Fuzzy Graph And Their Applications: A Review

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Abstract- *Following the introduction and development of fuzzy sets, various investigations on this subject were conducted, with the end result appearing as a Fuzzy Graph (Combination of graph theory and fuzzy set theory). In this post, we will look at some of the most important papers on various types of existing fuzzy graphs.*

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I. INTRODUCTION

Euler presented the notion of graph theory for the first time in 1736. In the history of mathematics, Euler's answer to the well-known Konigsberg bridge issue is regarded as the first theorem of graph theory. This is now widely considered as a part of combinatory mathematics. The theory of graphs is a powerful tool for addressing combinatorial issues in a variety of fields, including geometry, algebra, number theory, topology, and operations research, optimization, and computer science.

Zadeh released his fundamental article on "Fuzzy sets" in 1965, which defined fuzzy set theory and, as a result, fuzzy logic. The goal of Zadeh's study was to provide a theory that could deal with the ambiguity and imprecision of some types of sets in human thought, notably in the realms of pattern recognition, information transfer, and abstraction.

Rosenfeld (1975) proposed the idea of fuzzy graphs as well as various fuzzy analogues of graph theoretic notions such as route, cycles, connectedness, and so on. The notion of fuzzy relations was presented by Zadeh (1987). The notion of fuzzy automorphism graphs was examined by Bhattacharya (1987) and Bhutani (1989). McAlester (1988) proposed a fuzzy intersection graph extension of intersection graphs. Mordeson (1993) discovered the essential features of fuzzy line graphs and presented the notion. Sunitha and Vijayakumar studied the notion of a full fuzzy graph (2002). Somasundaram studied the idea of dominance in fuzzy graphs (1998). Product fuzzy graphs were presented by Ramaswamy and Poornima (2009).

Atanassov (1999) proposed the first definition of intuitionistic fuzzy relations and intuitionistic fuzzy graphs, which were subsequently investigated in [2009]. In intuitionistic fuzzy graphs, Parvathi and Thamizhendhi (2010) developed the idea of dominance number. Domination in product fuzzy graphs was presented by Mahioub Shubatah (2012). Vinoth Kumar and Geetha Ramani (2011) defined product intuitionistic fuzzy graphs.

A graph is a nonempty set V with symmetric binary relations. A fuzzy graph, on the other hand, is a symmetric binary fuzzy relation on a fuzzy subset. Kaufmann[1] defined the first fuzzy graph in 1973, based on Zadeh's fuzzy relations [2]. Azriel Rosenfeld [3] was the first to investigate fuzzy relations on fuzzy sets and create the idea of fuzzy graphs in 1975. R.T. Yeh and S.Y. Bang established several connectivity ideas in fuzzy graphs at the same time.

Zimmermann [4] has studied various fuzzy graph characteristics. The book "Fuzzy graphs and Fuzzy hypergraphs" [5] by Mordeson and Nair is a good source for study on fuzzy graphs and fuzzy hypergraphs.

This paper provides an overview of the fuzzy graph and its different types. We will simply discuss fuzzy graphs and their types in this section, not their operations and attributes.

II. PRELIMINARIES

2.1 Graph

A graph G is defined as an ordered pair: $G=(V,E)$ where V is the set of Vertices. A vertex is also known a node or component (elements) and E :Set of edges. An boundary is an unordered pair (x,y),of vertices in V.

2.2 Fuzzy Graph

A fuzzy graph G is defined as an ordered pair: *G =(V,E)* where *V* denotes the set of Vertices. A vertex is also known a node or component (elements) and E stands for set of edges. An edge is an component of the fuzzy set E: $X \times Y \rightarrow [0,1]$.

III. TYPE OF FUZZY GRAPHS

Regular Fuzzy Graph (RFG)

In their study "On Regular Fuzzy Graphs," A. Nagoor Gani and K. Radha (2008) proposed regular fuzzy graphs. They introduced regular fuzzy graphs, total degree fuzzy graphs, and absolutely regular fuzzy graphs in their article. Various examples are used to compare regular fuzzy graphs with entirely regular fuzzy graphs. A necessary and sufficient condition for regular fuzzy graph equivalence and characterization was established. [6]

Complementary Fuzzy Graph (CFG)

Moderson (1994) established the idea of complement of fuzzy graphs, while M.S.Sunitha and A. Vijayakumar (2001) provided a modified definition [7]. R. Sattanathan and S. Lavanya (2009) conducted research on complementary fuzzy graphs and fuzzy chromatic number. In their article, they find the fuzzy chromatic number of a fuzzy graph's complement and give bounds for the sum and product of the fuzzy chromatic number of the fuzzy graph and its complement. [8]

Antipodal Fuzzy Graph (AFG)

The notion of Antipodal Graph was established by A. Nagoor Gani and J. Malarvizhi (2010). Smith[8] proposed the idea of antipodal graph of a given graph G in crisp graph theory. Aravamudhan and Rajendran[9] address the condition on the graph G for A(G) = G and A(G)= G. In their article, they defined and explored the antipodal fuzzy graph as a fuzzy counterpart of this.

Constant Intuitionistic Fuzzy Graphs (CIFG)

Constant intuitionistic fuzzy graphs were presented by M. G. Karunambigai, R. Parvathi, and R. Buvaneshwari (2011). Constant Intuitionistic Fuzzy Graphs (IFGs) and absolutely constant IFGs were presented in their study. A criterion for equivalence was established that was both required and sufficient. A description of constant IFGs on a cycle was also provided. Some features of constant IFGs were also examined, with appropriate demonstrations. [10]

Fuzzy Graph Structures (FGS)

Based on the notion of graph structure, T. Dinesh and T.V. Ramakrishnan (2011) presented the concept of a fuzzy graph structure. E. Sampathkumar created a new concept, graph structure, which is an extension of concepts such as graphs, signed graphs, and edge-coloured graphs with colorings. G = (V,R1, R2,..., Rk) is a graph structure if V is a nonempty set and R1, R2,..., Rk are mutually disjoint relations on V such that each Ri, I = 1,2,3,...,k is symmetric and irreflexive. This is the driving force for the investigation of fuzzy graph structures. New notions such as iedge, icycle, itree, iforest, fuzzy icycle, fuzzy itree, fuzzy iforest, iconnectedness, and so on are presented and researched.[11]

Product Intuitionistic Fuzzy Graph (PIFG)

Atanassov [12] presented the first definition of intuitionistic fuzzy graphs. The notion of product fuzzy graph is introduced by Dr. V. Ramaswamy and Poornima.B. N. Vinoth Kumar and G. Geetha Ramani (2011) introduce the notion of Product Intuitionistic Fuzzy Graphs. Look at the characteristics of Product Intuitionistic Fuzzy Graphs further.

Bipolar Fuzzy Hypergraphs (BFH)

Zhang [13] pioneered the notion of bipolar fuzzy sets as an extension of fuzzy sets in 1994. Bipolar fuzzy sets are an extension of fuzzy sets with a membership degree range of [-1,1]. In a bipolar fuzzy set, an element with membership degree 0 is irrelevant to the corresponding property, an element with membership degree (0,1) is somewhat fulfils the property, and an element with membership degree [-1,0] is somewhat satisfies the implicit counter property.

S.Samanta and M.Pal (2012) define certain fundamental notions of bipolar fuzzy hypergraphs, such as cut level bipolar fuzzy hypergraphs, dual bipolar fuzzy hypergraphs, and bipolar fuzzy transversals. In addition, certain fundamental theorems relating to the aforementioned graphs have been provided. [14]

Irregular Fuzzy Graphs (IFG)

S. R. Latha and A. Nagoor Gani (2012) defined There were three types of fuzzy graphs introduced: neighbourly total irregular fuzzy graphs, highly irregular fuzzy graphs, and highly total irregular fuzzy graphs. A comparison of neighbourly irregular and extremely irregular fuzzy graphs was performed. In addition, certain results on neighbourly irregular fuzzy graphs were investigated.[15]

Irregular Bipolar Fuzzy Graphs (IBFG)

S.Samanta and M.Pal (2012) define irregular bipolar fuzzy graphs and the different classifications of these graphs. The size of normal bipolar fuzzy graphs was calculated. It was discovered that highly and neighbourly irregular bipolar fuzzy graphs have a relationship. Some fundamental theorems relating to the graphs in question have also been given. [16]

Fuzzy Labeling Graph (FLG)

A novel notion of fuzzy labelling was proposed by A.Nagoor Gani and D. Rajalaxmi (a) Subahashini (2012). If a graph has fuzzy labelling, it is said to be a fuzzy labelling graph. We've spoken about fuzzy subgraphs, unions, fuzzy bridges, fuzzy end nodes, fuzzy cut nodes, and the weakest arc of fuzzy labelling graphs. In addition, the number of weakest arcs, fuzzy bridges, cut nodes, and end nodes in a fuzzy labelling cycle has been determined. (G) is proven to be a fuzzy cut node and (G) is proven to be a fuzzy end node of a fuzzy labelling graph. It was further demonstrated that if G is a linked fuzzy labelling graph, there exists a strong path between any two nodes. [17]

Strong Intuitonistic Fuzzy Graphs (SIFG)

Muhammad Akram and Bijan Davvaz (2012) define and analyse the features of strong intuitionistic fuzzy graphs. They address several self complementary and self weak complementary strong intuitionistic fuzzy graph notions. They pioneered the idea of intuitionistic fuzzy line graphs. [18]

Interval-valued Fuzzy Graph (IVFG)

Muhammad Akram and Wieslaw A. Dudek (2012) define and explore the operations of Cartesian product, composition, union, and join on interval-valued fuzzy graphs. They introduced the idea of isomorphism (or weak isomorphism) between interval-valued fuzzy networks. They also presented various features of self complementary and self weak complementary interval-valued fuzzy complete graphs and developed the concept of interval-valued fuzzy complete graphs. [19]

Fuzzy Middle Graph (FMG)

The notion of fuzzy middle graphs and its attributes were presented by Veena Mathad and B. Sharada (2012).[20]

Complete Interval-Valued Fuzzy Graph (CIVG)

Young Bae Jun and Hossein Rashmanlou (2013) They presented three new operations on interval-valued fuzzy graphs in their paper: direct product, semi strong product, and strong product. They provided enough circumstances for each of them to be complete.[21]

Balanced Intuitionistic Fuzzy Graph(BIFG)

Al-Hawary [1] developed the notion of balanced fuzzy graphs and investigated several fuzzy graph operations. Atanassov [6] defined intuitionistic fuzzy relations and intuitionistic fuzzy graphs (IFGs). Parvathy and Karunambigai[8] further on the notion of IFG and examined its components. Articles [24, 25, 26] inspired us to investigate balanced IFGs and their features. Their research focuses on the important aspects of balanced IFG. Section 2 discusses the fundamental definitions and theorems required. The need for an IFG to be a Balanced IFG if the graph G is full, strong, regular, and self complementary IFG were explored. They also talked about the features of complementary and self-complementary balanced IFGs. It also discusses the direct product, semi-strong product, and strong product of intuitionistic fuzzy graphs, as well as their features, with appropriate illustrations. [22]

Fuzzy Dual Graph (FDG)

Nuha abdul-jabbar, Jehan H. Naoom, and Eman H. Ouda proposed the Fuzzy dual graph in December 2009. In their study, they studied the notion of fuzzy dual graphs and established the following properties: the dual of the dual of fuzzy graph is the fuzzy graph itself, and the dual of fuzzy bipartite graph is Eulerian fuzzy graph. [23]

Maximal Product of Fuzzy Graph (MPFG)

Sambathkumar proposed on graph structure(GS) $G=(V,R_1,\ldots,R_n)$ consists of a nonempty set V with relations $R_1,R_2,\ldots,R_n$ on set V which are mutually disjoint such that each relation R_i, $1\leq i\leq n$ is symmetric and irreflexive.

Graph structure $G^*=(V,R_1,\ldots,R_n)$ can be represented just like a graph where each edge is labeled as R_i, $1\leq i\leq n$.

Application of Fuzzy Graphs

Identification of the most contentious problems between countries: Nowadays, the globe has shrunk to the size of a hamlet, and all countries are interconnected. Relationships between various countries are not all the same. Some nations have good ties with one another; for example, Pakistan and

China have had a long-standing friendship. However, certain countries do not have excellent connections, and as a result, world peace is jeopardised. Some contentious problems between those countries are to blame for the unfavourable ties. Many contentious concerns, such as line of control issues, counterterrorism actions, nuclear weapon proliferation, power conflicts, religious difficulties, and other nations' occupations, pose a perpetual threat to world peace.

Different nations have numerous concerns with one another, but during this historical period, one topic is particularly contentious and requires the attention of peace-loving groups to be resolved in order for combat actions among those countries to be restrained. India and Pakistan, for example, have several challenges with one another, including the subject of Kashmir, water concerns, religious conflicts, terrorism, power disputes, and line of control issues. The India–Pakistan relationship has been strained at various points in time owing to a variety of concerns. The line of control is now the most contentious issue between Pakistan and India, and it must be resolved as soon as possible.

With the aid of the membership function, we can utilise a fuzzy-graph structure to highlight the most contentious topic between any two nations within a specific time period, and we can also indicate the severity degree of the problem at that time. The fuzzy-graph structure of the most contentious topics can be extremely beneficial to peacekeeping groups and the United Nations in maintaining world peace.

Consider the following eight powerful countries:

S={ North Korea, America, South Korea, Pakistan, Iran, Russia, India, and Afghanistan}.

Many relations may be defined on set S. Let us define the following S relations: R_1 = nuclear weapons proliferation, R_2 = counter-terrorism actions, R_3 = line of control concerns, R_4 = becoming more powerful, R_5 = religious issues, and R_6 = occupying another nation, such that $(S,R_1,R_2,R_3,R_4,R_5,R_6)$ is a graph structure. Each factor in the relationship represents a certain type of most contentious topic between the two countries. Because the network structure is $(S, R_1,R_2,R_3,R_4,R_5,R_6)$, a pair of nations can appear in only one relation. As a result, it would be deemed an element of that connection, whose membership value is higher than that of other relations. Using the above-mentioned data, elements in relations are coupled with their membership values, and the resultant sets are the fuzzy sets on R_1,R_2,R_3,R_4,R_5,R_6, accordingly.

Let

R1={(America,NorthKorea),(America,Iran),
(NorthKorea,Russia),(Pakistan,NorthKorea)},
R2={(Pakistan,America)},
R3={(Pakistan,India),(Afghanistan,India)},
R4={(America,Russia),(SouthKorea,NorthKorea)
},
R5={(Iran,India),(Afghanistan,SouthKorea)},
R6={(America,Afghanistan)}. And the corresponding fuzzy sets are:
μ1={((America,NorthKorea),0.8),((America,Iran),0.7),
((NorthKorea,Russia),0.8),((Pakistan,NorthKorea),0.8)},
μ2={((Pakistan,America),0.8)}
μ3={((Pakistan,India),0.8),((Afghanistan,India),0.5)},
μ5={((Iran,India),0.7),((Afghanistan,SouthKorea),0.5)},
μ6={((America,Afghanistan),0.5)}.
Clearly, (σ,μ1,μ2,μ3,μ4,μ5,μ6) is a fuzzy-graph structure and is shown in below Figure.

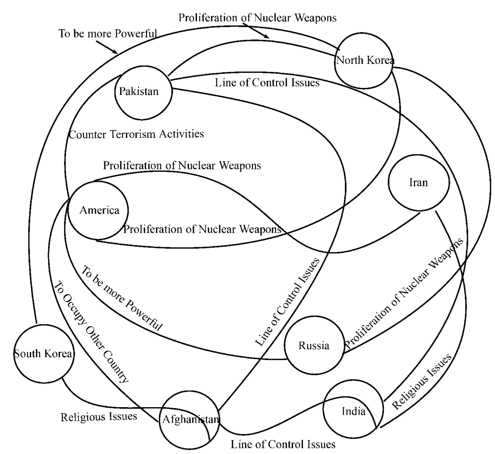

Fig 1: Fuzzy-graph structure depicting most controversial issues among different countries.

Figure 1 illustrates the FGS with each edge depicting the most contentious subject in the associated country. For example, the spread of nuclear weapons is the most contentious issue between the United States and North Korea, and its membership value is 0.8. It should be highlighted that vertex America has the greatest vertex degree in terms of nuclear weapon proliferation. This implies that America's nuclear weapons proliferation is a contentious topic with other countries. Furthermore, according to this fuzzy-graph structure, the line of control problem is now the most contentious topic between Pakistan and India, with a membership value of 0.8. A FGS of all countries can be extremely beneficial to the UN and other organizations in maintaining world peace. It would draw attention to the contentious problems that needed to be resolved as soon as possible.

IV. CONCLUSION

Graph theory has a wide range of applications in a variety of fields, including networking, communication, data mining, clustering, image capture, image segmentation, planning, and scheduling. In other cases, though, certain characteristics of a graph-theoretical system may be unclear. It is quite natural to use fuzzy-graphical strategies to cope with ambiguity and ambiguous concepts. Fuzzy-graph theory offers a wide range of applications in simulating diverse real-time systems with varying levels of accuracy.

In this work, we intend to summaries kinds of fuzzy graphs, fuzzy graph structures, and types of intuitionistic fuzzy graphs, as well as people associated with the corresponding field. We believe that this publication will assist scholars in getting a quick overview of the topic of Fuzzy Graph.

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