Fractal Dimension Calculation of The Image: A Survey

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Abstract- *Fractal dimension (FD) is the ratio for providing the index of complexity by comparing a image or pattern can change in detail with the scale with which it is measured. It can also be defined as the measure of space filling capacity of a image or pattern that explains how the fractal scales differently. In this work we have explained the fractal geometry as well as some well-known methods for calculating the FD for a grey scale image. Fractal geometry provides mathematical representation for many complex objects such as mountain, clouds etc. and for many natural objects also. The fractal analysis has been improved for digital image analysis and mostly applied for texture analysis and segmentation analysis. Fractal Dimension (FD) is mainly used to calculate the minimum reducing factor at which the image or objects can be divided into multiple numbers of self-similar objects.*

Keywords- FD, Grey scale image, Color image, Fractal geometry, Self-similar object.

I. INTRODUCTION

Artificial objects have flat surface which can be defined by the help of polygons or smooth curve surface as per the requirement. But the natural objects often have rough, jagged, random edges which are very difficult to represent with different curve and polygons. For the natural objects or image we require both curve and polygons simultaneously. Trying to draw the natural objects by means of straight lines is difficult.It is must to draw the natural objects the computer or machine should draw the jagged lines. Two end point will be given to the machine then the machine will try to draw the straight line and putting all the points for the objects around the line. Finally by using any best fit algorithm we need to find fount two closest point to the line for calculation of approximate value of the FD [1]. Then by applying the Euclidian algorithm the distance between the two points will be calculated which is known. Now a days it is possible to find out the self-similarity of the natural image by the help of the above said concept called as *fractals,* and in this section I have given the brief description regarding FD and fractal geometry.

Role of scaling: The concept of a fractal dimension is in unconventional views of scaling and dimensions per the research done in [2]. **Figure 1** illustrates, traditional notions of geometry dictate that shapes scale predictably according to intuitive and familiar ideas about the space they are contained within, such that, for instance, measuring a line using first one measuring stick then another 1/3 its size, will give for the second stick a total length 3 times as many sticks long as with the first .This holds in 2 dimensions, as well. If one measures the area of a square then measures again with a box of side length 1/3 the size of the original, one will find 9 times as many squares as with the first measure. Such familiar scaling relationships can be defined mathematically by the general scaling rule in Equation 1, where the variable N stands for the number of new sticks, ϵ for the scaling factor, and D for the fractal dimension:

$$N \alpha \epsilon \qquad (1)$$

The same rule applies to fractal geometry but less intuitively. To elaborate, a fractal line measured at first the line needs to be divided into one part and then 2 part and so on. Finally one condition will be there where it is impossible to divide the whole line into self similar parts. That condition is called as self similarity if the given image with a defined reduction factor. to be one length, when remeasured using a new stick scaled by 1/3 of the old may not be the expected 3 but instead 4 times as many scaled sticks long. In this case, N=4 when ϵ=1/3, and the value of D can be found by rearranging

Equation 1:

$$log_{\epsilon}N = -D = \frac{\log N}{\log \epsilon} \qquad (2)$$

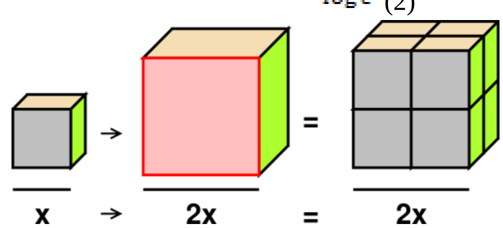

Fig 1 Scaling behavior with the reduction scale

Self similarity: In the mathematics field the self similarity is nothing but the concept of dividing the whole image or object into equal part. After the reduction all the portions of the image will be then added together with a proper magnification factor by means of which the final image can be achieved. Self-similarity is nothing but the concepts which will be used whenever the fractal is to be calculated of a give image of some specified length and pixel. For instance, a well known example of Koch snowflake having both symmetrical and scale-invariant is shown in the fig 1.2 for understanding the self-similarity concept. It can be continually magnified 3x without changing shape. The non-trivial similarity evident in fractals is distinguished by their fine structure, or detail on arbitrarily small scales. As a counter example, whereas any portion of a straight line may resemble the whole [20].

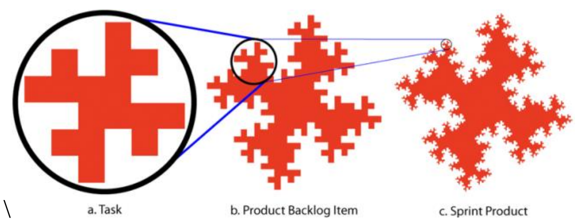

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Fig 2Self similarity of image

Fractals are patterns that feature geometric elements at ever smaller scales to produce both self-similar and irregular shapes and surfaces. Fractal shapes are often self-similar (segments look like each other and like the whole object) and independent of scale (they look similar, no matter how close you zoom in).

II. RELATED WORK

In [2] an investigation of acknowledgment of plant leaf pictures is a significant and troublesome errand. Extricating the surface component of leaf pictures turns into the way to tackle this issue as of late. Considering some wavelet strategies just spotlight on low-recurrence sub-groups of pictures and some fractal measurement strategies utilizing a solitary type additionally can't recognize the pictures well, a novel wavelet fractal highlight based methodology for plant leaf pictures acknowledgment is proposed. Right off the bat, the preprocessed leaf pictures are pyramid decayed with 5/3 lifting wavelet change and sub pictures are acquired. At that point fractal measurements of each sub pictures are determined to be the wavelet fractal highlight of leaf pictures. At last back proliferation fake neural system is utilized to order plant leaf pictures. The exploratory outcomes show that the proposed strategy can improve the presentation for plant picture acknowledgment contrasted and strategies utilizing just wavelet or then again fractal measurement. In [3] an investigation of Fractal measurements of leaves from Cercis canadensis L., Robinia pseudoacacia L., Amelanchier arborea (F.Michx.) Fernald, Prunus persica (L.) Batsch, Quercus alba L., Carpinus caroliniana Walter, Ficus carica L., Morus rubra L., Platanus orientalis L., and Ulmus rubra Muhl. were determined. The qualities were then affirmed and thought about by those acquired from box-checking strategy and the example estimations of thickness relationship work (first time in the writing). It is currently proposed just because that there is a connection between a fractal measurement of the leaf and a surface thickness of the picture and was presumed that along with different measures, the fractal measurements with surface thickness capacity could be utilized as another way to deal with taxonomical investigation of plants. In [4] talked about on life forms bolster constant trade with the earth so they keep up in a state a long way from their thermodynamic balance.

The plants keep up themselves under low entropy conditions, a vital essential to life. The idea of fractal measurement to portray structures, which appear to be identical at all length scales, was first proposed by Mandelbrot Objects are normally alluded to as self-like demonstrate their scale-invariant structure. The normal attribute of such fractal objects is that their length relies upon the length scale used to measure it, and the fractal measurement reveals to us the exact idea of this reliance. Estimation of fractal measurement of leaf shape was as of late performed structure different creators. We evaluated Fractal Dimension of various types of leaves taking a gander at their internal structure until to the cell core. In [5] a survey talked about a picture examination strategy dependent on the case tallying calculation was assessed for its latent capacity to portray grapevine leaves. In spite of the fact that vine leaves come up short on the self-comparability of the hypothetical fractals, leaves are contender for portrayal utilizing fractal investigation on account of their exceptionally mind boggling structure.

In the field of image processing different researchers done their experiments using fractal by using different method In [3,4] Gangepain and Roques-carmes describe the reticular cell counting method for calculating the FD of grey scale image**.** But in [5] that was improved by Voss by incorporating probability theory. Ivanovic in [6] had described the idea of random midpoint displacement algorithm using Gaussian random variable to calculate the luminance of images and describe the h-parameter for differentiating the scaling behavior of the image. Sarkar in [7-9] had introduced one more method for calculating FD that is known as Differential box counting (DBC) method. Due to its simplicity, automaticity and computability the above said method is being used in various application field [10].

Pentland in [11-12] had proposed a method for calculating the FD by using the Fourier power spectrum of surface intensity, with the assumption FD=2 for smooth surface and FD=3 for maximum rough surface.

In [13] several evaluation methods has been proposed by Stefan Jansson for estimating the fractal properties of high intensity image i.e., for the natural image. In the research work provided in [13] nine different methods had been introduced for calculating the FD.

III. CALCULATION OF FRACTAL DIMENSION

Fractal dimension of an image is the self-similar aspects as shown in the fig 1.2. Fractal dimension is defined as the set, for which the Hausdroff dimension is larger as compared to the topological dimension of the same image. FD is is nothing but the approximate smallest size of the self similar image which has been broken down from the original image of some specified length. The self similarity is the method of reducing the actually image of some specified length into smallest pieces of equal length. This concept is being used for calculating the FD of a grey scale image and of a color image as well.

A fractal set A is the union of N_r self similar non overlapping copies of the total image with a reducing factor r. The non overlapping copies will be scaled up or down according to the reducing factor r. The Fractal dimension (FD) is calculated by using the given formula.

$$FD = \frac{\log(N)}{\log(\frac{1}{r})} \qquad (3)$$

Where N is the total number of boxes of size L of an image of size L X L. 1/r is the reduction factor to which the image has been divided into small boxes of equal size. FD is the fractal dimension of the specified image.

To understand the concept of fractal dimension we need to elaborate the dimension first. The line has the dimension as 1, a plane has the dimension as 2 where as a cube has the dimension value as 3.

The dimension of a line is 1 because there is only one axis or it can be said that there is only 1 direction is there through which one can move on line. Where in case of a plane there are 2 different axis present to travel one is length and another is breadth. Where as if we will consider a cube then we can say that there are 3 different direction or 3 different dimension length, height and breadth are present. What should be the genuine reason behind the concept a line has the dimension as1, a plane has 2 where as a cube has the dimension as 3?

Why the line is called as 1-dimensional and the plane is called as multi dimensional? Both of the objects are self similar in nature. This is because the line can be divided into multiple small parts having equal length. Similarly we can divide the same line 5,6,7….. infinite times, but whenever all the small parts will be magnified the original image can be obtained. Hence the line can be divided into N self similar parts having reducing factor N and magnifying factor N.

But a square is a bit different. We can reduce a square into 4 self similar squares. Hence while calculating the self similarity we need to consider the reducing factor and the magnifying factor as 2. Alternatively, the square can be broken into 9 self-similar pieces with magnification factor 3, or 25 self-similar pieces with magnification factor 5. So, the square may be broken into **N^2** self-similar pieces, each of which must be having the magnifying factor N to yield the original figure. In fig 3.1 the square has been divided into N^2 self similar objects each having the magnifying factor as N.

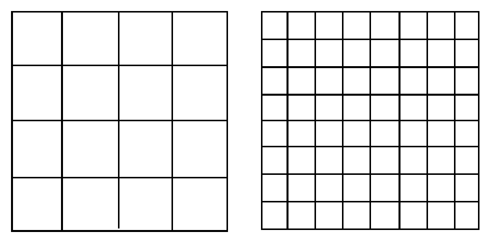

Fig 3 A square broken into N^2 self-similar pieces, each with magnification factor N

There is also alternative way to specify the dimension of a self-similar object. The dimension can be defined as the exponent of the number of self-similar copies each having the magnification factor **N** into which the figure may be broken.

Fractal dimension is a measure of how "complicated" a self-similar figure is. In a rough sense, it measures "how many points" lie in a given set. A plane is "larger" than a line, while **S** sits somewhere in between these two sets.

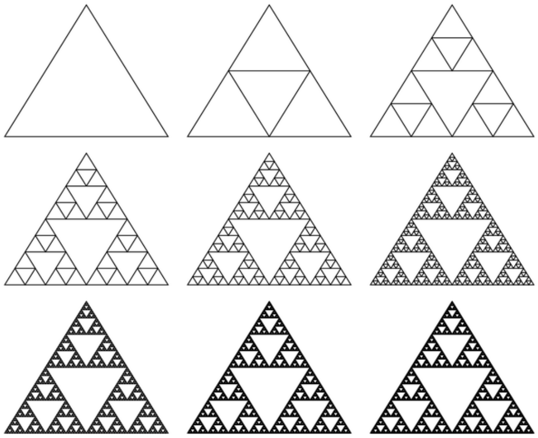

Fig 4 Sierpinski triangle having self-similar pieces.

IV. EXISTING METHOD

A) Differential Box Counting Method

In [4] B.B. Chaudhuri and Nirupam Sarkar had introduced the method for calculating the fractal dimension known as differential box counting (DBC) method. In DBC method the box counting is calculated as the difference between the maximum and minimum grey level in intensity values of the image having some specified length. Taking the assumption that image of M×M pixels and are scaled down to the different box size of L×L where $M/2 \geq L > 1$. Here the reduction value r = L/M and the reduction factor is calculated as 1/r=M/L. Now here considered the image of 3D space, where (*x*, *y*) denoting 2D space which are partitioned into different cells and third coordinate denoting the gray levels of each particular cell. The (x, y) 2D space is partitioned in the cell size of L×L. Then the maximum and minimum gray-level of image has been calculated at $(i,j)^{th}$ cell fall l^{th} *and* k^{th} pixel of that cell respectively.

Then

$$n_{r(i,\,j)} = l - k + 1 \qquad (6)$$

N_r in the $(I, j)^{th}$ cell. Taking contribution from all the cells having different gray levels and we can got the complete image as the bellow equation

$$\mathrm{Nr} = \sum_{i,j} n_r\,(i,j) \qquad (7)$$

N_r is countered for different values of r with different value of L, by using the equation (1). The FD can be estimated from the list square fit of $\log(N_r)$ against log(1/r). By counting N_r as the above manner we can got the approximate fractal dimension.

B) Reticular cell counting method

In [13] Gangepain and Roques-Carmes introduced the reticular cell counting method [13]. If the image having size M×M and the box size of that image L×L, then the entire image is covered by the boxes of side $L \times L \times L'$.

Here $L' = [L \times G/M]$, from L' the gray level per unit can be found out, where G is the total no of gray levels. N is calculated by counting the total number of boxes which contain at least one gray intensity level. From different box size of length L, the dimension D having reduction factor 1/r can be calculated as 1/r= M/L where M will have constant value for different L values. From equation (3) we have

$$N \propto L^{-D}$$
$$\text{i.e. } N(L) \propto L^{-D} \qquad (5)$$

For each box length L, the value of N is calculated where N stands for the self-similar pieces of the same object and a log-log plot of N vs. 1/r is made, so the best-fit positions at several points from where we able to calculate more approximate slop for dimension. The least square linear fit line will be –D.

V. CONCLUSION

In this research article we have studied two popular FD calculation method such as DBC and Reticular cell counting method. Both of the algorithm utilizes different approaches in calculating the fractal dimension of a image. FD of a grey scale image and color image can be determined by using these two algorithm.

REFERENCES

[1] B.B. Mandelbrot, The Fractal Geometry of Nature, Freeman, San Francisco, CA, 1982
[2] A.P. Pentland, “Fractal-based description of natural scenes,” IEEETransactions on Pattern Analysis and Machine Intelligence, vol. 6, pp.661–674, 1984.
[3] H.O. Peitgen, H. Jurgens, D. Saupe, Chaos and Fractals: New Frontiers of Science, first ed, Springer, Berlin, 1992.
[4] B.B. Chaudhuri, N Sarkar, “Texture segmentation using fractal dimension,” IEEE ransactions on Pattern Analysis and Machine Intelligence, vol. 17, pp. 72–77, 1995.
[5] S. Liu, S. Chang, “Dimension estimation of discrete-time fractional Brownian motion with applications to image texture classification,” IEEE Transactions on Image Processing, vol. 6, pp. 1176–1184, 1997.
[6] T. Ida, Y. Sambonsugi, “Image segmentation and contour detection using fractal coding,” IEEE Transactions on

Circuits System Video Technology, vol. 8, pp. 968–977, 1998.

[7] G. Neil and K. M. Curtis. "Shape recognition using fractal dimension," Pattern Recognition, vol. 30, no. 12, pp. 1957- 1969, 1997.

[8] G. Wu, D. Liang, and Y. Tian, "Texture image segmentation using fractal dimension," Chinese Journal of Computers, vol. 22, no.10, pp.1109-1113, 1999.

[9] K.H. Lin, K.M. Lam, W.C. Siu, “Locating the eye in human face images using fractal dimensions,” IEEE Proceedings on Vision, Image and Signal Processing, vol. 148, pp. 413–421, 2001.

[10] P. Asvestas, G.K. Matsopoulos, K.S. Nikita, “A power differentiation method of fractal dimension estimation for 2-D signals,” Journal of Visual Communication and Image Representation, vol. 9, pp. 392–400, 1998.

[11] A. P. Pentland, “Fractal based description of natural scenes,” IEEE Trans. Pattern Anal. Mach. Intell., vol. 6, pp. 661–674, 1984.

[12] A. P. Pentland,”Shading into texture,” Artificial Intell., vol. 29, pp. 147- 170, 1986.

[13] N. Sarkar, B.B. Chaudhuri, “An efficient differential box-counting approach to compute fractal dimension of image,” IEEE Transactions on Systems, Man, and Cybernetics, vol. 24, pp. 115–120, 1994.