Explanation of A Scientific Theory For A Change – In General And Special Form – By Using Metaphysics

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Abstract*- The paper gives a methodology to find the single solution to disambiguation, by one in-determinate, using an arbitrary constant; and one of the infinitely many in-determinates is obtained, invariably, by a process of reverse engineering, using six generalized laws.*

Keywords- in-determinate, Entropy, Uncertainty, Certainty, Reliability

I. INTRODUCTION

(i) Referring to mathematics, the term Integration means a process of assimilation or combining infinitesimal data and the inverse process is Differentiation which measures a change of a function output with infinitesimal change in input. Quoting from the Fundamental Theorem of Calculus which is followed, states that, firstly, one of the antiderivatives of a function may be obtained as the integral of the function with a variable bound of integration; and secondly, that the integral of a function over some interval can be computed by any one of its infinitely many antiderivatives. It is to be noted, that in the first part of the theorem, there is no mention of any constant, though the integration is bound yet variable, which implies there is arbitration; and in the second part, the computation is not defined and that can be numerical or left to possible symbolic formulations, as may be.

(ii) Referring to philosophy, three laws of logic are followed, the first of which is the law of non-contradiction, which states that contradictory propositions cannot both be true in the same premises; the second is the law of excluded middle, which states that for a proposition, either this or the negation is to be true; and the third is the law of identity, which states that a proposition is identical with itself. It is to be noted, the first law indicates that a proposition is either true or not true; the second law indicates that there is at least one proposition as true that is all the propositions cannot be not true; and the third law indicates that each proposition is unique. The three laws describe the summation.

(iii) The formalization by mathematics using philosophy provide not only clearer but also a bigger picture, in the representations of information and probability. The formulations by linearised approximations for analytic functions relating the partial and total measures of probability -- given wholly by the six laws as is propounded -- ensure faster computations with lesser complexities, compared to the conventional methods of integration and differentiation that depict a change. The project gives a solution to the indeterminacies and disambiguation and the reverse engineering
by the new general and special theory as is established -- that hence provide a quantum model to explain every real world physical phenomena.

(iv) In the following sections, there is a review of a project by Maitra [2021, 2020], titled ‘In Continuation to Quantum of Time’ and ‘Quantum of Time’ that is a new theory on probability relations using philosophical science, references [1],[2]. In the section (1), the paper by Friend and Molinini [2015], reference [3], is shown, which serves as a guidance material towards a feasibility of this particular study. Next, in section (2), the references [1],[2] are examined. The section (3) explains the way the new theory, as propounded by the project, [1] and [2], can model a physical phenomena. The last is section (4), that gives a conclusion.

II. FEASIBILITY STUDY

The question whether a physical phenomenon or a physical theory can be wholly explained by mathematics, is already fully answered [3], where mathematics have been shown, necessarily, as indispensable.

Proceeding in the same lines, the very first question that surfaces is, whether mathematics is enough to explain a whole scientific phenomenon, even without a physical theory to exist. To find an answer to this question, intuitively though, it is expected that one must not dwell into a possibly infinite realm of philosophy, because philosophy is a study about reality and existence. One is expected, rather to limit a correspondence of a particular physical phenomenon in question and mathematics as in a one-to-one relation. Here it appears that, though a physical theory might exist, yet it may be insignificant before the known laws of nature, if those can be directed right into mathematical practice. The meaning of the term whole used to describe a scientific phenomenon or a theory, is such as it is to be wholly (other than partially) explainable using mathematics. Again, a whole scientific theory or phenomenon means it is universal, as is a general theory ; while a partial scientific theory or phenomenon would mean empirical observations by a particular observer alone having a definition, as special that is given by a relativistic theory. It needs to be highlighted, that a partial explanation in the present context itself creates space for partial use of some physical theory, while the physical phenomenon is taken wholly. Moreover, any physical theory is indisputably based on some laws of nature.

Hence, the question raised first, that if mathematics is enough to explain a whole scientific phenomenon, even without any existence of a physical theory, loses its relevance.

A change is something, that happens, for which, either the cause or the explanation comes into question; as there exists the inertial forces to bring a physical change, there ought to be a mathematical justification for the same. There is a conjunction or superposition, as apparent, if taken beyond the real plane. And this is just defined as metaphysics, a branch of philosophy, which is abstract and deals with the first principles in general; that further means, logic ought to have a part there as well.

The observation is: mathematics and physics are necessary, where each is connected, partially or sufficiently, by using metaphysics, in order to wholly explain a scientific phenomenon.

Thus, with the feasibility study being done, we are quite assured that we are heading on-track and that this project is clearly the right candidate for examination and explanation.

III. EXAMINATION

(i) Prelude to the first principles

Extract from Reference [4].

The prelude is regarding entropy, which is very well known characteristic of disorderliness, for over half a century, especially in the science of information theory in data communications, as the information content and the entropy relate to the uncertainty involved; that is also analogous to the physical property of every pure substance called thermodynamic entropy or disorder; but there is an ambiguity herein and this need necessarily be reconsidered once more.
The entropy is also being referred to as ‘surprisal’, which implies a result that is randomly variable and so indeterminate about the information contained therein. And the analogy drawn for the information content with the thermodynamic property of entropy, calls for their characteristics to be similar that is increasing and monotonically as well, with no violations, such as the thermodynamic entropy of the universe is known to be ever-increasing, in general. However, the ambiguity becomes apparent, as and when the surprisal element refers to the information content, as the entity of uncertainty which is known to exist.

The formula, which is not derived, rather adopted for the function of entropy, is given as: {summation [-x.ln(x)]}, where (x) is a variable and (ln) is the natural logarithm with a base of the exponential number (e). The summation used in the equation implies for all the information taken together and is a discrete function, in general.

As for a well-defined system of independent, identical and distributed units is subjected to testing and the failures are noted in a cumulative manner that is in summation, the probability function will be found to be increasing over time. And if the number of individual units in the system are finitely large, then the failure rate in terms of probability that is the probability density function tends to become uniform or constant and so the cumulative failure probability will be found to assume a linear approximation as such.

And for a probability density function, which is formed for a system of finitely large number of units, or a mass function that is discrete, such as for the point probabilities also known as a ‘Poisson’ process, then it is supposedly a linearly increasing function. And this is the originating function for any normal distribution as well.

The entropy function that is for the information content is found to be bound within the field of probability that is from zero to one, where the beginning and the end points on the boundary are both at zero values and the function reaches a maximum about the middle of domain.

This observation of the information content is not consistent with the definition of thermodynamic entropy, which is supposed to be strictly monotonic alone.

Referring to the known entropy function that is also called the information content, as propounded by ‘Shannon’ in information theory, is given as: {summation[-x.log(x)]}, or {-x.log(x)}, for a single variable (x), which is equally applicable for a normal distribution also, written as:

{(1/2).log(2.pi.exp(sigma square))}, where sigma square is the variance and (exp) is the exponential function. This entropy relation in probability, is found to be not only linearly increasingfrom zero to one, rather of a ‘sigmoid’ shape lying within the bounded region.

The Gauss-Normal form of the standard normal distribution, which is given as the exponential of minus square of (x), also generates the error function (erf) that is given in an integral form of the function variable (t) ranging from zero to (x), with a coefficient of two by square root of two (pi), where the value of (pi) is 3.141…, for the exponential of minus (t) square in terms of an incremental term (dt). The error function represents the difference that exists between the normal distribution and the generalized form as such.

Now, the entropy function for the information content is a form of power function as is defined in logarithmic transformation that is given as: {- x.log(x)}, which is for a single variable (x); where the corresponding root function in logarithmic transformation is defined as: {(-1/x). log(x)}.

The real problem is that the entropy, defined by the information theory, shows a function curve that is increasing-decreasing only and an error function that is continuously increasing cannot be derived thereof.

Well, since entropy for the information content is to be bound by the field axioms, which is for the probability, then the only function curve that may be found to exist is as given by the formula of Shannon that is: {-x.log(x)}.

As for instance, functions such as: {-log(x)}; {-x.log(1/x)} and {-(1/x).log(x)} are all out of bounds and hence to be rightly ignored.

And so the problem becomes self evident, for there cannot be a root function defined in terms of logarithmic transformations alone, which can be bound within the given field of probability that is for the real numbers between zero and one.

Furthermore, the entropy formula given by Shannon has a logarithmic base of 2, supposedly for the information content to be present in ‘bits’ that is of zeroes and ones, considering the binary system of numbers.

Now, then, we have the entropy for the information content as continuous, which is also meant to be differentiable in general.

The information content that is given by the entropy function as: [f(x) = -x.log(x)], is similar to the function: {f(x) = a(x).b(x)}. And the derivative denoted by (’) is: {f’(x) = a(x).b’(x) + b(x).a’(x)}. And so for: {a(x) = x} and {b(x) = log(x)}; we have: {a’(x) = 1} and {b’(x) = 1/x}. Therefore: {f’(x) = (x.(1/x)) + (log(x).1} ; that gives: [f’(x) = 1 + log(x)] as the solution.

The integration of the information content as given by: [-x.log(x)], is achieved by parts, first by taking: { u = log(x) }, which gives the differential as: { du = (1/x). dx }; and { dv = x.dx } that is: { v = (x.x)/2 }. In the second step: {integral of (x.log(x).dx) = integral of (u.dv) }; which gives: { uv - integral of (v.du) }. That gives: {((x.x)/2).log(x) – (1/2).integral of
(x.dx)}. And the solution is: [((x.x)/2).log(x) – (x.x)/4 + C]; where (C) is known as the constant of integration.

It is to be recalled that logarithm values are necessarily to be positive numbers, which cannot be zero or negative and so logarithms are meant to be expressed in modulus as such. Thus, log(0) and log(1) are both to be taken as zero. With that, the solution of the definite integral: [((x.x)/2).log(x) – (x.x)/4 + C]; between the boundaries of one and zero yields: [1/4] – [0]. And for the initial condition, where(x) is zero, the entropy is zero and the constant of integration (C) is zero. Therefore, we have the final solution as: [1/4], which gives the area under the entropy function [-x.log(x)].

The known entropy function for the information content that is given as: [-x.log(x)], is found to increase to a maximum value beginning from zero and then decrease to reach zero value at the end of the domain of the variable. It is hence an increasing-decreasing function that is not monotonic in general. And the maximum value of the function is observed to be at 0.367, which is reached at 35 percent from beginning of the domain scale, approximately.

And if the exponential of the entropy function is taken as: [exp(-x.log(x))], or more precisely with an inverse of the natural logarithm given by the exponential (e), such as: [exp(- x.ln(x))], in order to facilitate the back-transform, then the curve is found to be exactly similar to the original function curve, though the maximum value reached is placed higher, beyond the bounds of the real number field of zero and one. The maximum value is formed exactly at the same point on the domain scale that is at 0.35 unit from the origin taken at zero and that maximum value is 1.44, approximately.

Hence, if the exponential function of the entropy is reduced by unity, then the curve matches close to the originating function, such as for any higher level ordering is found in general. The corresponding maximum value is about

0.44, which is placed at 35 percent on the domain, approximately.

Now, the maximum value reached by the exponential function of the entropy is much higher than that of the originating function and so does not co-relate fully with the area under the entropy function: [-x.ln(x)], which is found to be [1/4] or 0.25, approximately.

The implication here is that, the maximum value reached by a curve, which originates at zero and ends at zero -- reasonably supposed to lie in close proximity to the hypotenuses of the two triangles -- such as the total area, as given by the product of base, which is 1.0 and perpendicular that is 0.25, comes to about 0.25 only.

The entropy function that is the information content, is given as a summation of finitely large number of signals given by a finite collection of binary digits, measured in 'bits', as is represented by the independent variable. The entropy is supposedly a discrete function that is not continuous. The entropy function is neither differentiable, nor has any definite integral. And the continuity of the function in the neighborhood is not known as such. Therefore, the entropy function is not mathematically analytic by definition, in general.

The implication here is such that, the entropy for the information content relates to the uncertainty part in signal processing of data communications; where a loss of information is by attenuation and interference from other sources in the neighbourhood of a given system. The concept of entropy in information science is elucidated by lossless compression that is an ideal process of decompression of the data to generate the original data as is, though there are losses involved in the process is already well known.

And for the information content represented by a finitely large collection of binary digits, there is an association of probability in the given sample space. It is therefore conclusive, that the entropy for information content stands for the entity of uncertainty, which is an established measure of probability alone.

Now, a lossless compression implies that the entropy should be algebraically associative, such as addition to the information that is hence regardless the order of the operation, by a process of compression and decompression, which thereby would restore the original information as received from the known source.

The exponent taken as: [exp(x.ln(x))], equals the power function: [(p) to-the-power (p)]; and the complementary function: [1 – exp(x.ln(x))] equates to: { 1- [(p) to-the-power (p)] }.

A minus sign introduced to the entropy formula for information content, written as: [-x.ln(x)], gives an equivalent reciprocal form of logarithmic equation as: [x.ln(1/x)]. However, the reason for taking the reciprocal of a random variable (x) as : (1/x), which goes out of bounds for the field of real numbers between zero and one, is not explicit. Therefore, there is not being enough reason for the entropy for information content to be accepted as is. As a matter of fact, logarithm is a transcendental function, by definition, that so is not fit for further algebraic operations. And the only way out is to consider the inverse process of the logarithmic transformation, which is given by the exponentiation only.

The one and only method of determining the power of a variable is by programming a logarithmic transformation, which is also known as a power function, having a base and an exponent that are known variables for the given system.

The root function for the logarithmic transformation is written as: [(1/x).ln(x)], which is interpreted as: [(x) to-the- power (1/x)]. It is important to note here, that the base of the transformation needs to be kept invariant.

The root function forms the maximum variations or fluctuations of the random variable, in general, which implies a reduction in probability value, such as efficiency of a system that is susceptible to losses. Thus, the fall in probability for any given system is marked by the lower bound of the entropy function. The other entropy bounds, such as the upper limits of fluctuations, are formed by the complementary functions, as such.

Now, referring to classical thermodynamics, the properties of any pure substance, which are 'intensive' in nature, exhibits no change with the amount of substance being present. There exist a number of microscopic states, which are called 'micro-states'. Entropy is a property of every pure substance and is a 'state' function, which depends on the initial and the final states of the system alone. There exists a relation between the specific entropy (s) and probability (w), where the entropy is additive, but the probability multiplicative. Now, if two parts (a) and (b) of a system in equilibrium are considered, such as in mixing of gases in two chambers, then: (S = sa + sb) and (w = wa.wb); and for { S = s.(w) }, { S.(w) = S.(wa) + S.(wb) = s. (wa.wb) }; which is a well known functional relation for the logarithm. And for the compatibility of energy and mass among the micro-states, which are hence consistent and so, reversible in general. And a system state that is reversible is meant to be in 'equilibrium', such that the number of micro-states (w), expected to be equally likely or equiprobable, is related by the formula of Boltzmann for entropy given as: [s = k.log(w)], where (k) is known as Boltzmann constant.

The entropy for a pure substance is determined by keeping the temperature and pressure constant at various values, which is plotted on a scale of Entropy and Temperature, known as T-S plot. The function curves show increasing nature in general. This is consistent with the law of thermodynamics which states that the entropy reaches a constant value only at absolute zero temperature. Therefore, it is conclusive that the entropy is to be strictly increasing.

The entropy used in information theory has a similarity with the thermodynamic entropy. The entropy, or disorder, is a measure of uncertainty involved with the association of probability. As for instance, in the information theory, which is binary, uncertainty is involved with probability between the two states of outcome; such that, at probability{ p=0 }, the term { p.log(p) } is also zero and so there is no uncertainty; and at { p=1 }, the entropy is zero as well. This in principle agrees with the thermodynamic property of entropy, which is a state function that is decided by the initial and the final states of equilibrium of the given system, as is a pure substance alone.

Thus, the entropy function is a bell-shaped curve, reaching a maximum value at about the middle, originating and ending at zero. The argument is justified with the example of an event either not happening or happening. The entropy is viewed akin to uncertainty, such as with the toss of a coin and throw of a six faced dice, where the later is supposedly more for the number of probable states being more as compared to the former.

Thus: there is a similarity as well as a difference existing between the entropy and the uncertainty. The entropy is a measure of disorderliness, about the indeterminate, which constitute the various states of a given system and that must be clearly distinguishable. This is unlike the uncertainty, which is supposed to be complex and hence an inseparable entity in relation to the probability concerned. That is the reason the uncertainty is also known as a measure of probability. Whereas, the entropy is a function of the system state and is fixed for a particular instance. And every pure substance, as is a system, is identifiable by any two physical properties and the entropy as a property is invariable at that point in space. Though the uncertainty is related to the probability and so is time varying. And since the entropy in information content is analogous to the thermodynamic property of entropy for any pure substance, the difference is considered equally alike. This is the most important distinction, which needs necessarily to be resolved. And so the question remaining to be answered is that, if uncertainty is defined as a measure of probability, whereas entropy is a physical property of state for a known system, then what is the functional relation that must exist between the two entities at all times. And then, what are the other related measures of probability and how these can be of significance, in gaining precision and accuracy, remains to be found.

The theory of knowledge stems from the conceptualization of truth, the formalization of the problem statement from scepticism, analysis of the source of knowledge and determination of criteria for the knowledge. The requirement, as such, is to attain perfect knowledge, where there are no errors to be present and this ensures no doubts to exist at all. And so there is subjectivity as a psychological belief, by itself. Thus, the certainty has different degrees, which stands as a measure of the possibility.

The uncertainty is the lack of certainty, resulting from either partial observations and/or ignorance due to the imperfect or unknown, which gives a measure of the probability as well. And the uncertainty gives an error margin from the observation values, which indicates an addition or subtraction of the random variables. The measurement of uncertainty is by taking the set of all possible outcomes, where probabilities are assigned to each of the states. The acceptable definition of risk by the international community, is the effect of uncertainty on objectives, which includes the probability of events and the benefit or loss resulting thereof. And the internationally acceptable definition of entropy, in information content as given by Shannon is: [s = -x.ln(x)]; similarly, the thermodynamic entropy as given by Boltzmann is: [s = -k.log(w)]. The uncertainty is synonymous with vagueness and ambiguity and so on, as portrayed by subjective logic that is given by the well known 'fuzzy' logic introduced by 'Zadeh', Reference [5].

Thus: even without any contradiction or inconsistencies to be present, there may be many a theory that we all well agree; but here is the single theory, the candidature of which is mathematically correct and logically right and this ought to be formalized using the philosophy of science alone.

(ii) Basic assumptions

Extract from Reference [4].

The definition of a system is that at which all our attention is focused; and a logical complement gives the environment surrounding the system. The unreliability of a system stands for the reliability of the surroundings as another system. The instantaneous system reliability is expressed as the exponential of the failure probability, where a minus sign is introduced by convention. The initial condition, at the origin, gives the system reliability as one that is a system at start is good as new.

The mean time between failure of a known system is a mathematical inverse of the failure probability, that can be expressed as the cumulative failure rate, which is found to be linearly increasing with time. The failure probability of a system is relatively a constant.

However, if the failure probability exhibits a change, possibly ranging from (0.50x) to (2.0x), where (x) is the failure probability for a single unit, then the rate of change or gradient of the system reliability curve will vary accordingly. This implies that for gradual changes in the failure rates, the system reliability shows comparatively gradual changes over time.

(iii) The axioms

Extract from Reference [4].

A1 : The certainty is given as: [c = exp(p. ln(p))], which is known as the power function, with the base and exponent as (p), where (p) is the probability.

The certainty, as apart from the probability of an event, which is due to the existence of belief as some function of possibility.

The certainty is unity when the probability of an event is fully known that is when it is either one or zero. The certainty is comparatively lesser when the outcome of an event is not known.

The certainty of unreliability is akin to the certainty of the surroundings. Likewise, there is the uncertainty for the system and for the surroundings as well.

A2 : The uncertainty is a complement of the certainty, as: [u = 1 – c], such that the sum is unity.

The uncertainty, as apart from the probability of an event, is due to the presence of ambiguity or vagueness about the complexity of the environmental conditions. The uncertainty does not exist when the probability of an event is fully known that is either one or zero, depending upon the event to have or have not occurred. The uncertainty is all apart from the certainty. The uncertainty is maximum in between the event happening and not happening, which implies there is a normal distribution, having a central tendency, as is the most usual.

The certainty and the uncertainty are bound by the axioms of the field of real numbers between 0 and 1, as for the subsets of probability.

Therefore, the maximum of the uncertainty corresponds to the minimum of the certainty on the same domain scale.

The probability and all the related measures are guided by the field axioms so formed by the super-set of possibility.

The probability of 0 and 1 stand for the possibilities with the same value as well.

The second order certainty is placed higher valued than the first order certainty.

The implication here is that, the power of a power is given by multiplication of the power terms, which are therefore placed higher valued, respectively.

The minimum of certainty, for any value of underlying failure probability, is not arbitrary, but a constant.

A3 : The minimum of certainty is the least upper bound that is known as supremum; and the maximum of uncertainty is the highest lower bound or infemum.

The minimum of certainty is observed to be a universal constant at 0.69, approximately.

The maximum of uncertainty is a universal constant at 0.31, approximately.

The supremum of the certainty and the infemum of the uncertainty do not intersect.

The supremum of the certainty and the infemum of the uncertainty coincide on the domain scale, at the point of equality of the underlying functions of failure probability and improbability, respectively.

The point of equality of the reliability and unreliability is not same as the point of equality of the failure probability and improbability, respectively.

A4 : For a single unit system, the power function of the probability intersects the exponential function of probability that is the reliability function at the point of supremum; and the corresponding uncertainty function intersects the unreliability function at the point of infemum.

For a single unit as a system, the power function of the failure probability is same as the system reliability at the point of supremum, which is at 0.7, approximately.

Likewise, the complement of the power function of probability and the unreliability hold the same value at the point of infemum that is at 0.3, approximately.

However, if the failure rate changes from unit value, the system behaves as a system of two units, connected either in series or in parallel and the power function, is accordingly placed as higher valued than the originating function, which is given by the instantaneous system reliability.

A5 : The power function of reliability gives the certainty, denoted as: [cr]; also, the power function of unreliability gives the second function of certainty, which is denoted as: [cur].

A6 : The [cr] is always greater than the reliability, but the [cur] is lesser than the reliability for the initial part and that intersects and exceeds the reliability value of 70 percent, which is at about 35 percent on the domain scale starting from the origin, for a single unit system.

A7 : The certainty of the reliability and the certainty of the unreliability intersect at a value of 70 percent that is at about 70 percent on the time scale from the origin, for a single unit system.

A8 : The reliability for a system of two identical independent units, connected in parallel, does not reach the minimum value corresponding to the exponential of -1, hence, the certainty function does not reach a minimum, anywhere within the system reliability life.

A9 : The reliability for a system of two identical units connected in series reaches a value corresponding to the exponential of -1, at the mid life that is 50 percent on the domain scale starting from 0, where the certainty function also reaches a minimum.

Therefore, there is a definite advantage for a system of two identical units connected in series, that the certainty is relatively high, comparable to that of a single unit system, until reaching the mid-life, or about 50 percent on the domain scale, which demarcates the useful reliability life of the system.

A10 : Risk is inverse of the reliability, written as: [k = 1/r]. The risk is always beyond the value of one, which is the upper bound that defines the closed field of probability.

A11 : The complementary function of the risk is the development, which is in the negative quadrant, written as: [d = k -1], such that the difference is always unity.

A12 : The highest value of risk is 2.718, or 2.7 approximately, which is reached at the end of the reliability life for a single unit system; and the corresponding highest value for the development is -1.7, approximately.

It is to be noted, that the limiting value for the risk function corresponds to the exponential of -1, taken for all systems in general.

As the reliability function shows a shallow gradient or the rate of change, the risk function also show a similar change.

A13 : The risk reaches the limiting value of 2.7, for a system of two identical units in series, which is at the mid life or 50 percent reliability life from the origin; and for a system of two identical units connected in parallel, the limiting value of risk that is 2.7, is not reached during the system reliability life.

A14 : The development limit is set at -1.7, which is at the end of the reliability life for a single unit; and this value is reached for a system of two identical units in series at the mid- life; and not reached anywhere within the reliability life for a system of two identical units in parallel.

A15 : The stability is inverse of the unreliability, written as: [t = 1/ur], which is at finitely large value at the beginning and decreases, converging towards the end of the reliability life.

The stability for a system of two identical units in parallel is higher valued than that of a single unit, which is again placed higher, when viewed in comparison to a system of two identical units in series.

A16 : The limiting value of the stability given as 1.58, which is for a single unit at the end of the reliability life, reaches early for a system of two identical units in series, at about the mid-life and never attains for a system of two identical units in parallel. A17 : The ideal safety function, as is defined by the Euler equation, written as: [SFi = cos(x) + ***i.***sin(x)], where the argument (x) take the values of system unreliability [ur].

It is to be noted, that the safety function is placed within a complex field, which is bound at both ends between one and two.

It is also to be noted, that the Euler formula equates to: [exp(***i.***x)], where (x) is the unreliability value, but that leads to an even higher valued function relative to the actual and hence is discarded.

A18 : The actual safety function is written as: [SFa = 1 + uc], where [uc] is the system uncertainty. Thus: [SFa = 2 – c], which is known as the twos-complement of certainty. And the actual safety function is placed lower valued than the ideal safety function.

A19 : The ideal safety function has a limiting value, which is reached at the end of reliability life for a single unit, that is at 1.39, approximately.

A20 : The ideal safety function given by the Euler formula for the argument as the failure probability [p], reaches a maximum value of 1.414, which is a global constant, located at about 80 percent on the domain scale starting from zero and that decreases thereafter.

Whereas, the actual safety function, for a single unit system, has a limiting value of 1.3, approximately, which is reached only at the end of the system reliability life.

A21 : The difference between the maximum of the ideal safety function and the actual safety function value, gives the safety factor [SF], at each instant of time; and that begins at a value of 40 percent at the origin and decreases to about 10 percent by the end of reliability life, for a single unit system.

It is to be noted, that systems with varying failure rates will follow the same principle, where the limiting value, or the safety factor, is taken as that given by a single unit system only.

A22 : The safety margin is defined as the difference between the maximum uncertainty and the instantaneous value of the average uncertainty, written as: [SM = Umax – Uav].

A23 : The likelihood is defined as the ratio of safety margin and the maximum value of uncertainty that is 0.3, taken as a global constant, that is written as: [L = SM/ Umax]. The unlikelihood is defined as the difference between the uncertainty of the system and uncertainty of the surroundings, given as: [UL = Usystem – Usurroundings].

It is to be noted that the likelihood and unlikelihood are not complementary functions, as such.

A24 : For a system to be functioning satisfactorily, the likelihood and unlikelihood are both supposed to be positive. The likelihood is always positive for a system, but unlikelihood can have negative values also, during a system operational life. A25 : The unlikelihood function gives rise to time stipulations, thereby dividing the reliability life for a known system into intervals that is hence expressible in the form of a two-dimensional array.

A26 : The root function, as is the logarithmic transformation, gives the new entropy function, written as: [s = exp.((1/x). ln(x))], where (x) is the system reliability.

It may be noted that for any variable (x), a root function in the logarithmic transformation gives the same result as a power function with the exponent as an inverse of the base variable.

A27 : The other entropy function is given by the variable (x) as the unreliability; and their respective complements result in two pairs, forming the upper and lower bound each, for the reliability and unreliability functions; the average entropy function, thereby resulting from the upper and the lower bounds, give rise to the new reliability and unreliability functions, respectively.

A28 : The point of intersection of the reliability and the unreliability is same as the point of intersection of the average entropy functions, therefore, there is no entropy change at the point where the reliability and the unreliability are equal in magnitudes.

Further, the certainty function also gets modified from the average entropy function which so form the new reliability and the unreliability functions.

Furthermore, with a modification to the originating function of reliability, the underlying failure probability gets changed and so the normal distribution function also changes shape, accordingly.

A system output may also be viewed akin to the probability variable, with a failure as the only measurable entity, where the input serves as complementary, such that their sum is always unity, which holds true by the conservation of matter and energy. The exponential of the output-to-input ratio, with a negative sign added, is well known as the efficiency parameter for a given system.

The system efficiency function is a decreasing function over time, within the field of real numbers between one and zero, as is analogous to a system reliability function also, in general.

It may further be noted, that if a power function is taken with the base and exponent as the efficiency, symbolized by [eta] and then the exponential is taken with a minus sign, then the resulting function is multi-valued and therefore complex. And such a function may either return the same value for two different inputs or constantly generate a single value, or else may be tending to infinity but truncated at the boundary of the field limited to one, for infinitely large number of inputs and so on.

And so the exponential of exponential is a complex valued function, which can be substantiated if and only if there is necessary and sufficient reason and not otherwise.

The entropy difference gives the new function for the probability of success or exponential of output-to-input ratio with a minus sign, that also represents the standard distribution function of the Gauss-Normal form, that is a generalized form of the standard Normal distribution alone, as is defined by a unit coefficient and exponential of the square of the variable with a minus sign, where the variable is the probability term.

The assumption here is that the probability is a linear function, such as the output function and is decreasing as well, whereas the exponential of the variable with a minus sign that is the output-to-input ratio, is curvy-linear and decreasing, such as a non-linear squared variable also.

A29 : The integral area under the ideal uncertainty function, defined as: [- x.ln(x)], is the area which is lesser than that of the standard Gauss-Normal distribution, which is also akin to a reduction in the reliability by the uncertainty term, in general and is found to be: (1/4) or 0.25 square units, approximately.

The reduction in area under the reliability curve is to be lesser than or equal to 0.25 square units as is observed by the actual uncertainty function given by: [u = 1 – exp(x.ln(x))] , which begins at zero and reaches a global maximum at 0.3, approximately, that again reaches zero value at the end of reliability life for a system.

Corollary to A29 : The area under a non-linear function of uncertainty is supposedly equal to or less than 0.25 square units as is given by the integration; but not less than the area under the linearised form of the curve, which sets the limit as 0.15 square units.

A30 : The actual uncertainty, which is a complement of the certainty, is lower valued than the ideal uncertainty given by the well known function: [- x.ln(x)], implying that their infemum or global maximum are lesser valued, respectively, since both the functions are bound alike at the two ends.

Corollary to Axiom 30 : The area under the entropy difference function is proportionate to the area under the average uncertainty function; such as these functions are always analogous.

(iv)The five laws

Extract from Reference [2].

There are the five rules -- as laws that govern the linear relations for the partial measures -- that have been well formalized for the linear relations.

First law: gives the maximum values, for the average uncertainty and the entropy difference as 0.3 and 0.5, respectively.

Second law: is for the complement of the range of probability that is the difference between the averages of the certainty and the uncertainty, which is given as twice in value of the average uncertainty. This means, the average uncertainty is half of the difference between the averages of the certainty and the uncertainty.

Third law: gives a limiting value of 0.5, for the difference function and also its one’s complement that is formed between the averages of the certainty and the uncertainty functions each. Fourth law: states that the average uncertainty value is 0.58 times the entropy difference.

Fifth law: states that the equation for the line of entropy difference for linear systems is: ($ds \leq dx$) and for non-linear systems is: ($ds \geq dx$) ; where (ds) is the entropy difference and (dx) is the distance on the x-axis, taken from the origin at zero. A corollary to the fifth law: states that the cumulative probability qualifies both as a linear and a non-linear system.

And this corollary provides justification in the approximation of the entropy difference in the form of a linear equation.

It may well be noted here, that there is an exception to these laws that is the exponential system reliability, which cannot be approximated as to be linear, even though the cumulative probability as the originating function is supposedly linear.

And it is also to be noted, that the coefficient of the exponential which is written for the system reliability, is the deciding factor for the linearity or non-linearity, whether that coefficient be less than or greater than one.

And it may be recollected from the earlier proofs that, the coefficient for the exponential system reliability represents the nature of configuration of the constituent units as the sub- systems, according to the present load-sharing mechanism among these individual units; as is also known as an independent and identically distributed system.

And now, we have right before us, a relationship that exists between the partial measures, as dual functions, for the sub-set of possibility.

(v)The sixth law.

Extract from Reference [1].

There are the five laws that give us a linear relationship among the partial measures of the sub-set of possibility.

Well, we do admit, that the greatest difficulty faced so far, is to find a connection -- that exists and is also to be unique-- between the partial and the total measures in the field of probability. And we ourselves realize, that if there be any such connection with the total measures also, then and only then will we be able to reach a complete closure.

A new law, that can relate the average entropy with the entropy difference, appears to be a near possibility indeed.
The reason for this proposition arises from the fact that, the difference function of entropy, instead of being afar, rather lies in close proximity to the average function of the entropy.
The sixth law that is discovered, gives the average entropy and the entropy difference to be exactly matching, which is though from the beginning up to reaching the point of equilibrium for a system.

Now, at the point of equilibrium, there is formed a singularity, which is the point where the average entropy and the originating functions are found to coincide.

And if we refer to mathematics, we find the well known mean value theorem, which states that, there is supposed to be at least one point where the slope of a closed curve that is between two fixed points, or the boundary points as in our case,must be equal to the slope of the straight line joining these two points.

Moreover, since there exists a singularity, which is apart from the end points at the boundaries that is under consideration, as is for a third point, the mean value theorem naturally becomes twice applicable. That means, there are at least two points between the initial and the final boundary points where the slopes are same as that of the line joining the two end points.

And this implies, there must be three points on the curve joining the end points at the boundaries, where they equate in values, with the straight line that connect the points.

And so, we have a justification right here, in an assumption of linearity for the function of average entropy, which we have propounded earlier.

And then moving further, we find the average entropy function to be equal in slope with the originating function, which is given by the cumulative probability.

And further, the entropy difference is found equal in slope with the originating function as well.

And equating the two, mathematically, we arrive at the lower bound of the entropy as equal to one-third in value of the upper bound of entropy; and here forms a law which is entirely different.

The new law states that, the entropy function value of the upper bound is thrice the lower bound; at any given point and for all systems. This is written as: ($s1 = 3.s2$).

The most important deduction, which algebraically follow the sixth law, is that, the average entropy is twice the lower bound of entropy in value.

And this deduction is valid for the initial part of the system reliability life only that is starting from the origin till reaching the equilibrium state.

The same rule that is valid for the initial part starting from the origin till reaching the equilibrium, is naturally to hold true for the later part of a system reliability life as well; where, the real reason for that, may be found from the very symmetry of the particular function curves that exist, in general.

Now, it is well observed, that for the later part of a system reliability life, which is starting from the equilibrium till the end, the entropy difference is decreasing from a value of0.50 to 0 ; and the corresponding average entropy is increasing to reach a value of 1 at the end point of the boundary.

So then, for the later part of the system reliability life, we may logically take both the functions to be increasing, with the average entropy equating to the one's complement of the entropy difference.

Here, the inference that follows, for the later part of the system reliability life, may be expressed as: (s2 = 3.s1 – 2) ; where (s1) is the upper bound and (s2) is the lower bound given by the entropy functions.
Sixth Law: The entropy function values given by the upper and lower bounds are : [s1 = 3.s2] for the initial part ; and [s1 = (s2+ 2) / 3] for the later part of a system life ; where (s1 > s2).

And checking, at the point of equilibrium, where the sixth law is supposed to be valid for both the initial and the final parts, we get: (3.s2 = (s2 + 2) / 3) ; that gives: (s2 = 0.25) ; and (s1 = 0.75) ; and the average entropy as: 0.50 ; that hence proves the singularity.

Well, then, the justification that we mentioned above, becomes also evident from the mean value theorem which holds true for the two segments, equally alike, as is given by a single line through three points, which are the initial, the singularity and the final point as such.

And that justification gets further strengthened, by the fact that a complementarity provides for a mirroring of the images in all such instances.

Thus, there is a complete closure, by the five laws and now the sixth one, for the partial measures and the total measures also, in the given sub-set of possibility, in general.

Let us analyse the sixth law by contradiction. If we consider the functions of difference and the average, we will find that the average remains unchanged if the difference is either increasing by divergence, or decreasing by convergence, where each are supposed to be in equal proportions. The average function changes in values, which means it is non-constant, if and only if the difference changes in a dis-proportionate manner, such as, if the infemum increases but the supremum decreases other than in proportion. But this finding is in contravention to the logical complementarity that exists in a well defined real field.

As a proof of what is just said, if we consider a function of power of the power function, then the infemum shifts higher and the supremum, which is a one's complement, decreases to a lower value, equally alike; where the average of these remains indifferent at 0.50, as a uniform constant.
Hence, the supposition, that the difference equals the average, for so long as a ratio that is linear, is maintained, gets itself proven.

Hence, starting from a single value -- as given by the lower bound of entropy taken at an instant of time -- the analysis begins, is proven.

Now, if we consider the augmentation as a recursive process, the average entropy is observed to take on values which tends to more and more non-linearity; where the non-linearity arises from the originating function being linear.
And as we have stated before, successive recursions, as in iterative loops, even if the generation of which might be uncontrolled or automatic, these we must leave.

And the reason for that is, the linearity ought to be retained, in order for the linear relations to hold.

And so, we have found: a limit for the sixth law: for the induced function of the average entropy alone, where all the recursive functions that might get successively generated are by all means to be ignored.

It is once again to be noted, that this limit for the sixth law, valid for the total measure, does not exist for the first five laws, which are for the partial measures only, in the given sub- set of possibility.

Hence, although we are neglecting the successive recursions of the total measure that is by the process of augmentation, still, the successive steps for the partial measures that is by the process of compression are to be retained; where, as may be recalled, that a compression process tends to attain more and more linearity and this cannot be left at all.

And with that, we have reached a full and fair closure -- to the picture given by the most single theory thus established by the six general laws -- wherein, beginning with a single instance and then proceeding to all the others, right from the initial to the final states, get generated analytically, for all real time systems which are memoryless and bounded as well.

III. EXPLANATION

(i) Generation of data

The data used in the project, [1] and [2], have been generated from real numbers approximated to three decimal places in a bounded interval between 0 and 1, with a uniform spacing of about 0.5 percent and that is not taken from any empirical database for which a citation may be called for.

(ii) Rules of logic

To site an instance, the statistical information based on all available resources, on Covid-19 Coronavirus pandemic by the end of 2021, the number of deaths reported in the world population have steadily risen to approximately 5.2 million.
The project, [1] and [2], presumes the cumulative frequency function to follow the rules of logic, in either increasing or uniform manner, where the system failures have been taken into consideration.

As it is an accepted fact, that probability is associated with information and so a cumulative frequency should have a failure probability that is attached. Hence, the cumulative frequency is expressed as a ratio, such that it lies in the bounded interval given by the probability. Further, the bounds of the probability interval, which is inclusive and closed, is as defined by the super-set of possibility.

(iii) Theory of evidence and the belief

As per the same instance of statistical information based on all available resources, on the effect of Covid-19 by the end of 2021, the number of world population with Covid-19 cases is about 267 million and the number recovered is 240 million, approximately. It so reveals, from the evidences, that there are no deaths among the already recovered.

The theory of evidence, popularly known as Dempster-Shafer theory (DST) draws a connection between uncertainty and probability, where each (point) is an evidence assigned with a degree, which is analogous to a mass that is attached, based on all the available evidences. There are the two non-additive continuous measures, which bind the probability interval, termed as Plausibility and Belief; where the Plausibility gives the upper bound and the Belief gives the lower bound for the probability. The two measures are the logical complements, Reference [5].

(iv) Conflict of interest

The theory of evidence is not explicit about the mass, attached to an evidence, if the evidence is zero; where the supposition is, the degree of evidence, or the Belief is a continuous measure of the probability. There appears a conflict, with the Belief of an evidence and the Belief of negation of the evidence, which may be written as: Bel(A) and Bel(~A), where (A) is a non-zero element of the set of probability. The conflict of interest appears as the Plausibility is shown to be complement of the Belief of negation of the evidence. The fact remains, that the evidence theory gives the measures of Plausibility and Belief to be non-additive, where Plausibility is greater than Belief, as is the two bounds of probability.

(v) Resolution of conflict and the uncertainty function

The conflict of interest is resolved by a presumption that the measures of Plausibility and Belief are one and same, where a logical complement is necessarily the uncertainty in general, stands to reason. In justification, there are the various synonyms for the uncertainty term, that have been stated to be used, Reference [5]; as for example, a complement of the belief or plausibility, and the likes.

The uncertainty and the complement, as used in the project, are the logarithmic power functions and the complementary parts thereof.

The negation of evidences, as is suggested by the evidence theory, that have also been taken into account.

(vi) The entropy function

The entropy or disorder is found to exist in the universe, as is propounded by the second law of thermodynamics, which is meant to ever increasing. Likewise, there is the entropy in information theory, where it is shown as a strictly increasing function, Reference [5].

The entropy functions, used in the project, [1] and [2], are the logarithmic root functions and the complementary pairs, including the respective negations, where applicable. The entropy functions are shown as the total measures of probability, that is unlike the uncertainty functions which are the partial measures only.

(vii) Linearity assumption

In the project, [1] and [2], the difference functions are found to be resulting from the corresponding entropy functions, the uncertainty functions and the originating functions of probability, respectively. There is the linearity assumption, with the underlying cumulative probability taken as linear, that holds inasmuch true. The difference functions, have given rise to a framework for a connection between the partial and the total measures, of the super-set of possibility, that is enumerated by the laws that govern the field axioms, as is sited in the examination, section {2}.

(viii) Average functions

In the project, [1] and [2], with a systemic viewpoint, the average functions are computed, for the reliability and the unreliability, such as for the measures of probability of failures, as well as the successes. The special theory evolves from the finding of a single value of supremum or infemum as a global maximum or minimum, that serves to be a universal constant, which is for the partial measures. Likewise, for the total measures, there is found to exist a single point of intersection that is an arbitrary constant in time, which is the point of equilibrium and that is decided by the system reliability for all the average functions, as is a point of entanglement. Further, there is a finding for recursions or successive loops of iterations for the average functions, where though the total measures exhibit augmentation or enhancement that approaches away from linearity; but the partial measures show a compression or contraction that tends more towards linearity. And this finding justifies the assumption of linearity, which is taken for the partial measures.

(ix) Mathematical representation of six generalized laws

For the average uncertainty (du), average certainty (dc), entropy difference (ds) between upper bound (s1) and lower bound (s2), distance (dx) from origin at zero:

(L1) : Max (du) = 0.3 ; Max (ds) = 0.5 ;
(L2) : (1 – dp) = 2(du) ;
du = (dc – du) / 2 ;
(L3) : Max (1 – dp) = 0.5 ; Max (dc – du) = 0.5 ;
(L4) : du = 0.58(ds)
(L5) : ds $\leq$ dx for linear systems ;
ds $\geq$ dx for non-linear systems ; (L6) : [s1 = 3.(s2)] ; for 0 $\leq$ dx $\leq$ q ;
[s1 = (s2 + 2) / 3] ; for q $\geq$ dx $\geq$ 1 ;
where s1 > s2, and point of entanglement (q) on domain.

(x) General and special considerations

The system reliability have been taken into account in the project, [1] and [2], at each step, where the standard form of exponential equations are used in place of the cumulative failures for probability. The general form of the theory is applicable for the originating probability function having uniform failures. The theory in the special form is meant for specific system reliability with a variable failure rate, depending upon the configuration or the architecture.

IV. CONCLUSION

This paper is not a comparative study of a new theory or any contradiction to contemporary theories, rather, this gives the methodology, arising from an indeterminate that is the probability in the real world scenario and the disambiguation that is the uncertainty and the entropy prevailing, by the six generalized laws; the ingenuity and uniqueness of the single solution, hence determined, is fully proven.

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