# Fuzzy Graph Coloring Using $\alpha^+$ Cuts

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***Abstract-*** *Total coloring is a function which assigns colors the vertices and edges so that adjacent vertices and incident edges receive different colors. Here we consider the fuzzy graphs with fuzzy set of vertices and fuzzy set of edges and fuzzy total coloring using* $\alpha^+$ *cuts.*

***Keywords***- Chromatic number, Chromatic index, Total chromatic number, Fuzzy set,$\alpha^+$ cuts.

## I. INTRODUCTION

The total coloring of a graph is an assignment of colors to the vertices and edges such that no two adjacent elements receive the same color.Generally for a given graph G=(V,E), a coloring function is a mapping $\pi: V \to N \ni \pi(i) \neq \pi(j)$where i and j are adjacent vertices in G. But if we use only k-colors to color a graph we define a k coloring $\pi^k$:V$\to \{1,2, \dots .k\}$.A graph is colourable if it admits a k proper coloring. The chromatic number $\chi(G)$,of a graph G is the minimum k for which G is k colorable.

Fuzzy graph coloring is one of the most important problem of fuzzy graph theory. It is mainly studied in combinatorial optimization like traffic light control,exam scheduling,register allocation etc.

**Definition1.1:**

A fuzzy set A defined on a non empty set X is the family A={x,$\mu_A(x)/x\epsilon X$ }where $\mu_A: X \to I$is the membership function.In classical fuzzy set theory the set I is usually defined on the interval [0,1]$\ni \mu_A(x)$=0 if $x \notin A$.$\mu_A(x) = 1 \; if \; x \in A$and any intermediate value represents the degrees in which x could belong to A. The set I could be discrete set of the form I={0,1,…k} where $\mu_A(x) < \mu_A(x')$indicates that the degree of membership of x toA is lower than the degree of membership of $x'$.

## II. FUZZY GRAPHS WITH CRISP VERTICES AND FUZZY EDGES

**Definition 2.1:**

The graph $\hat{G} = (V, \hat{E})$ is a fuzzy graph where V is the vertex set and E is the fuzzy edge set which is characterized by the matrix $\mu = [\mu_{ij}]$,i,j $\in V, \mu_{ij} = \mu_{\check{E}}(i,j)$ for every i,j$\in V \ni i \neq j$ and $\mu_{\check{E}}: V \times V \to I$ is the membership function.

Each element $\mu_{ij} \in I$ represents the intensity level of the edge {i,j} for any i,j$\in V$ with $i \neq j$. The fuzzy graph can also be denoted by $\hat{G} = (V, \mu)$

The set I is linearly ordered in such a way that the expression on $\mu_{i,j} < \mu_{i'j'}$ stands for the intensity level of edge {i,j} is lower than the intensity level of edge {$i', j'$}.The fuzzy graph $\hat{G}$can be considered as generalization of the crisp graph G, since taking I=[0,1],$\hat{G}$ becomes a crisp graph.

## III. FUZZY VERTEX COLORING: [3,6]

**Definition3.1:**

A fuzzy set A defined on X can be characterized from its family of $\alpha^+ -$cuts $A_{\alpha^+} = \{x \in X/\mu_A(x) > \alpha\}, \alpha \in I$.

Hence any crisp k-coloring $\pi^k{}_{\alpha^+}$can be defined on $G_{\alpha^+}$. The k coloring function $\hat{G}$is defined through this sequence for each $\alpha\epsilon I$. Here $\chi_{\alpha^+}$denote the chromatic number of $G_{\alpha^+}$.

**Definition3.2:**

For a fuzzy graph $\hat{G} = (V, \mu)$, its chromatic number is the fuzzy number $\chi(\hat{G}) = \{x, v(x)/x \in X\}$, wherex={1,..$|V|$},v(x)={sup{$\alpha \in I/x \in A_{\alpha^+}$}} $x \in X$ and $A_{\alpha^+} = \{1, \dots \chi_{\alpha^+}\}, \alpha \in I$

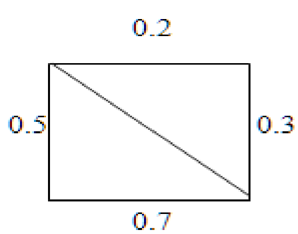
The chromatic number of fuzzy graph is a normalized fuzzy number whose model value is associated with the empty edge set graph. Its meaning depends on the sense of index $\alpha^+$. It can be interpreted that for lower value of $\alpha$ there are many incompatible edges between the vertices so that more colors are needed in order to consider the incompatibilities. On the other hand for higher value of $\alpha^+$ there are fewer incompatible edges and less colors are needed. The fuzzy coloring problem conists of determining the chromatic number of a fuzzy graph and an associated coloring function.

For any level $\alpha$,the minimum number of color needed to color the crisp graph $G_{\alpha^+}$ will be computed is defined. In this way the fuzzy chromatic number is defined as fuzzy number through its $\alpha^+$-cuts.

**Example:1**

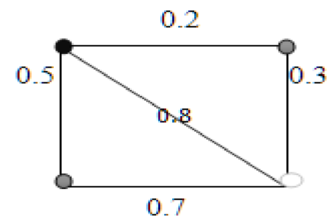
Consider the fuzzy graph $\hat{G} = (V, \mu)$ where V={1,2,3,4} and the matrix of $\mu$ is defined as

$$\mu = \begin{bmatrix} 0 & 0.2 & 0.8 & 0.5 \\ 0.2 & 0 & 0.3 & 0 \\ 0.8 & 0.3 & 0 & 0.7 \\ 0.5 & 0 & 0.7 & 0 \end{bmatrix}$$

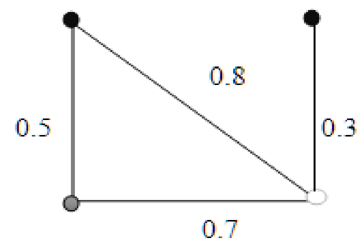



For this example ,six crisp graphs $G_{\alpha^+} = (V, E_{\alpha^+})$ are obtained by considering the values $\alpha \epsilon I$. For each $\alpha \epsilon I$,

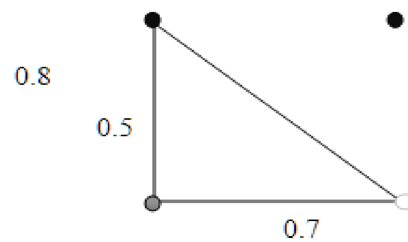
$\alpha = 0 \chi'_{\alpha^+} = 3$



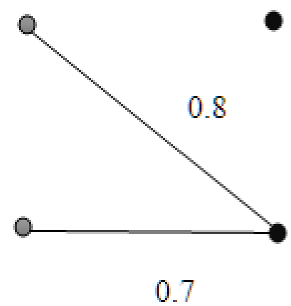

$\alpha = 0.2 \chi'_{\alpha^+} = 3$



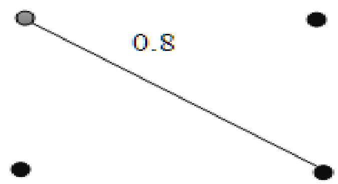

$\alpha = 0.3\ \chi'_{\alpha^+} = 3$




$\alpha = 0.5\ \ \chi'_{\alpha^+} = 2$



$\alpha = 0.7 \ \chi'_{\alpha^+} = 2$




$\alpha = 1 \ \chi'_{\alpha^+} = 1$

Thus the fuzzy chromatic number of $\hat{G}$ is $\chi(\hat{G})$={(1,1),(2,0.7),(3,0.3)}

Here we defined fuzzy chromatic number as a fuzzy number as followes

| $\alpha$ | V | $\chi$ | $c_{\alpha^+}(12)$ | $c_{\alpha^+}(13)$ | $c_{\alpha^+}(14)$ | $c_{\alpha^+}(23)$ | $c_{\alpha^+}(34)$ |
|---|---|---|---|---|---|---|---|
| 0 | 12,13,14,23,34 | 3 | 1 | 2 | 3 | 3 | 1 |
| 0.2 | 13,14,23,34 | 3 | 0 | 2 | 1 | 1 | 1 |
| 0.3 | 14,13,34 | 3 | 0 | 2 | 1 | 0 | 1 |
| 0.5 | 13,34 | 2 | 0 | 1 | 0 | 0 | 1 |
| 0.7 | 13 | 2 | 0 | 1 | 0 | 0 | 0 |
| 1 | Φ | 1 | 0 | 0 | 0 | 0 | 0 |

## IV. FUZZY EDGE COLORING

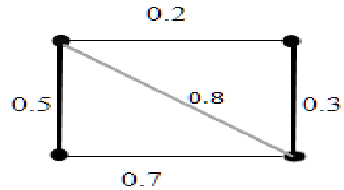
Let $G_{\alpha^+} = \{(V, E_{\alpha^+})/\alpha\epsilon I\}$ be the family of $\alpha^+$ cuts of $\hat{G}$ where the $\alpha^+$ cut of a fuzzy graph is the crisp graph $G_{\alpha^+} = (V, E_{\alpha^+})$ with $E_{\alpha^+} = \{(i,j)/i,j\epsilon V, \mu_{ij} > \alpha\}$.

In crisp case the edge chromatic number of a graph is either $\Delta$ or $\Delta + 1$ where $\Delta$ is the maximum of vertex degree

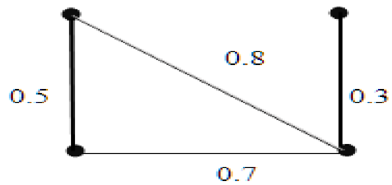
**Definition4.1:**

For a fuzzy graph $\hat{G} = (V, \mu)$ its edge chromatic number is the fuzzy number $\chi'_f(\hat{G}) = \{x, \lambda(x)/x\epsilon X\}$ where X={1,…$\Delta + 1$},$\lambda(x) = sup\ \{(\alpha\epsilon I/x\epsilon A_{\alpha^+})\}$ and $A_{\alpha^+} = (1, \ldots \chi'_{\alpha^+}), \alpha\epsilon I$. The crisp graph $G_{\alpha^+} = (V, E_{\alpha^+})$ are obtained by considering the values of $\alpha\epsilon I$, for each $\alpha\epsilon I$

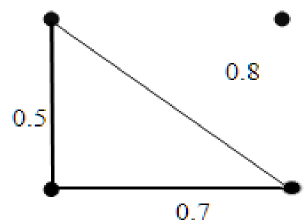
$\alpha = 0 \chi'_{\alpha^+} = 3$



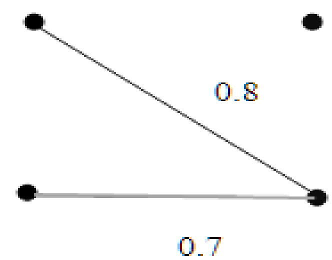

$\alpha = 0.2 \chi'_{\alpha^+} = 3$



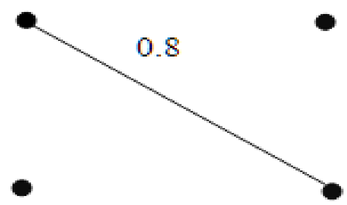

$\alpha = 0.3 \ \chi'_{\alpha^+} = 3$




$\alpha = 0.5 \chi'_{\alpha^+} = 2$




$\alpha = 0.7 \ \chi'_{\alpha^+} = 1$



$$\alpha = 1\ \chi'_{\alpha^+} = 0$$

Since $\Delta = 3$ for the given graph the fuzzy edge chromatic number of $\hat{G}$ is

$$\chi'_{\alpha^+} = \{(1,0.7),(2,0.5),(3,0.3)\}$$

If $\mu = 1$ for all edge of the graph $\hat{G}$ then it becomes the crisp graph G then $\chi'_{\alpha^+} = \chi'$

| $\alpha$ | $E_{\alpha^+}$ | $\chi$ | $c_{\alpha^+}(12)$ | $c_{\alpha^+}(13)$ | $c_{\alpha^+}(14)$ | $c_{\alpha^+}(23)$ | $c_{\alpha^+}(34)$ |
|---|---|---|---|---|---|---|---|
| 0 | 12,13,14,23,34 | 3 | 1 | 2 | 3 | 3 | 1 |
| 0.2 | 13,14,23,34 | 3 | 0 | 2 | 1 | 1 | 1 |
| 0.3 | 14,13,34 | 3 | 0 | 2 | 1 | 0 | 1 |
| 0.5 | 13,34 | 2 | 0 | 1 | 0 | 0 | 1 |
| 0.7 | 13 | 1 | 0 | 1 | 0 | 0 | 0 |
| 1 | Φ | 0 | 0 | 0 | 0 | 0 | 0 |

## V. FUZZY TOTAL COLORING:

Let $G_{\alpha^+} = \{(V, E_{\alpha^+})/\alpha\epsilon I\}$ be the family of $\alpha^+$ cuts of $\hat{G}$ where the $\alpha^+$ cut of a fuzzy graph is the crisp graph $G_{\alpha^+} = (V, E_{\alpha^+})$ with $E_{\alpha^+} = \{(i,j)/i,j\epsilon V, \mu_{ij} > \alpha\}$.

In crisp case the edge chromatic number of a total graph is atmost $\Delta + 2$ where $\Delta$ is the maximum vertex degree.So we define fuzzy chromatic number as a fuzzy number as follows:

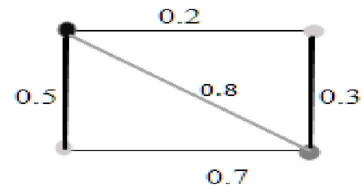
**Defintion5.1:**

For a fuzzy graph $\hat{G} = (V,\mu)$,its total chromatic number is the fuzzy number $\chi^T{}_f(\hat{G}) = \{x, \tau(x)/x\epsilon X\}$, where x={1,….$\Delta + 2$},$\tau(x) = \sup\{\alpha\epsilon I/x\epsilon A_{\alpha^+}\}$ and $A_{\alpha^+} = \{1, \ldots \chi^T{}_f\}$,$\alpha\epsilon I$.

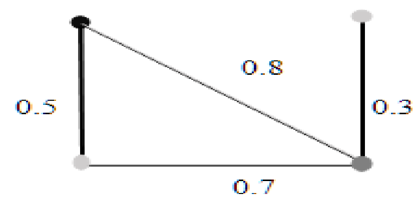
Example:1

For each $\alpha\epsilon I$,the $C_{\alpha^+}: V \cup E \rightarrow \{1,2, \ldots k\}$ and the chromatic number $\chi_f{}^T$ or $G_{\alpha^+}$.

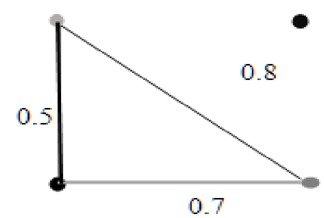
$$\alpha = 0\chi^T{}_f = 5$$



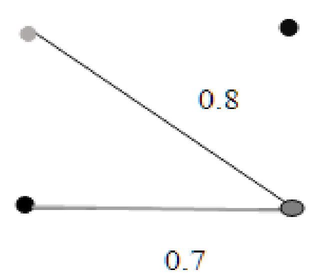

$$\alpha = 0.2\chi^T{}_f = 5$$



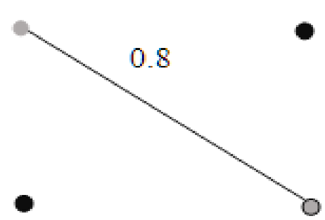

$$\alpha = 0.3\ \chi^T{}_f = 5$$




$$\alpha = 0.5\ \chi^T{}_f = 4$$




$$\alpha = 0.7\ \chi^T{}_f = 3$$



$$\alpha = 1 \ \chi^{T}{}_{f} = 1$$

Since $\Delta = 3$ for the given the fuzzy total chromatic number of $\hat{G}$ is

$$\chi^{T}{}_{f} = \{(1,1),(3,0.7),(4,0.5),(5,0.3)\}$$

| $\alpha$ | $\chi^{T}{}_{f}$ | $c_{\alpha^+}(12)$ | $c_{\alpha^+}(13)$ | $c_{\alpha^+}(14)$ | $c_{\alpha^+}(23)$ | $c_{\alpha^+}(34)$ |
|---|---|---|---|---|---|---|
| 0 | 5 | 1 | 2 | 3 | 3 | 2 |
| 0.2 | 5 | 0 | 2 | 1 | 1 | 2 |
| 0.3 | 5 | 0 | 2 | 1 | 0 | 2 |
| 0.5 | 4 | 0 | 1 | 0 | 0 | 1 |
| 0.7 | 3 | 0 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 |

## VI. CONCLUSION

In this paper we defined the fuzzy chromatic number, chromatic index, fuzzy total chromatic number as fuzzy numbers through the $\alpha^{+}$ cuts of the fuzzy graph which are crisp graphs.

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