}$$

From the stream function and velocity potential we obtain

$$V_x = \frac{\partial \varnothing}{\partial x} = \frac{\partial \psi}{\partial y} \tag{5}$$

$$V_y = \frac{\partial \varnothing}{\partial y} = -\frac{\partial \psi}{\partial x} \tag{6}$$

By differentiating the complex velocity function we get

$$\frac{dw}{dz} = \frac{\partial \varnothing}{\partial x} + i\frac{\partial \psi}{\partial y} = V_x - iV_y \tag{7}$$

From the above equation we can say that the derivative of the complex potential is the complex conjugate of the velocity which is thus an analytic function of z.

By combining equation 3 with different basic flows we get

$$Uniform\ stream: w = U^* z \tag{8}$$

$$Source\ or\ sink\ at\ z_0: w = \frac{m}{2\pi}\ln(z - z_0) \tag{9}$$

$$Doublet\ at\ z_0: w = \frac{B}{(z - z_0)} \tag{10}$$

$$Vortex\ at\ z_0: w = -\frac{i\Gamma}{2\pi}\ln(z - z_0) \tag{11}$$

In the above equations the asterisk denotes a complex conjugate. The vortex in equation (XX) is counterclockwise if r>0 and clockwise if r<0. The power of complex variable theory is that, since an analytic function of an analytic function is analytic, we can solve a flow involving a simple geometry and then use an analytic function to transform, or ***map***, that geometry into a much more complicated one.

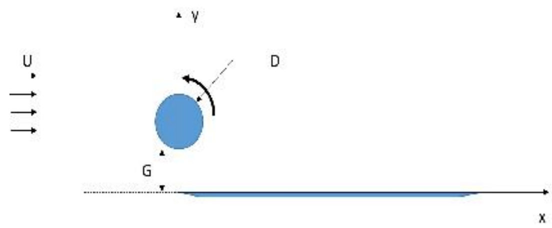

Figure 4: Schematic diagram of experimental set-up for generating Adverse Pressure Gradient (APG) using a rotating cylinder and location of measurement window

Above figure has a rotating cylinder over a flat plate. A flow of velocity U passes around the cylinder. We have to derive the equations for complex potential and velocity field. The flat pate in the above figure can be considered as a wall which causes the formation of images. For flow past a circular cylinder there is formation of a vortex, doublet and uniform flow. The images of the vortex and doublet are created on the other side of the plate.

Let z_0 be the centre of the cylinder and z_1 be the centre of the imaged cylinder. The equation for the doublet and vortex at point z_0 can be given as

$$Doublet\ at\ z_0: w = \frac{B}{(z - z_0)} \tag{12}$$

$$Vortex\ at\ z_0: w = -\frac{i\Gamma}{2\pi}\ln(z - z_0) \tag{13}$$

Similarly, the equation at point z_1 can be given as

$$Doublet\ at\ z_1: w = \frac{B}{(z - z_1)} \tag{14}$$

$$Vortex\ at\ z_1: w = -\frac{i\Gamma}{2\pi}\ln(z - z_1) \tag{15}$$

The overall potential function for the above flow is given by

$$w = U^* z + \frac{Ua^2}{(z - z_0)} - \frac{i\Gamma}{2\pi}\ln\frac{(z - z_0)}{a} + \frac{Ua^2}{(z - z_1)} + \frac{i\Gamma}{2\pi}\ln\frac{(z - z_1)}{a} \tag{16}$$

Taking the derivative of the potential function 'w' with respect to 'z', we get complex conjugate of velocity

$$\frac{dw}{dz} = U' - \frac{Ua^2}{(z - z_0)^2} - \frac{i\Gamma}{2\pi}\frac{1}{(z - z_0)} - \frac{Ua^2}{(z - z_1)^2} + \frac{i\Gamma}{2\pi}\frac{1}{(z - z_0)} \tag{17}$$

In order to predict the separation points two methods were used in our study

Flow separation point prediction using Thwaite's Criteria

Inserting the Karman Pohlhausen approximation values into the equations we get expressions for displacement thickness, momentum thickness and stress at the wall. Substituting these values in the integrated Navier Stoke's equation we get

$$\frac{\rho}{\mu U^5}\frac{d(U^6\delta_m^2)}{dx} \approx 0.47 \qquad (18)$$

Integrating the above equation

$$\delta_m^2 = \delta_m^2| + \frac{0.47}{U^6}\int_0^x U^5\,dx \qquad (19)$$

Where δ_m represents the momentum thickness of the boundary layer

Thwaite's correlated a number of solutions of the integrated boundary layer equations and found that separation could be easily predicted by the result

$$\delta_m^2 = -\frac{0.09\mu}{\rho\frac{dU}{dx}} \qquad (20)$$

Combining equations 19 and 20 we get

$$-\frac{0.09\mu}{\rho\frac{dU}{dx}} = \delta_m^2| + \frac{0.47}{U^6}\int_0^x U^5 dx \qquad (21)$$

The integration in the above equation has been started from the stagnation point. Also,

$$\lambda = \frac{\delta_M^2}{\nu}\frac{dU}{dx} \qquad (22)$$

Flow separation point prediction using Stratford's Criteria

Stratford's separation criterion is an old classical way to assess if a turbulent boundary layer is likely to separate or not. According to Stratford's criteria a boundary layer is about to separate when

$$C_p'.\sqrt{\frac{x'dC_p'}{dx}} = k.\left(\frac{Re}{10^6}\right)^{0.1} \qquad (23)$$

$$C_p' = 1 - \left(\frac{U}{U_{max}}\right)^2 \qquad (24)$$

Also Stratford suggested that the separation will occur at a point

$$(x - x_0)^2 C_p\left(\frac{dC_p}{dx}\right)^2 = 0.0104 \qquad (25)$$

Both the Thwaite's and Stratford's criteria require the knowledge of 'U' at the boundary. As the Stratford criteria does not contain any integration it is somewhat easier to use. Even though the Thwaite's and Stratford's criteria are for 2D flows they can be applied to 3D flows which have a 2D behaviour.

NOTE: The above equations and derivations were referred from (Graebel, 2007).

III. RESULTS AND DISCUSSIONS

Pressure gradient is the key factor in the study of flow around an object or body. When the pressure gradient takes up a positive value it is known as the adverse pressure gradient which causes the separation of flow. For our study a graph for pressure coefficient gradient was plotted against the position along the plate.

Case 1: Pressure coefficient gradient for G/D = 1.5 and variable V/R values:

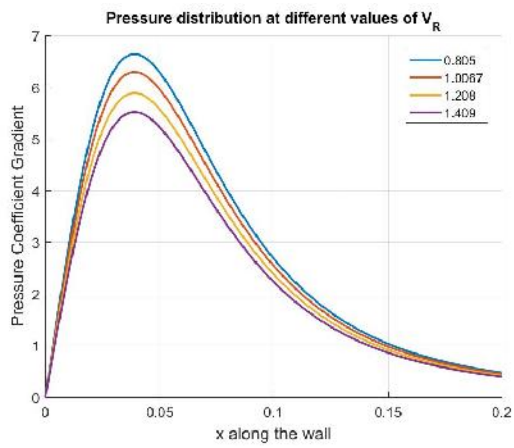

Figure 5: Pressure distribution at different values of VR.

The plot represents the pressure distribution from the leading edge of the plate or from the stagnation point. The pressure coefficient value was the highest for VR = 0.805 and the pressure coefficient value decreased with increasing VR ratio. The pressure value keeps on increasing from the leading edge of the cylinder and the separation of flow occurs at the point where the adverse pressure gradient is at the peak. The increased pressure on the surface of the airfoil causes a decreased lift.

Case 2: Pressure Coefficient Gradient for VR = 1.208 and variable G/D values:

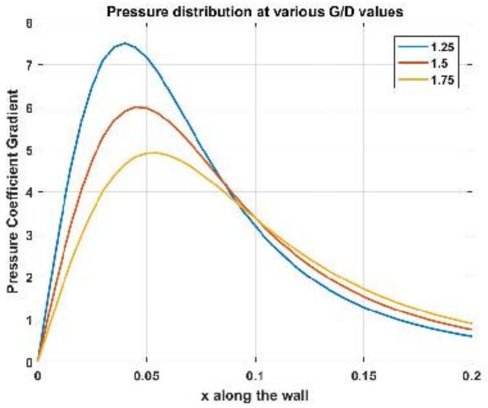

Figure 6: Pressure distribution at different values of G/D

The above plot represents the pressure distribution at different values of G/D. It was observed that the coefficient of pressure was the highest for G/D = 1.25 and it decreased as G/D value increases. It states that as the cylinder is placed near to the flat plate the pressure coefficient value is higher and it causes the separation of flow at a location closer to the leading edge. As the distance between the cylinder and the flat plate increases the separation point moves more downstream. This happens because of the viscous effect. The boundary layer closer to the wall starts moving away from the surface of the wall due to no-slip boundary condition while the layers which are farther from the wall take time to separate which is exactly what is observed from the plot.

Hysteresis is said to occur at higher angle of attacks (near to the stall angle). Hysteresis loop obtained in the study of flow around an object can be either clockwise or anticlockwise and in the lift coefficient profiles or drag coefficient profiles. In the hysteresis loop the pressure distribution on the lower surface of the airfoil is considerably lower than the pressure distribution on the upper surface of the airfoil at a specific angle of attack.

Streamlines are family of curves that are tangent to the velocity vector of the flow. They show the direction in which the massless fluid element will travel at any point in time (Streamlines, Streaklines and Pathlines, 2017). For the given parameters in our study streamlines were generated around the cylinder. Plots for different G/D values were generated keeping VR constant.

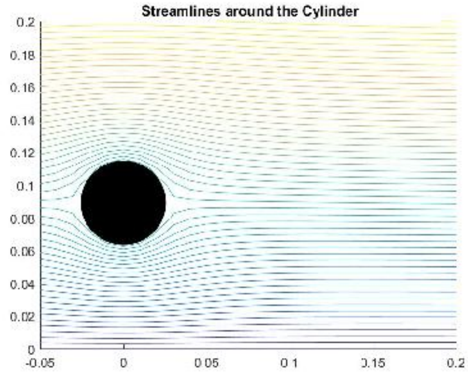

Figure 7: Streamlines for G/D = 1.25

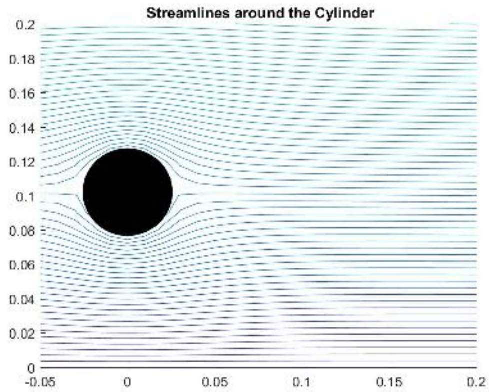

Figure 8: Streamlines for G/d = 1.5

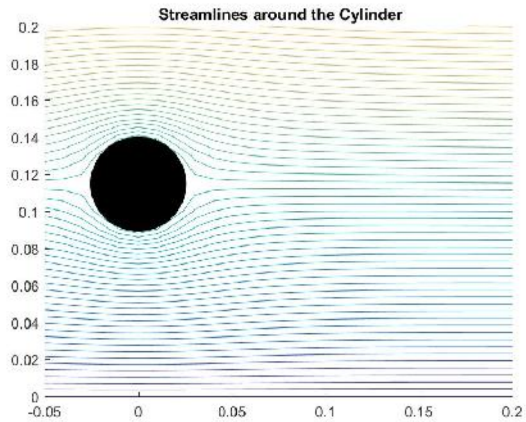

Figure 9: Streamlines for G/D = 1.75

Separation point prediction using Thwaite's criteria:

The separation points were predicted theoretically based on the given values for constant G/D and varying VR and vice versa. These values were compared with the experimental values. The tabulated form of the results is shown below :

Table 1: Comparison of experimental separation point with the predicted value using Thwaite's criteria

VR (G/D=1.5)	Xs (predicted) (mm)	Xs (Exp.) (mm)
0.805	48	47
1.0067	46	43
1.208	44	39
1.409	42	34

G/D (VR=1.208)	Xs (predicted) (mm)	Xs (Exp.) (mm)
1.75	52	54
1.50	44	39
1.25	36	38

The predicted separation points resemble a decreasing value as the VR value is increased, it states that the separation point for a high velocity flow is located at a location further upstream that the flow with reduced velocity. As the velocity of flow increases the separation point moves further upstream in the x-direction. Similarly for constant VR and varying G/D values the predicted separation point values were obtained to be descending order. It states that as the cylinder moves away from the wall the separation point moves further downstream as the adverse pressure gradient also decreases as the G/D value increases.

A plot of lambda vs the position of plate(x) was plotted using the Thwaite's criteria for the separation point prediction

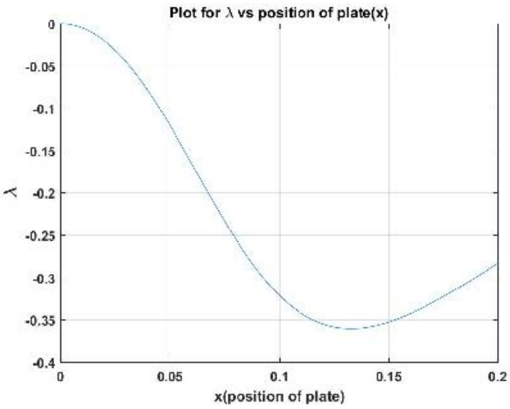

Figure 10: Plot of λ vs position of plate (x)

Separation point prediction using Stratford criteria:

The Stratford's criteria is easier to use than Thwaite's criteria as it does not include any integrations. The separation points predicted using this criteria are as follows:

Table 2: Comparison of experimental separation point with the predicted value using Stratford's criteria

VR (G/D=1.5)	Xs(predicted) (mm)	Xs(Exp.) (mm)
0.805	49	47
1.0067	47	43
1.208	45	39
1.409	43	34

G/D (VR=1.208)	Xs(Predicted) (mm)	Xs(Exp) (mm)
1.75	54	54
1.50	45	39
1.25	36	38

The separation points predicted using the Thwaite's criteria and Stratford criteria do not much difference hence it can be concluded that both the criteria's are good to predict the values of separation points. When the predicted values are compared with the experimental values it is seen that an early separation was predicted for the above flow.

Table 3: Comparison of separation points predicted using Thwaite's criteria and Stratford's criteria

VR (G/D=1.5)	Xs(predicted using Thwaite's) (mm)	Xs(Predicted using Stratford) (mm)
0.805	48	49
1.0067	46	47
1.208	44	45
1.409	42	43

G/D (VR=1.208)	Xs(predicted using Thwaite's) (mm)	Xs(Predicted using Stratford) (mm)
1.75	52	54
1.50	44	45
1.25	36	36

IV. CONCLUSION

From this study it was seen that pressure gradient plays an important role in flow separation. The adverse pressure gradient causes the flow to separate which increases the pressure drag on the airfoil thereby decreasing its lift and affecting its performance. The flow separates at some point on the object surface and reattaches back on its surface, these separation points can be predicted both experimentally and theoretically. Thwaite's and Stratford's criteria were used in our study to predict the separation points. The values obtained from both the criteria's are almost similar which states that both the criteria's provide the same results. When these values are compared with the experimental results it is seen that an early separation was obtained by the predicted values and the difference in the values is considerable, the reason in the difference of the values is because of different parameters. Research is being done on how to reduce the effect of flow separation in case of airfoils to increase the aerodynamic performance and efficiency. NASA had done research on using active control in flow separation in which a synthetic jet was used increase the momentum of the flow thereby causing it to stay attached to the airfoil surface and increasing the lift coefficient (Ravindran, 1999)Also suction and injection control method can be used to reduce the effect of flow separation by suppressing the separation bubble and reducing the pressure on the upper surface of the airfoil thereby increasing the lift and reducing the drag (Experimental Study of Flow Separation Control of an Airfoil by Suction and Injection, 2010). Vortex generators are also used to reduce the effect of flow separation. The only drawback of this study was that the reattachment point could not be predicted with the given values. With advanced research the problem of flow prediction has been reduced considerably since the past few decades and more research is going on to make it more efficient.

Appendix

```
%%Program on Prediction of flow separation over a
flat plate
%%Code to determine the pressure co-efficient at
constant VR and varying
%%G/D values

%Given values
%flow velocity
U_inf = 0.132;
%Diameter of cylinder
D = 0.051;
%Distance from the leading edge of the plate to the
end of the cylinder
L = 0.0255;
a = D./2;
R = D./2;
G = 1.25.*D;
G1=1.5.*D;
G2=1.75.*D;
%Assigning system variables
syms x y;
y = 0;
%Point in complex plane
z =x+i.*y;
%Co-rdinates in the x-y plane
z0 = (G+a).*1i;
z1 = (G1+a).*1i;
z2 = (G2+a).*1i;
%Co-ordinates in the negative x-y plane
Zp = -z0;
Zp1=-z1;
Zp2=-z2;
VR =1.208;
%The circulation at VR=1.208
gamma = 2.*pi.*R.^2.*((2.*VR.*U_inf)./D);

%Uniform stream
w = (z).*U_inf;
%Doublet at point z0
doublet = (a.^2.*U_inf/(z-z0));
%Doublet at point z1
doublet1 = (a.^2.*U_inf/(z-z1));
%Doublet at point z2
doublet2 = (a.^2.*U_inf/(z-z2));
%Doublet at point z3
doublet3 = a.^2.*U_inf/(z-Zp);
%Vortex at point z0
vortex = (i.*gamma.*log(z-z0))./(2.*pi);
%Vortex at point z1
vortex1 = (i.*gamma.*log(z-z1))./(2.*pi);
%Vortex at point z2
vortex2 = (i.*gamma.*log(z-z2))./(2.*pi);
%Vortex at point zp
vortex4 = (i.*gamma.*log(z-Zp))./(2.*pi);
%flow past a uniform stream
W = w+doublet-vortex+doublet3+vortex4;
W1 = w+doublet1-vortex1+doublet3+vortex4;
W2 = w+doublet2-vortex2+doublet3+vortex4;
%Differentiating wrt x
U_inf = diff(W,x);
U1 = diff(W1,x);
U2 = diff(W2,x);
%the velocity expression for a flow around a
cylinder is given by
% Umax = conj(U)-((U.*r.^2)./(-i.*z0).^2)-
((i.*gamma)./(-2.*pi.*i.*z0))-
((U.*r.^2)./(i.*z0).^2)+((i.*gamma)./(-
2.*pi.*i.*zp))
Umax=subs(U_inf,x,0);
Umax1=subs(U1,x,0);
Umax2=subs(U2,x,0);

%Pressure Coefficient
Cp=(1-(U_inf./Umax).^2);
Cp1=(1-(U1./Umax1).^2);
Cp2=(1-(U2./Umax2).^2);

X=0:0.003:0.2;

dp=diff(Cp,x);
dp1=diff(Cp1,x);
dp2=diff(Cp2,x);
%Making substitutions
P=subs(dp,x,X);
P1=subs(dp1,x,X);
P2=subs(dp2,x,X);
```

```
figure (3)
hold on
plot(X,P,'linewidth',1.5);
plot(X,P1,'linewidth',1.5);
plot(X,P2,'linewidth',1.5);
legend('1.25','1.5','1.75')
title('Pressure coefficient at different G/D')
ylabel('Pressure Coefficient
Gradient','fontsize',12)
xlabel('x along the wall','fontsize',12)
grid on


%%Code to determine pressure coefficient at constant
G/D and varying VR
%%values

%Given values
%Flow Velocity
U_inf = 0.132;
%Diameter of cylinder
D = 0.051;
%Length from the leading edge of the plate to the
end of cylinder
L = 0.0255;
a = D./2;
R = D./2;
G = 1.25.*D;
G1=1.5.*D;
G2=1.75.*D;
%Assigning system variables
syms x y;
y = 0;
%Point in complex plane
Z =x+i.*y;
%Co-rdinates in the x-y plane
Z0 = (G+D./2).*1i;
Z1 = (G1+D./2).*1i;
Z2 = (G2+D./2).*1i;

%Co-ordinates in the negative x-y plane
Zp = -Z0;
Zp1=-Z1;
Zp2=-Z2;

%The different values of VR
VR =0.85;
VR1=1.0067;
VR2=1.208;
VR3=1.409;

%The circulation at different velocities
gamma = 2.*pi.*R.^2.*((2.*VR.*U_inf)./D);
gamma1 = 2.*pi.*R.^2.*((2.*VR1.*U_inf)./D);
gamma2 = 2.*pi.*R.^2.*((2.*VR2.*U_inf)./D);
gamma3 = 2.*pi.*R.^2.*((2.*VR3.*U_inf)./D);

%Uniform stream
w = (Z).*U_inf;
%Doublet at point Z0
doublet = (a.^2.*U_inf/(Z-Z0));
%Doublet at point Zp
doublet1 = a.^2.*U_inf/(Z-Zp);
%Vortex at point Z0
vortex = (i.*gamma.*log(Z-Z0))./(2.*pi);
vortex1 = (i.*gamma1.*log(Z-Z0))./(2.*pi);
vortex2 = (i.*gamma2.*log(Z-Z0))./(2.*pi);
vortex3 = (i.*gamma3.*log(Z-Z0))./(2.*pi);

%Vortex at point Zp
vortex4 = (i.*gamma.*log(Z-Zp))./(2.*pi);

%flow past a uniform stream
W = w+doublet-vortex+doublet1+vortex4;
W1 = w+doublet-vortex1+doublet1+vortex4;
W2 = w+doublet-vortex2+doublet1+vortex4;
W3 = w+doublet-vortex3+doublet1+vortex4;

%Differentiating wrt x
U = diff(W,x);
U1 = diff(W1,x);
U2 = diff(W2,x);
U3 = diff(W3,x);

%the velocity expression for a flow around a
cylinder is given by
% Umax = conj(U)-((U.*r.^2)./(-i.*z0).^2)-
((i.*gamma)./(-2.*pi.*i.*z0))-
((U.*r.^2)./(i.*z0).^2)+((i.*gamma)./(-
2.*pi.*i.*zp))
Umax=subs(U,x,0);
Umax1=subs(U1,x,0);
Umax2=subs(U2,x,0);
Umax3=subs(U3,x,0);

%Pressure Coefficient
Cp=(1-(U./Umax).^2);
Cp1=(1-(U./Umax1).^2);
Cp2=(1-(U./Umax2).^2);
Cp3=(1-(U./Umax3).^2);

X=0:0.001:0.2;

dp=diff(Cp,x);
dp1=diff(Cp1,x);
dp2=diff(Cp2,x);
dp3=diff(Cp3,x);

%Making substitutions
P=subs(dp,x,X);
P1=subs(dp1,x,X);
P2=subs(dp2,x,X);
P3=subs(dp3,x,X);
figure (2)
```

```
hold on
plot(X,P,'linewidth',1.5);
plot(X,P1,'linewidth',1.5);
plot(X,P2,'linewidth',1.5);
plot(X,P3,'linewidth',1.5);
legend('0.805','1.0067','1.208','1.409')
title('Pressure distribution at different values of
V_R')
ylabel('Pressure Coefficient
Gradient','fontsize',12)
xlabel('x along the wall','fontsize',12)
grid on
```

Code to generate streamlines around the cylinder using the given values

```
%Given values
%Flow Velocity
U_inf = 0.132;
%Cylinder Diameter
D = 0.051;
%Radius
r = D/2;
%Length from the leading edge of the plate to the
end of cylinder
L = 0.0255;

GD = 1.5;
G = GD.*D;
VR = 0.805;
%Circulation
gamma = (VR.*2.*U_inf/D).*2.*pi*r.^2;

%Points in complex plane
z0 = i*((GD*D)+D/2);
z1 = conj(z0);


tolerance = +5e-2;
x = meshgrid(-0.05:.001:.2);
y = x';
z = x+i*y;
for i = 1:length(x)
    for j = 1:length(y)
        if abs(z(i,j)-z0) <=  r - tolerance
          z(i,j)=NaN;

         end
    end
end

%Flow velocity around the cylinder
w = conj(U_inf)*z+((r.^2./(z-
z0)).*U_inf)+((r.^2./(z-z1)).*U_inf)-
(((i*gamma)./(2.*pi)).*(log((z-
z0)./r)))+(((i*gamma)./(2.*pi)).*(log((z-z0)./r)));

theta = 0:.1:2*pi;
Z_circle = r*(cos(theta)+i*sin(theta)) + z0;


%Streamline plot
figure(1)
hold on
contour(real(z),imag(z),imag(w),[0:0.001:0.2]);
fill(real(Z_circle),imag(Z_circle),'r')
title('Streamlines around a cylinder')
axis([-0.05 0.2 0 0.2])
```

Separation point prediction using Thwaite's criteria with varying VR and G/D = 1.5

```
%flow velocity
U = 0.132;
%diameter of cylinder
D = 0.051;
%Length from the leading edge of the plate to the
end of cylinder
L = 0.0255;
r = D./2;
R = D./2;
G = 1.5.*D;
%Assigning system variables
syms x y;
y = 0;
%Points in the complex plane
z = x+i.*y;
z0 = (G+D./2).*i;
zp = -z0;
VR = 1.208;
%Circulation
gamma = 2.*pi.*R.^2.*((2.*VR.*U)./D);
%Uniform stream
w = (z).*U;
%Doublet at point z0
doublet = (r.^2.*U/(z-z0));
%Doublet at point zp
doublet1 = r.^2.*U/(z-zp);
%Vortex at point z0
vortex = (i.*gamma.*log(z-z0))./(2.*pi);
%Vortex at point zp
vortex1 = (i.*gamma.*log(z-zp))./(2.*pi);
%Flow past a uniform stream
W = w+doublet-vortex+doublet1+vortex1;
%Differentiating wrt x
U = diff(W,x);
%Using the equation for Thwaite's criteria
Momentumthickness =
diff(U,x).*(0.47./U.^6).*int(U.^5,x,0,x)
X = 0:0.002:0.2;
Y = subs(Momentumthickness,x,X);
vpa(X,3);
```

```
j=1;
for i = 1:length(Y)
    if vpa(Y(i),3)<-0.09
        val(j)=vpa(Y(i),3);
        loc(j) = X(i);
        j=j+1;
    end
end
loc(1)
plot (X,Y);
xlabel('x(position of plate)');
ylabel('\lambda','fontsize',14);
title('Plot for \lambda vs position of plate(x)')
grid on
```

Separation point prediction using Thwaite's criteria with varying G/D and VR=1.208

```
%flow velocity
U = 0.132;
%Cylinder diameter
D = 0.051;
%Length from the leading edge of the plate to the
end of cylinder
L = 0.0255;
r = D./2;
R = D./2;
%Entering the given G/D values
G_D=input('enter G/D value');
G = G_D.*D;
%Assigning system variables
syms x y;
y = 0;
%Point in the complex plane
z = x+i.*y;
z0 = (G+D./2).*i;
zp = -z0;
VR = 1.208;
%circulation is given as
gamma = 2.*pi.*R.^2.*((2.*VR.*U)./D);
%Uniform stream
w = (z).*U;
%Doublet at z0
Doublet = (r.^2.*U/(z-z0));
%Doublet at zp
Doublet1 = r.^2.*U/(z-zp);
%Vortex at z0
Vortex = (i.*gamma.*log(z-z0))./(2.*pi);
%Vortex at zp
Vortex1 = (i.*gamma.*log(z-zp))./(2.*pi);
%Flow past the uniform stream
W = w+Doublet-Vortex+Doublet1+Vortex1;
%Differentaiting wrt x
U = diff(W,x);
Momentumthickness =
diff(U,x).*(0.47./U.^6).*int(U.^5,x,0,x)
```

```
X = 0:0.002:0.2;
Y = subs(Momentumthickness,x,X);
vpa(X,3);
j=1;
for i = 1:length(Y)
    if vpa(Y(i),3)<-0.09
        val(j)=vpa(Y(i),3);
        loc(j) = X(i);
        j=j+1;
    end
end
loc(1)
plot (X,Y);
xlabel('x(position of plate');
ylabel('\lambda');
title('Plot for \lambda vs position of plate(x)')
grid on
```

```
%%Prediction of separation point by Stratford'd
criteria with constant G/D
%%= 1.5 and varying VR

%Flow Velocity
U_inf = 0.132;
%Cylinder Diameter
D = 0.051;
%Length from the leading edge of the plate to the
end of cylinder
L = 0.0255;
a = D./2;
R = D./2;
G = 1.5.*D;
%Assigning system variables
syms x y;
y = 0;
%Points in the complex plane
z = x+i.*y;
z0 = (G+D./2).*i;
zp = -z0;
%Velocity
VR = input('enter VR Value');

%gaama = ((2.*VR.*U)./D);
%Circulation
gamma = 2.*pi.*R.^2.*((2.*VR.*U_inf)./D);
%Uniform stream
w = (z).*U_inf;
%Doublet at point z0
doublet = (a.^2.*U_inf/(z-z0));
%Doublet at point zp
doublet1 = a.^2.*U_inf/(z-zp);
%Vortex at point z0
vortex = (i.*gamma.*log(z-z0))./(2.*pi);
%Vortex at point zp
vortex1 = (i.*gamma.*log(z-zp))./(2.*pi);
%Flow past a uniform stream
```

```
W = w+doublet-vortex+doublet1+vortex1;
%Differentaiting wrt x
U_inf = diff(W,x);
syms x;
%The velocity expression for flow around a cylinder
is given by
% Umax = conj(U)-((U.*r.^2)./(-i.*z0).^2)-
((i.*gamma)./(-2.*pi.*i.*z0))-
((U.*r.^2)./(i.*z0).^2)+((i.*gamma)./(2.*pi.*i.*zp))
Umax = subs(U_inf,x,0);
Cp=(1-(U_inf./Umax).^2);

c=x.^2;
d=diff(Cp,x);
N=c.*Cp.*d.^2
X=0:0.001:0.2
M=subs(N,x,X);
j=1;
for i=1:length(M);
    if vpa(M(i),3)>=0.0104
        val(j)= vpa(M(i),3);
        loc(j)= X(i);
        j=j+1;
    end
end
loc(1)
```

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