Some Results of K-G Intuitionistic Fuzzy Matrices

A.Ruban[1], J.P. Indhumathi[2]
[1, 2] Dept of Mathematics
[1, 2] G.T.N Arts College (Autonomous), Karur Road, N.S. Nagar, Dindigul, Tamilnadu-624005

Abstract- *Hashimoto [1] introduced the concept of fuzzy matrices and studied the canonical form ofa transitive matrix. Fuzzy matrices arise in many applications, one of which is as adjacency matrices of fuzzy relations and fuzzy relational equations have important applicationsin pattern classification and in handing fuzziness in knowledge based systems. In this paper, some properties of k-g Intuitionistic fuzzy matrices are derived over the operations* $\oplus$ *and* $\odot$.

Keywords- k-g Intuitionistic Fuzzy Matrices, $\alpha-cut$, complement

I. INTRODUCTION

Hashimoto [1] introduced the concept of fuzzy matrices. It is known that matrices play very major role in various areas such as mathematics, physics, statistics, engineering, social sciences and many others. But in daily life situations, the problems do not always involve classical data. Consequently, we cannot successfully use traditional classical matrices because of various types of uncertainties present in daily life problems. Kim et.al. [4] studied the canonical form of an idempotent matrix. Ragab et. al. [10] presented some properties of the min-max composition of fuzzy matrices. Kolodziejczyk [5] presented the canonical form of a strongly transitive matrix. Xin [14,15] studied controllable fuzzy matrices. Thomason [12] and Kim [6] defined the ad-joint of square fuzzy matrix. Tian et. al. [13] studied power sequence of fuzzy matrices. Kim et. al. [7] studied determinant of square fuzzy matrices. Hemasinha et. al. [2] investigated iterations of fuzzy circulants matrices. Ragab et. al. [9] presented some properties on determinant and adjoint of a square fuzzy matrix. Shyamal and Pal et al. introduced two operations on fuzzy matrices. some exponential based results on fuzzy matrices. In this paper, some exponential based results on Intuitionistic fuzzy matrices are studied. Some theorems and results related to the above concepts are derived. Denote the $k \in \mathbb{N}. \mathbb{N}$ is natural number and $g \in [0,1]$

II. PRELIMINARIES

Definition 2.1

An intuitionistic fuzzy set (IFS) A in E (universe of discourse) is defined as an object of the following form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\}$, where the functions $\mu_A : E \to [0,1]$ and $\nu_A : E \to [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in E$ in A, respectively and for every $x \in E$, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$. Let I be the set of all real numbers lying between 0 and 1, i.e., $I = \{x : 0 \leq x \leq 1\}$. Also let $\langle F\rangle$ be the set of tuples $\langle a, b\rangle$, where $a, b \in I$ and $0 \leq a + b \leq 1$ i.e., $\langle F\rangle = \{\langle a, b\rangle : 0 \leq a + b \leq 1, a, b \in I\}$. The addition and multiplication between any two elements of are defined bellow.

Definition 2.2

Let $x = \langle x_\mu, x_\nu\rangle$ and $y = \langle y_\mu, y_\nu\rangle$ are any two elements of $\langle F\rangle$ The addition $(+)$ and multiplication $(\cdot)$ between x and y are defined as

$x + y = \langle x_\mu, x_\nu\rangle + \langle y_\mu, y_\nu\rangle =$
$\langle max(x_\mu, y_\mu), min(x_\nu, y_\nu)\rangle = \langle x_\mu \vee y_\mu, x_\nu \wedge y_\nu\rangle$

and

$x \cdot y = \langle x_\mu, x_\nu\rangle + \langle y_\mu, y_\nu\rangle =$
$\langle min(x_\mu, y_\mu), max(x_\nu, y_\nu)\rangle = \langle x_\mu \wedge y_\mu, x_\nu \vee y_\nu\rangle$

In arithmetic operations (such as addition, multiplication etc.) only the values of membership and nonmembership are needed. So from now we denote IFS as $A = \{x = \langle x_\mu, x_\nu\rangle | x \in E\}$.

Definition 2.3

Let R be an IFI and $x, y \in R$ where $x = \langle x_\mu, x_\nu\rangle$ and $y = \langle y_\mu, y_\nu\rangle$ then $x = y$ if and only if $x_\mu = y_\mu$ and $x_\nu = y_\nu$.

Definition 2.4

Let R be an IFI and $x, y \in R$ where $x = \langle x_\mu, x_\nu\rangle$ and

$y = \langle y_\mu, y_v \rangle$ then $x \leq y$ if and only if $x_\mu \leq y_\mu$ and $y_v \leq x_v$.

Definition 2.5

Let R be an IFI and $x, y \in R$ where $x = \langle x_\mu, x_v \rangle$ and $y = \langle y_\mu, y_v \rangle$ then $x < y$ if and only if $x \leq y$ and $x \neq y$.

Definition 2.6

Let R be an IFI and $A \in M_n(R)$. A is said to be power-convergent if $A^p = A^{p+1}$ for some positive integer p. If A is power-convergent the least positive integer p such that $A^p = A^{p+1}$ is called the index of A and is denoted by $i(A)$.

Definition 2.7

Let $x, y \in [0,1]$. Then the operation $\ominus$ defined as $x \ominus y = \begin{cases} x, & if\ x > y \\ 0, & if x \leq y \end{cases}$

Definition 2.8

$$x^{(\alpha)}(upper\ \alpha - cut) = \begin{cases} 1, & if\ x \geq \alpha \\ 0, & if x < \alpha \end{cases}$$

$$x_{(\alpha)}(lower\ \alpha - cut) = \begin{cases} x, & if\ x \geq \alpha \\ 0, & if x < \alpha \end{cases}$$

Definition 2.9

Let $x \in [0,1]$. The complement of an element x is defined by $x^c = 1 - x$.

Definition 2.10

Let $x, y \in [0,1]$. The two operators $\oplus$ and $\odot$ are defined by $x \oplus y = x + y - x.y$ and $x \odot y = x.y$. It is obvious that (i) $1 \oplus x = 1$, (ii) $1 \odot x = x$,(iii) $0 \oplus x = x$ and $0 \odot x = 0$.

Definition 2.11

Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be two fuzzy matrices of order $m \times n$. Then

1) $A \oplus B = [a_{ij} + b_{ij} - a_{ij}.b_{ij}]$
2) $A \odot B = [a_{ij}.b_{ij}]$
3) $A \vee B = [(a_{ij} \vee b_{ij}), (a'_{ij} \wedge b'_{ij})]$
4) $A \wedge B = [(a_{ij} \wedge b_{ij}), (a'_{ij} \vee b'_{ij})]]$
5) $A^{[k+1]} = A^{[k]} \odot A,\ A^{[1]} = A, k = 1,2, \ldots$
6) $A^{(\alpha)} = [a_{ij}^{\alpha}, (a'_{ij})^{\alpha}]$ (upper α-cut fuzzy matrix)
7) $A_{(\alpha)} = [a_{ij_{(\alpha)}}, (a'_{ij})_{\alpha}]$ (lower α-cut fuzzy matrix)
8) $A' = [a_{ji}]$ (the transpose of A)
9) $A^c = [1 - a_{ij}]$(the complement of A)
10) $A^{[k\oplus 1]} = A^{[k+1-k.1]}$, $A^{[1]} = A, k = \mathbb{N}$, $ie)\ \mathbb{N}$ is a natural number.
11) $A^{[k\odot 1]} = A^{[k.1]}$,$A^{[1]} = A, k = \mathbb{N}$, $ie)\ \mathbb{N}$ is a natural number.
12) $[g \oplus 1]A = [g + 1 - g.1]A$, $[1]A = A, 0 \leq g \leq 1$
13) $[g \odot 1]A = [g.1]A$,$[1]A = A, 0 \leq g \leq 1$

III. MAIN RESULTS

Theorem 3.1

Let A be a intuitionistic fuzzy matrix, then

i) $\langle A^{[k\oplus 1]}, (A')^{[k\odot 1]} \rangle = \langle (A^k \vee A), ((A')^k \wedge A') \rangle$
ii) $\langle A^{[k\odot 1]}, (A')^{[k\oplus 1]} \rangle = \langle (A^k \wedge A), ((A')^k \vee A') \rangle$
iii) $\langle (g \oplus 1)A, (g \odot 1)A' \rangle = \langle (gA \vee A), (gA' \wedge A') \rangle$
iv) $\langle (g \odot 1)A, (g \oplus 1)A' \rangle = \langle (gA \wedge A), (gA' \vee A') \rangle$
v) $\langle (A^c)^c, ((A')^T)^T \rangle = \langle (A^T)^T, ((A')^c)^c \rangle$

Proof :

i) Let $A = \langle a_{ij}, a'_{ij}\rangle$, where $0 \le a_{ij} \le 1,\ 0 \le a'_{ij} \le 1\ \forall\ i,j = 1,2,\dots,n$ and k be any positive integer such that $\langle a_{ij}^{k\oplus 1}, (a'_{ij})^{k\odot 1}\rangle = \langle (a_{ij}^{k} \vee a_{ij}), (a'_{ij})^{k} \wedge (a_{ij})\rangle$ and then, $\langle A^{[k\oplus 1]}, (A')^{[k\odot 1]}\rangle = \langle (A^{k} \vee A), ((A')^{k} \wedge A)\rangle$.

ii) Let $A = \langle a_{ij}, a'_{ij}\rangle$, where $0 \le a_{ij} \le 1,\ 0 \le a'_{ij} \le 1\ \forall\ i,j = 1,2,\dots,n$ and k be any positive integer such that $\langle a_{ij}^{[k\odot 1]}, (a'_{ij})^{[k\oplus 1]}\rangle = \langle (a_{ij}^{k} \wedge a_{ij}), \left((a'_{ij})^{k} \vee a_{ij}\right)\rangle$ and then, $\langle A^{[k\odot 1]}, (A')^{[k\oplus 1]}\rangle = \langle (A^{k} \wedge A), ((A')^{k} \wedge A)\rangle$.

iii) Let $A = \langle a_{ij}, a'_{ij}\rangle$ where $0 \le a_{ij} \le 1,\ 0 \le a'_{ij} \le 1\ \forall\ i,j = 1,2,\dots,n$ an$0 \le g \le 1$. such that $\langle [g \oplus 1]a_{ij}, [g \odot 1]a'_{ij}\rangle = \langle (g a_{ij} \vee a_{ij}), (g a_{ij} \wedge a'_{ij})\rangle$ and then $\langle [g \oplus 1]A, [g \odot 1]A'\rangle = \langle (gA \vee A), (gA' \wedge A')\rangle$.

iv) Let $A = \langle a_{ij}, a'_{ij}\rangle$ where $0 \le a_{ij} \le 1,\ 0 \le a'_{ij} \le 1\ \forall\ i,j = 1,2,\dots,n$ and $0 \le g \le 1$. such that $\langle [g \odot 1]a_{ij}, [g \oplus 1]a'_{ij}\rangle = \langle (g a_{ij} \wedge a_{ij}), (g a'_{ij} \vee a'_{ij})\rangle$ and then $\langle [g \odot 1]A, [g \oplus 1]A'\rangle = \langle (gA \wedge A), (gA' \vee A')\rangle$.

v) Let $A = \langle a_{ij}, a'_{ij}\rangle$ where $0 \le a_{ij} \le 1,\ 0 \le a'_{ij} \le 1\ \forall\ i,j = 1,2,\dots,n$ and $0 \le k \le 1$. such that $\langle (g+1)a_{ij}, (g.1)a'_{ij}\rangle \ne \langle (g a_{ij} \oplus a_{ij}), (g a'_{ij} \odot a'_{ij})\rangle$ and then $\langle (g+1)A, (g.1)A'\rangle = \langle (gA \oplus A), (gA \odot A)\rangle$.

vi) Let $A = \langle a_{ij}, a'_{ij}\rangle$ where $0 \le a_{ij} \le 1,\ 0 \le a'_{ij} \le 1\ \forall\ i,j = 1,2,\dots\dots n$. such that $\langle (a_{ij}^{c})^{c}, \left((a'_{ij})^{T}\right)^{T}\rangle = \langle (a_{ij}^{T})^{T}, (a'^{c}_{ij})^{c}\rangle$ and then $\langle (A^{c})^{c}, ((A')^{T})^{T}\rangle = \langle (A^{T})^{T}, ((A')^{c})^{c}\rangle$.

Example 3.1

Let $A = \begin{pmatrix} \langle 0.2, 0.8\rangle & \langle 0.5,0.5\rangle \\ \langle 0.1,0.9\rangle & \langle 0.8,0.2\rangle \end{pmatrix}$ be a intuitionistic fuzzy matrix, the

i) Let us consider $k = 2$

$$A = \begin{pmatrix} \langle 0.2\rangle & \langle 0.5\rangle \\ \langle 0.1\rangle & \langle 0.8\rangle \end{pmatrix}$$

$$A^2 = \begin{pmatrix} \langle 0.04\rangle & \langle 0.25\rangle \\ \langle 0.01\rangle & \langle 0.64\rangle \end{pmatrix}$$

$$A' = \begin{pmatrix} \langle 0.8\rangle & \langle 0.5\rangle \\ \langle 0.9\rangle & \langle 0.2\rangle \end{pmatrix}$$

$$(A')^2 = \begin{pmatrix} \langle 0.64\rangle & \langle 0.25\rangle \\ \langle 0.81\rangle & \langle 0.04\rangle \end{pmatrix}$$

$$A^c = \begin{pmatrix} \langle 0.8\rangle & \langle 0.5\rangle \\ \langle 0.9\rangle & \langle 0.2\rangle \end{pmatrix}$$

$$(A')^c = \begin{pmatrix} \langle 0.2\rangle & \langle 0.5\rangle \\ \langle 0.1\rangle & \langle 0.8\rangle \end{pmatrix}$$

<u>L.H.S</u>

$$\langle A^{[k\oplus 1]}, (A')^{[k\odot 1]}\rangle = \begin{pmatrix} \langle 0.2,0.64\rangle & \langle 0.5,0.25\rangle \\ \langle 0.1,0.81\rangle & \langle 0.8,0.04\rangle \end{pmatrix} \rightarrow \textbf{(1)}$$

<u>R.H.S</u>

$$\Longrightarrow \langle (A^2 \vee A)\rangle = \begin{pmatrix} \langle 0.2\rangle & \langle 0.5\rangle \\ \langle 0.1\rangle & \langle 0.8\rangle \end{pmatrix} \rightarrow \textbf{(a)}$$

$$\Longrightarrow (A')^2 \wedge A' = \begin{pmatrix} \langle 0.64\rangle & \langle 0.25\rangle \\ \langle 0.81\rangle & \langle 0.04\rangle \end{pmatrix} \rightarrow \textbf{(b)}$$

From equation (a) & (b) we have

$$= \begin{pmatrix} \langle 0.2,0.64\rangle & \langle 0.5,0.25\rangle \\ \langle 0.1,0.81\rangle & \langle 0.8,0.04\rangle \end{pmatrix} \rightarrow \textbf{(2)}$$

From equation (1) & (2), we get,

$$\begin{pmatrix} \langle 0.2,0.64\rangle & \langle 0.5,0.25\rangle \\ \langle 0.1,0.81\rangle & \langle 0.8,0.04\rangle \end{pmatrix} = \begin{pmatrix} \langle 0.2,0.64\rangle & \langle 0.5,0.25\rangle \\ \langle 0.1,0.81\rangle & \langle 0.8,0.04\rangle \end{pmatrix}$$

Hence $\langle A^{[k\oplus 1]}, (A')^{[k\odot 1]}\rangle = \langle (A^{k} \vee A), ((A')^{k} \wedge A')\rangle$

ii) Let us consider $k = 2$

$$\langle A^{[k\odot 1]},(A')^{[k\oplus 1]}\rangle = \begin{pmatrix}\langle 0.04,0.8\rangle & \langle 0.25,0.5\rangle\\ \langle 0.01,0.9\rangle & \langle 0.64,0.2\rangle\end{pmatrix}$$

$$\langle (A^2 \wedge A),((A)^2 \vee A')\rangle = \begin{pmatrix}\langle 0.04,0.8\rangle & \langle 0.25,0.5\rangle\\ \langle 0.01,0.9\rangle & \langle 0.64,0.2\rangle\end{pmatrix}$$

$$\begin{pmatrix}\langle 0.04,0.8\rangle & \langle 0.25,0.5\rangle\\ \langle 0.01,0.9\rangle & \langle 0.64,0.2\rangle\end{pmatrix} = \begin{pmatrix}\langle 0.04,0.8\rangle & \langle 0.25,0.5\rangle\\ \langle 0.01,0.9\rangle & \langle 0.64,0.2\rangle\end{pmatrix}$$

Hence $\langle A^{[k\odot 1]},(A')^{[k\oplus 1]}\rangle = \langle (A^k \wedge A),((A')^k \vee A')\rangle$

iii) Let us consider $g = 0.2$

$$(0.2)A = \begin{pmatrix}\langle 0.04\rangle & \langle 0.1\rangle\\ \langle 0.02\rangle & \langle 0.16\rangle\end{pmatrix}$$

$$(0.2)A' = \begin{pmatrix}\langle 0.16\rangle & \langle 0.1\rangle\\ \langle 0.18\rangle & \langle 0.04\rangle\end{pmatrix}$$

$$\langle (g \oplus 1)A,(g \odot 1)A'\rangle = \begin{pmatrix}\langle 0.2,0.16\rangle & \langle 0.5,0.1\rangle\\ \langle 0.1,0.18\rangle & \langle 0.8,0.04\rangle\end{pmatrix}$$

$$\langle (0.2(A) \vee A),(0.2(A') \wedge A')\rangle = \begin{pmatrix}\langle 0.2,0.16\rangle & \langle 0.5,0.1\rangle\\ \langle 0.1,0.18\rangle & \langle 0.8,0.04\rangle\end{pmatrix}$$

$$\begin{pmatrix}\langle 0.2,0.16\rangle & \langle 0.5,0.1\rangle\\ \langle 0.1,0.18\rangle & \langle 0.8,0.04\rangle\end{pmatrix} = \begin{pmatrix}\langle 0.2,0.16\rangle & \langle 0.5,0.1\rangle\\ \langle 0.1,0.18\rangle & \langle 0.8,0.04\rangle\end{pmatrix}$$

Hence

$$\langle (g \oplus 1)A,(g \odot 1)A'\rangle = \langle (gA \vee A),(gA' \wedge A')\rangle$$

iv) Let us consider $g = 0.2$

$$\langle (g \odot 1)A,(g \oplus 1)A'\rangle = \begin{pmatrix}\langle 0.04,0.8\rangle & \langle 0.1,0.5\rangle\\ \langle 0.02,0.9\rangle & \langle 0.16,0.2\rangle\end{pmatrix}$$

$$\langle (0.2(A) \wedge A),(0.2(A') \vee A')\rangle = \begin{pmatrix}\langle 0.04,0.8\rangle & \langle 0.1,0.5\rangle\\ \langle 0.02,0.9\rangle & \langle 0.16,0.2\rangle\end{pmatrix}$$

$$\begin{pmatrix}\langle 0.04,0.8\rangle & \langle 0.1,0.5\rangle\\ \langle 0.02,0.9\rangle & \langle 0.16,0.2\rangle\end{pmatrix} = \begin{pmatrix}\langle 0.04,0.8\rangle & \langle 0.1,0.5\rangle\\ \langle 0.02,0.9\rangle & \langle 0.16,0.2\rangle\end{pmatrix}$$

Hence $\langle (g \odot 1)A,(g \oplus 1)A'\rangle = \langle (gA \wedge A),(gA' \vee A')\rangle$

v) $\langle (A^{\varsigma})^{\varsigma},((A')^{\tau})^{\tau}\rangle = \begin{pmatrix}\langle 0.2,0.8\rangle & \langle 0.5,0.5\rangle\\ \langle 0.1,0.9\rangle & \langle 0.8,0.2\rangle\end{pmatrix} \rightarrow (3)$

$\langle (A^{\tau})^{\tau},((A')^{\varsigma})^{\varsigma}\rangle = \begin{pmatrix}\langle 0.2,0.8\rangle & \langle 0.5,0.5\rangle\\ \langle 0.1,0.9\rangle & \langle 0.8,0.2\rangle\end{pmatrix} \rightarrow (4)$

From equation (3) & (4) we have, $\begin{pmatrix}\langle 0.2,0.8\rangle & \langle 0.5,0.5\rangle\\ \langle 0.1,0.9\rangle & \langle 0.8,0.2\rangle\end{pmatrix} = \begin{pmatrix}\langle 0.2,0.8\rangle & \langle 0.5,0.5\rangle\\ \langle 0.1,0.9\rangle & \langle 0.8,0.2\rangle\end{pmatrix}$

Hence $\langle (A^{\varsigma})^{\varsigma},((A')^{\tau})^{\tau}\rangle = \langle (A^{\tau})^{\tau},((A')^{\varsigma})^{\varsigma}\rangle$

Theorem 3.2

For any intuitionistic fuzzy matrix A,

i) $\langle A^{[k\oplus 1]},(A')^{[k\odot 1]}\rangle \le A$
ii) $\langle A^{[k\odot 1]},(A')^{[k\oplus 1]}\rangle \le A$
iii) $\langle [g \oplus 1]A,[g \odot 1]A'\rangle \le A$
iv) $\langle [g \odot 1]A,[g \oplus 1]A'\rangle \le A$

Proof:

i) The ij^{th} element of $\langle A^{[k\oplus 1]},(A')^{[k\odot 1]}\rangle$ is $\langle a_{ij}^{k+1-k.1},(a'_{ij})^{k.1}\rangle$ and that A is $\langle a_{ij},a'_{ij}\rangle$. i.e., $\langle a_{ij}^{k+1-k-1},(a'_{ij})^{k-1}\rangle \le \langle a_{ij},a'_{ij}\rangle$. Hence $\langle A^{[k\oplus 1]},(A')^{[k\odot 1]}\rangle \le A$

ii) The ij^{th} element of $\langle A^{[k\odot 1]},(A')^{[k\oplus 1]}\rangle$ is $\langle a_{ij}^{k-1},(a'_{ij})^{k+1-k1}\rangle$ and that A is $\langle a_{ij},a'_{ij}\rangle$. i.e., $\langle a_{ij}^{k},(a'_{ij})^{k+1-k1}\rangle \le \langle a_{ij},a'_{ij}\rangle$. Hence $\langle A^{[k\odot 1]},(A')^{[k\oplus 1]}\rangle \le A$

iii) The ij^{th} element of $\langle [g \oplus 1]A,[g \odot 1]A'\rangle$ is $\langle (g+1-g.1)a_{ij},(g.1)a'_{ij}\rangle$ and that A is $\langle a_{ij},a'_{ij}\rangle$. i.e., $\langle (g+1-g.1)a_{ij},(g.1)a'_{ij}\rangle \le \langle a_{ij},a'_{ij}\rangle$. Hence $\langle [g \oplus 1]A,[g \odot 1]A'\rangle \le A$.

iv) The ij^{th} element of $\langle [g \odot 1]A,[g \oplus 1]A'\rangle$ is $\langle [g.1]a_{ij},[g+1-g.1]a'_{ij}\rangle$ and that A is $\langle a_{ij},a'_{ij}\rangle$. i.e., $\langle [g.1]a_{ij},[g+1-g.1]a'_{ij}\rangle \le \langle a_{ij},a'_{ij}\rangle$. Hence $\langle [g \odot 1]A,[g \oplus 1]A'\rangle \le A$.

Example 3.2

Let $A = \begin{pmatrix}\langle 0.2,0.8\rangle & \langle 0.5,0.5\rangle\\ \langle 0.1,0.9\rangle & \langle 0.8,0.2\rangle\end{pmatrix}$ be a intuitionistic fuzzy matrix,

i) Let us consider $k = 2$

$$\langle A^{[k\oplus 1]},(A')^{[k\odot 1]}\rangle = \begin{pmatrix} \langle 0.2,0.64\rangle & \langle 0.5,0.25\rangle \\ \langle 0.1,0.81\rangle & \langle 0.8,0.04\rangle \end{pmatrix}$$

$$\begin{pmatrix} \langle 0.2,0.64\rangle & \langle 0.5,0.25\rangle \\ \langle 0.1,0.81\rangle & \langle 0.8,0.04\rangle \end{pmatrix} \leq \begin{pmatrix} \langle 0.2,0.8\rangle & \langle 0.5,0.5\rangle \\ \langle 0.1,0.9\rangle & \langle 0.8,0.2\rangle \end{pmatrix}$$

Hence $\langle A^{[k\oplus 1]},(A')^{[k\odot 1]}\rangle \leq A$.

ii) Let us consider $k = 2$

$$\langle A^{[k\odot 1]},(A')^{[k\oplus 1]}\rangle = \begin{pmatrix} \langle 0.04,0.8\rangle & \langle 0.25,0.5\rangle \\ \langle 0.01,0.9\rangle & \langle 0.64,0.2\rangle \end{pmatrix}$$

$$\begin{pmatrix} \langle 0.04,0.8\rangle & \langle 0.25,0.5\rangle \\ \langle 0.01,0.9\rangle & \langle 0.64,0.2\rangle \end{pmatrix} \leq \begin{pmatrix} \langle 0.2,0.8\rangle & \langle 0.5,0.5\rangle \\ \langle 0.1,0.9\rangle & \langle 0.8,0.2\rangle \end{pmatrix}$$

Hence $\langle A^{[k\odot 1]},(A')^{[k\oplus 1]}\rangle \leq A$.

iii) Let us consider $g = 0.2$

$$\langle [g\oplus 1]A,[g\odot 1]A'\rangle = \begin{pmatrix} \langle 0.2,0.16\rangle & \langle 0.5,0.1\rangle \\ \langle 0.1,0.18\rangle & \langle 0.8,0.04\rangle \end{pmatrix}$$

$$\begin{pmatrix} \langle 0.2,0.16\rangle & \langle 0.5,0.1\rangle \\ \langle 0.1,0.18\rangle & \langle 0.8,0.04\rangle \end{pmatrix} \leq \begin{pmatrix} \langle 0.2,0.8\rangle & \langle 0.5,0.5\rangle \\ \langle 0.1,0.9\rangle & \langle 0.8,0.2\rangle \end{pmatrix}$$

Hence $\langle [g\oplus 1]A,[g\odot 1]A'\rangle \leq A$

iv) Let us consider $g = 0.2$

$$\langle [g\odot 1]A,[g\oplus 1]A'\rangle = \begin{pmatrix} \langle 0.04,0.8\rangle & \langle 0.1,0.5\rangle \\ \langle 0.02,0.9\rangle & \langle 0.16,0.2\rangle \end{pmatrix}$$

$$\begin{pmatrix} \langle 0.04,0.8\rangle & \langle 0.1,0.5\rangle \\ \langle 0.02,0.9\rangle & \langle 0.16,0.2\rangle \end{pmatrix} \leq \begin{pmatrix} \langle 0.2,0.8\rangle & \langle 0.5,0.5\rangle \\ \langle 0.1,0.9\rangle & \langle 0.8,0.2\rangle \end{pmatrix}$$

Hence $\langle [g\odot 1]A,[g\oplus 1]A'\rangle \leq A$

IV. RESULTS ON α-CUT OF INTUITIONISTIC FUZZY MATRIX

The upper α-cut intuitionistic fuzzy matrix is basically a boolean intuitionistic fuzzy matrix. It represents only two states 0 and 1. But the lower α-cut intuitionistic fuzzy matrix is a multi-graded intuitionistic fuzzy matrix. When the elements of this matrix are less than αthen lower α-cut intuitionistic fuzzy matrix represents same state 0 and all other cases it represent the actual states. It is also used for intuitionistic fuzzy matrix.

Let the intuitionistic fuzzy matrix A represents the crowdness status of the network N at any time period and date of year. If we consider the crowdness as two states i.e., if we consider the road is fully crowd when the crowdness gradation is greater than some value, say α,0 <α< 1, and the road is free from crowd when the gradation is less than α, then the intuitionistic fuzzy matrix Abecomes$A(\alpha)$, the upper α- cut intuitionistic fuzzy matrix.

Theorem 4.1

For any intuitionistic fuzzy matrix A,then,

i) $\langle A^{[k\oplus 1]},(A')^{[k\odot 1]}\rangle = \langle (A^k \vee A),((A')^k \wedge A')\rangle$

ii) $\langle A^{[k\odot 1]},(A')^{[k\oplus 1]}\rangle = \langle (A^k \wedge A),((A')^k \vee A')\rangle$

iii) $\langle (g\oplus 1)A,(g\odot 1)A'\rangle = \langle (gA \vee A),(gA' \wedge A')\rangle$

iv) $\langle (g\odot 1)A,(g\oplus 1)A'\rangle = \langle (gA \wedge A),(gA' \vee A')\rangle$

Proof:

i) Let $A = \langle a_{ij}, a'_{ij}\rangle$, where $0 \leq a_{ij} \leq 1, 0 \leq a'_{ij} \leq 1$ and $0 \leq \alpha \leq 1 \; \forall \, i,j = 1,2,\ldots,n$ and k be any positive integer such that $\langle a_{ij}^{k\oplus 1},(a'_{ij})^{k\odot 1}\rangle = \langle (a_{ij}^k \vee a_{ij}),(a'_{ij})^k \wedge (a_{ij})\rangle$ and then, $\langle A^{[k\oplus 1]},(A')^{[k\odot 1]}\rangle = \langle (A^k \vee A),((A')^k \wedge A)\rangle$.

ii) Let $A = \langle a_{ij}, a'_{ij}\rangle$, where $0 \leq a_{ij} \leq 1,\; 0 \leq a'_{ij} \leq 1$and $0 \leq \alpha \leq 1 \; \forall \, i,j = 1,2,\ldots,n$ and k be any positive integer such that $\langle a_{ij}^{[k\odot 1]},(a'_{ij})^{[k\oplus 1]}\rangle = \langle (a_{ij}^k \wedge a_{ij}),((a'_{ij})^k \vee a_{ij})\rangle$ and then, $\langle A^{[k\odot 1]},(A')^{[k\oplus 1]}\rangle = \langle (A^k \wedge A),((A')^k \wedge A)\rangle$.

iii) Let $A = \langle a_{ij}, a'_{ij}\rangle$ where $0 \leq a_{ij} \leq 1,\; 0 \leq a'_{ij} \leq 1$and $0 \leq \alpha \leq 1 \; \forall \, i,j = 1,2,\ldots,n$ and $0 \leq g \leq 1.$ such that $\langle [g\oplus 1]a_{ij},[g\odot 1]a'_{ij}\rangle = \langle (ga_{ij} \vee a_{ij}),(ga_{ij} \wedge a'_{ij})\rangle$ and then $\langle [g\oplus 1]A,[g\odot 1]A'\rangle = \langle (gA \vee A),(gA' \wedge A')\rangle$.

iv) Let $A = \langle a_{ij}, a'_{ij}\rangle$ where $0 \leq a_{ij} \leq 1,\; 0 \leq a'_{ij} \leq 1$and $0 \leq \alpha \leq 1 \; \forall \, i,j = 1,2,\ldots,n$ and $0 \leq g \leq 1.$ such that $\langle [g\odot 1]a_{ij},[g\oplus 1]a'_{ij}\rangle = \langle (ga_{ij} \wedge a_{ij}),(ga'_{ij} \vee a'_{ij})\rangle$ and then $\langle [g\odot 1]A,[g\oplus 1]A'\rangle = \langle (gA \wedge A),(gA' \vee A')\rangle$.

Example 4.1

Let $A = \begin{pmatrix} \langle 0.2,0.8\rangle & \langle 0.5,0.5\rangle \\ \langle 0.1,0.9\rangle & \langle 0.8,0.2\rangle \end{pmatrix}$ be a intuitionistic fuzzy matrix, the

i) Let us consider $k = 2, \alpha = 0.2$.

$$\langle A^{[k\oplus 1]}, (A')^{[k\odot 1]}\rangle^{\alpha} = \begin{pmatrix} \langle 0.2,0.64\rangle & \langle 0.5,0.25\rangle \\ \langle 0.1,0.81\rangle & \langle 0.8,0.04\rangle \end{pmatrix}^{0.2}$$

$$\langle A, (A')^2\rangle^{0.2} = \begin{pmatrix} \langle 1,1\rangle & \langle 1,1\rangle \\ \langle 0,1\rangle & \langle 1,0\rangle \end{pmatrix}$$

$$\langle (A^k \vee A), ((A')^k \wedge A')\rangle^{\alpha} = \begin{pmatrix} \langle 0.2,0.64\rangle & \langle 0.5,0.25\rangle \\ \langle 0.1,0.81\rangle & \langle 0.8,0.04\rangle \end{pmatrix}^{0.2}$$

$$\langle (A^2 \vee A), ((A')^2 \wedge A')\rangle^{0.2} = \begin{pmatrix} \langle 1,1\rangle & \langle 1,1\rangle \\ \langle 0,1\rangle & \langle 1,0\rangle \end{pmatrix}$$

$$\begin{pmatrix} \langle 1,1\rangle & \langle 1,1\rangle \\ \langle 0,1\rangle & \langle 1,0\rangle \end{pmatrix} = \begin{pmatrix} \langle 1,1\rangle & \langle 1,1\rangle \\ \langle 0,1\rangle & \langle 1,0\rangle \end{pmatrix}$$

Hence

$$\langle A^{[k\oplus 1]}, (A')^{[k\odot 1]}\rangle^{\alpha} = \langle (A^k \vee A), ((A')^k \wedge A')\rangle^{\alpha}.$$

ii) Let us consider $k = 2, \alpha = 0.2$.

$$\langle A^{[k\odot 1]}, (A')^{[k\oplus 1]}\rangle^{\alpha} = \begin{pmatrix} \langle 0.04, 0.8\rangle & \langle 0.25,0.5\rangle \\ \langle 0.01,0.9\rangle & \langle 0.64,0.2\rangle \end{pmatrix}^{0.2}$$

$$\langle A^2, (A')\rangle^{0.2} = \begin{pmatrix} \langle 0,1\rangle & \langle 1,1\rangle \\ \langle 0,1\rangle & \langle 1,1\rangle \end{pmatrix}$$

$$\langle (A^k \wedge A), ((A')^k \vee A')\rangle^{\alpha} = \begin{pmatrix} \langle 0.04, 0.8\rangle & \langle 0.25,0.5\rangle \\ \langle 0.01,0.9\rangle & \langle 0.64,0.2\rangle \end{pmatrix}^{0.2}$$

$$\langle (A^2 \wedge A), ((A')^2 \vee A')\rangle^{0.2} = \begin{pmatrix} \langle 0,1\rangle & \langle 1,1\rangle \\ \langle 0,1\rangle & \langle 1,1\rangle \end{pmatrix}$$

$$\begin{pmatrix} \langle 0,1\rangle & \langle 1,1\rangle \\ \langle 0,1\rangle & \langle 1,1\rangle \end{pmatrix} = \begin{pmatrix} \langle 0,1\rangle & \langle 1,1\rangle \\ \langle 0,1\rangle & \langle 1,1\rangle \end{pmatrix}$$

Hence $\langle A^{[k\odot 1]}, (A')^{[k\oplus 1]}\rangle^{\alpha} = \langle (A^k \wedge A), ((A')^k \vee A')\rangle^{\alpha}$.

iii) Let us consider $g = 0.2, \alpha = 0.2$.

$$\langle (g \oplus 1)A, (g \odot 1)A'\rangle^{\alpha} = \begin{pmatrix} \langle 0.2,0.16\rangle & \langle 0.5,0.1\rangle \\ \langle 0.1,0.18\rangle & \langle 0.8,0.04\rangle \end{pmatrix}^{0.2}$$

$$\langle A, 0.2A'\rangle^{0.2} = \begin{pmatrix} \langle 1,0\rangle & \langle 1,0\rangle \\ \langle 0,0\rangle & \langle 1,0\rangle \end{pmatrix}$$

$$\langle (gA \vee A), (gA' \wedge A')\rangle^{\alpha} = \begin{pmatrix} \langle 0.2,0.16\rangle & \langle 0.5,0.1\rangle \\ \langle 0.1,0.18\rangle & \langle 0.8,0.04\rangle \end{pmatrix}^{0.2}$$

$$\langle (0.2A \vee A), (0.2A' \wedge A')\rangle^{\alpha} = \begin{pmatrix} \langle 1,0\rangle & \langle 1,0\rangle \\ \langle 0,0\rangle & \langle 1,0\rangle \end{pmatrix}$$

$$\begin{pmatrix} \langle 1,0\rangle & \langle 1,0\rangle \\ \langle 0,0\rangle & \langle 1,0\rangle \end{pmatrix} = \begin{pmatrix} \langle 1,0\rangle & \langle 1,0\rangle \\ \langle 0,0\rangle & \langle 1,0\rangle \end{pmatrix}$$

Hence

$$\langle (g \oplus 1)A, (g \odot 1)A'\rangle^{\alpha} = \langle (gA \vee A), (gA' \wedge A')\rangle^{\alpha}.$$

iv) Let us consider $g = 0.2, \alpha = 0.2$.

$$\langle (g \odot 1)A, (g \oplus 1)A'\rangle^{\alpha} = \begin{pmatrix} \langle 0.04,0.8\rangle & \langle 0.1,0.5\rangle \\ \langle 0.02,0.9\rangle & \langle 0.16,0.2\rangle \end{pmatrix}^{0.2}$$

$$\langle 0.2A, A'\rangle^{0.2} = \begin{pmatrix} \langle 0,1\rangle & \langle 0,1\rangle \\ \langle 0,1\rangle & \langle 0,1\rangle \end{pmatrix}$$

$$\langle (gA \wedge A), (gA' \vee A')\rangle^{\alpha} = \begin{pmatrix} \langle 0.04,0.8\rangle & \langle 0.1,0.5\rangle \\ \langle 0.02,0.9\rangle & \langle 0.16,0.2\rangle \end{pmatrix}^{0.2}$$

$$\langle (0.2A \wedge A), (0.2A' \vee A')\rangle^{0.2} = \begin{pmatrix} \langle 0,1\rangle & \langle 0,1\rangle \\ \langle 0,1\rangle & \langle 0,1\rangle \end{pmatrix}$$

$$\begin{pmatrix} \langle 0,1\rangle & \langle 0,1\rangle \\ \langle 0,1\rangle & \langle 0,1\rangle \end{pmatrix} = \begin{pmatrix} \langle 0,1\rangle & \langle 0,1\rangle \\ \langle 0,1\rangle & \langle 0,1\rangle \end{pmatrix}$$

Hence

$$\langle (g \odot 1)A, (g \oplus 1)A'\rangle^{\alpha} = \langle (gA \wedge A), (gA' \vee A')\rangle^{\alpha}.$$

Theorem 4.2

For any intuitionistic fuzzy matrix A, then,

i) $\langle A^{[k\oplus 1]}, (A')^{[k\odot 1]}\rangle_{=} = \langle (A^k \vee A), ((A')^k \wedge A')\rangle_{=}$
ii) $\langle A^{[k\odot 1]}, (A')^{[k\oplus 1]}\rangle_{=} = \langle (A^k \wedge A), ((A')^k \vee A')\rangle_{=}$
iii) $\langle (g \oplus 1)A, (g \odot 1)A'\rangle_{=} = \langle (gA \vee A), (gA' \wedge A')\rangle_{=}$
iv) $\langle (g \odot 1)A, (g \oplus 1)A'\rangle_{=} = \langle (gA \wedge A), (gA' \vee A')\rangle_{=}$

Proof:

i) Let $A = \langle a_{ij}, a'_{ij}\rangle$, where $0 \le a_{ij} \le 1$, $0 \le a'_{ij} \le 1$ and $0 \le \alpha \le 1$ $\forall\, i,j = 1,2,\dots,n$ and k be any positive integer such that $\langle a_{ij}^{k\oplus 1}, (a'_{ij})^{k\odot 1}\rangle_{=} = \langle (a_{ij}^{k} \vee a_{ij}), (a'_{ij})^{k} \wedge (a_{ij})\rangle_{=}$ and then, $\langle A^{[k\oplus 1]}, (A')^{[k\odot 1]}\rangle_{=} = \langle (A^k \vee A), ((A')^k \wedge A)\rangle_{=}$.

ii) Let $A = \langle a_{ij}, a'_{ij}\rangle$, where $0 \le a_{ij} \le 1$, $0 \le a'_{ij} \le 1$ and $0 \le \alpha \le 1$ $\forall\, i,j = 1,2,\dots,n$ and k be any positive integer such that $\langle a_{ij}^{[k\odot 1]}, (a'_{ij})^{[k\oplus 1]}\rangle_{=} = \langle (a_{ij}^{k} \wedge a_{ij}), ((a'_{ij})^{k} \vee a_{ij})\rangle_{=}$ and then, $\langle A^{[k\odot 1]}, (A')^{[k\oplus 1]}\rangle_{=} = \langle (A^k \wedge A), ((A')^k \wedge A)\rangle_{=}$.

iii) Let $A = \langle a_{ij}, a'_{ij}\rangle$ where $0 \le a_{ij} \le 1$, $0 \le a'_{ij} \le 1$ and $0 \le \alpha \le 1$ $\forall\, i,j = 1,2,\dots,n$ and $0 \le g \le 1$. such that $\langle [g \oplus 1]a_{ij}, [g \odot 1]a'_{ij}\rangle_{=} = \langle (g a_{ij} \vee a_{ij}), (g a_{ij} \wedge a'_{ij})\rangle_{=}$ and then $\langle [g \oplus 1]A, [g \odot 1]A'\rangle_{=} = \langle (gA \vee A), (gA' \wedge A')\rangle_{=}$

iv) Let $A = \langle a_{ij}, a'_{ij}\rangle$ where $0 \le a_{ij} \le 1$, $0 \le a'_{ij} \le 1$ and $0 \le \alpha \le 1$ $\forall\, i,j = 1,2,\dots,n$ and $0 \le g \le 1$. such that $\langle [g \odot 1]a_{ij}, [g \oplus 1]a'_{ij}\rangle_{=} = \langle (g a_{ij} \wedge a_{ij}), (g a'_{ij} \vee a'_{ij})\rangle_{=}$ and then $\langle [g \odot 1]A, [g \oplus 1]A'\rangle_{=} = \langle (gA \wedge A), (gA' \vee A')\rangle_{=}$

Example 4.2

Let $A = \begin{pmatrix} \langle 0.2, 0.8\rangle & \langle 0.5, 0.5\rangle \\ \langle 0.1, 0.9\rangle & \langle 0.8, 0.2\rangle \end{pmatrix}$ be a intuitionistic fuzzy matrix, the

i) Let us consider $k = 2, \alpha = 0.2$.

$$\langle A^{[k\oplus 1]}, (A')^{[k\odot 1]}\rangle_\alpha = \begin{pmatrix} \langle 0.2,0.64\rangle & \langle 0.5,0.25\rangle \\ \langle 0.1,0.81\rangle & \langle 0.8,0.04\rangle \end{pmatrix}_{0.2}$$

$$\langle A, (A')^2\rangle_{0.2} = \begin{pmatrix} \langle 0.2,0.64\rangle & \langle 0.5,0.25\rangle \\ \langle 0,0.81\rangle & \langle 0.8,0\rangle \end{pmatrix}$$

$$\langle (A^k \vee A), ((A')^k \wedge A')\rangle_\alpha = \begin{pmatrix} \langle 0.2,0.64\rangle & \langle 0.5,0.25\rangle \\ \langle 0.1,0.81\rangle & \langle 0.8,0.04\rangle \end{pmatrix}_{0.2}$$

$$\langle (A^2 \vee A), ((A')^2 \wedge A')\rangle_{0.2} = \begin{pmatrix} \langle 0.2,0.64\rangle & \langle 0.5,0.25\rangle \\ \langle 0,0.81\rangle & \langle 0.8,0\rangle \end{pmatrix}$$

$$\begin{pmatrix} \langle 0.2,0.64\rangle & \langle 0.5,0.25\rangle \\ \langle 0,0.81\rangle & \langle 0.8,0\rangle \end{pmatrix} = \begin{pmatrix} \langle 0.2,0.64\rangle & \langle 0.5,0.25\rangle \\ \langle 0,0.81\rangle & \langle 0.8,0\rangle \end{pmatrix}$$

Hence $\langle A^{[k\oplus 1]}, (A')^{[k\odot 1]}\rangle_\alpha = \langle (A^k \vee A), ((A')^k \wedge A')\rangle_\alpha$.

ii) Let us consider $k = 2, \alpha = 0.2$.

$$\langle A^{[k\odot 1]}, (A')^{[k\oplus 1]}\rangle_\alpha = \begin{pmatrix} \langle 0.04, 0.8\rangle & \langle 0.25,0.5\rangle \\ \langle 0.01,0.9\rangle & \langle 0.64,0.2\rangle \end{pmatrix}_{0.2}$$

$$\langle A^2, A'\rangle_{0.2} = \begin{pmatrix} \langle 0,0.8\rangle & \langle 0.25,0.5\rangle \\ \langle 0,0.9\rangle & \langle 0.64,0.2\rangle \end{pmatrix}$$

$$\langle (A^k \wedge A), ((A')^k \vee A')\rangle_\alpha = \begin{pmatrix} \langle 0.04, 0.8\rangle & \langle 0.25,0.5\rangle \\ \langle 0.01,0.9\rangle & \langle 0.64,0.2\rangle \end{pmatrix}_{0.2}$$

$$\langle (A^2 \wedge A), ((A')^2 \vee A')\rangle_{0.2} = \begin{pmatrix} \langle 0,0.8\rangle & \langle 0.25,0.5\rangle \\ \langle 0,0.9\rangle & \langle 0.64,0.2\rangle \end{pmatrix}$$

$$\begin{pmatrix} \langle 0,0.8\rangle & \langle 0.25,0.5\rangle \\ \langle 0,0.9\rangle & \langle 0.64,0.2\rangle \end{pmatrix} = \begin{pmatrix} \langle 0,0.8\rangle & \langle 0.25,0.5\rangle \\ \langle 0,0.9\rangle & \langle 0.64,0.2\rangle \end{pmatrix}$$

Hence
$\langle A^{[k\odot 1]}, (A')^{[k\oplus 1]}\rangle_\alpha = \langle (A^k \wedge A), ((A')^k \vee A')\rangle_\alpha$.

iii) Let us consider $g = 0.2, \alpha = 0.2$.

$$\langle (g \oplus 1)A, (g \odot 1)A'\rangle_\alpha = \begin{pmatrix} \langle 0.2,0.16\rangle & \langle 0.5,0.1\rangle \\ \langle 0.1,0.18\rangle & \langle 0.8,0.04\rangle \end{pmatrix}_{0.2}$$

$$\langle A, 0.2A'\rangle_{0.2} = \begin{pmatrix} \langle 0.2,0\rangle & \langle 0.5,0\rangle \\ \langle 0,0\rangle & \langle 0.8,0\rangle \end{pmatrix}$$

$$\langle (gA \vee A), (gA' \wedge A')\rangle_\alpha = \begin{pmatrix} \langle 0.2,0.16\rangle & \langle 0.5,0.1\rangle \\ \langle 0.1,0.18\rangle & \langle 0.8,0.04\rangle \end{pmatrix}_{0.2}$$

$$\langle (0.2A \vee A), (0.2A' \wedge A')\rangle_{0.2} = \begin{pmatrix} \langle 0.2,0\rangle & \langle 0.5,0\rangle \\ \langle 0,0\rangle & \langle 0.8,0\rangle \end{pmatrix}$$

$$\begin{pmatrix} \langle 0.2,0\rangle & \langle 0.5,0\rangle \\ \langle 0,0\rangle & \langle 0.8,0\rangle \end{pmatrix} = \begin{pmatrix} \langle 0.2,0\rangle & \langle 0.5,0\rangle \\ \langle 0,0\rangle & \langle 0.8,0\rangle \end{pmatrix}$$

Hence
$\langle (g \oplus 1)A, (g \odot 1)A'\rangle_\alpha = \langle (gA \vee A), (gA' \wedge A')\rangle_\alpha$

iv) Let us consider $g = 0.2, \alpha = 0.2$.

$$\langle (g \odot 1)A, (g \oplus 1)A'\rangle_\alpha = \begin{pmatrix} \langle 0.04,0.8\rangle & \langle 0.1,0.5\rangle \\ \langle 0.02,0.9\rangle & \langle 0.16,0.2\rangle \end{pmatrix}_{0.2}$$

$$\langle 0.2A, A'\rangle_{0.2} = \begin{pmatrix} \langle 0,0.8\rangle & \langle 0,0.5\rangle \\ \langle 0,0.9\rangle & \langle 0,0.2\rangle \end{pmatrix}$$

$$\langle (gA \wedge A), (gA' \vee A')\rangle_\alpha = \begin{pmatrix} \langle 0.04,0.8\rangle & \langle 0.1,0.5\rangle \\ \langle 0.02,0.9\rangle & \langle 0.16,0.2\rangle \end{pmatrix}_{0.2}$$

$$\langle (0.2A \wedge A), (0.2A' \vee A')\rangle_{0.2} = \begin{pmatrix} \langle 0,0.8\rangle & \langle 0,0.5\rangle \\ \langle 0,0.9\rangle & \langle 0,0.2\rangle \end{pmatrix}$$

$$\begin{pmatrix} \langle 0,0.8\rangle & \langle 0,0.5\rangle \\ \langle 0,0.9\rangle & \langle 0,0.2\rangle \end{pmatrix} = \begin{pmatrix} \langle 0,0.8\rangle & \langle 0,0.5\rangle \\ \langle 0,0.9\rangle & \langle 0,0.2\rangle \end{pmatrix}$$

Hence
$\langle (g \odot 1)A, (g \oplus 1)A'\rangle_\alpha = \langle (gA \wedge A), (gA' \vee A')\rangle_\alpha$.

Lemma 4.1

For any intuitionistic fuzzy matrix A,

i) $\langle A^{[k\oplus 1]}, (A')^{[k\odot 1]}\rangle^{=} \geq \langle A^{[k\oplus 1]}, (A')^{[k\odot 1]}\rangle_{=}$

ii) $\langle A^{[k\odot 1]}, (A')^{[k\oplus 1]}\rangle^{=} \geq \langle A^{[k\odot 1]}, (A')^{[k\oplus 1]}\rangle_{=}$

iii) $\langle (g \oplus 1)A, (g \odot 1)A'\rangle^{=} \geq \langle (g \oplus 1)A, (g \odot 1)A'\rangle_{=}$

iv) $\langle (g \odot 1)A, (g \oplus 1)A'\rangle^{=} \geq \langle (g \odot 1)A, (g \oplus 1)A'\rangle_{=}$

Proof : This proof is similar to theorem 4.1 and 4.2.

Theorem 4.3

For any intuitionistic fuzzy matrix A,then,

i) $\langle \left(A^{[k\oplus 1]}\right)^{=}, \left((A')^{[k\odot 1]}\right)_{=}\rangle = \langle (A^k \vee A)^{=}, ((A')^k \wedge A')_{=}\rangle$

ii) $\langle \left(A^{[k\odot 1]}\right)^{=}, \left((A')^{[k\oplus 1]}\right)_{=}\rangle = \langle (A^k \wedge A)^{=}, ((A')^k \vee A')_{=}\rangle$

iii) $\langle \left(A^{[k\oplus 1]}\right)_{=}, \left((A')^{[k\odot 1]}\right)^{=}\rangle = \langle (A^k \vee A)_{=}, ((A')^k \wedge A')^{=}\rangle$

iv) $\langle \left(A^{[k\odot 1]}\right)_{=}, \left((A')^{[k\oplus 1]}\right)^{=}\rangle = \langle (A^k \wedge A)_{=}, ((A')^k \vee A')^{=}\rangle$

v) $\langle \left((g \oplus 1)A\right)^{=}, \left((g \odot 1)A'\right)_{=}\rangle = \langle (gA \vee A)^{=}, (gA' \wedge A')_{=}\rangle$

vi) $\langle \left((g \odot 1)A\right)^{=}, \left((g \oplus 1)A'\right)_{=}\rangle = \langle (gA \wedge A)^{=}, (gA' \vee A')_{=}\rangle$

vii) $\langle \left((g \oplus 1)A\right)_{=}, \left((g \odot 1)A'\right)^{=}\rangle = \langle (gA \vee A)_{=}, (gA' \wedge A')^{=}\rangle$

viii) $\langle \left((g \odot 1)A\right)_{=}, \left((g \oplus 1)A'\right)^{=}\rangle = \langle (gA \wedge A)_{=}, (gA' \vee A')^{=}\rangle$

Proof :

i) Let $A = \langle a_{ij}, a'_{ij} \rangle$, where $0 \le a_{ij} \le 1$, $0 \le a'_{ij} \le 1 \ \forall\, i,j = 1,2,\ldots,n$ and k be any positive integer such that $\langle (a_{ij}^{k\oplus 1})^{=}, ((a'_{ij})^{k\odot 1})_{=} \rangle = \langle (a_{ij}^{k} \vee a_{ij})^{=}, ((a'_{ij})^{k} \wedge a'_{ij})_{=} \rangle$ and then,

$\langle (A^{[k\oplus 1]})^{=}, ((A')^{[k\odot 1]})_{=} \rangle = \langle (A^{k} \vee A)^{=}, ((A')^{k} \wedge A)_{=} \rangle$.

Similarly,

ii) $\langle (A^{[k\odot 1]})^{=}, ((A')^{[k\oplus 1]})_{=} \rangle = \langle (A^{k} \wedge A)^{=}, ((A')^{k} \vee A')_{=} \rangle$

iii) $\langle (A^{[k\oplus 1]})_{=}, ((A')^{[k\odot 1]})^{=} \rangle = \langle (A^{k} \vee A)_{=}, ((A')^{k} \wedge A')^{=} \rangle$

iv) $\langle (A^{[k\odot 1]})_{=}, ((A')^{[k\oplus 1]})^{=} \rangle = \langle (A^{k} \wedge A)_{=}, ((A')^{k} \vee A')^{=} \rangle$

v) $\langle ((g \oplus 1)A)^{=}, ((g \odot 1)A')_{=} \rangle = \langle (gA \vee A)^{=}, (gA' \wedge A')_{=} \rangle$

vi) $\langle ((g \odot 1)A)^{=}, ((g \oplus 1)A')_{=} \rangle = \langle (gA \wedge A)^{=}, (gA' \vee A')_{=} \rangle$

vii) $\langle ((g \oplus 1)A)_{=}, ((g \odot 1)A')^{=} \rangle = \langle (gA \vee A)_{=}, (gA' \wedge A')^{=} \rangle$

viii) $\langle ((g \odot 1)A)_{=}, ((g \oplus 1)A')^{=} \rangle = \langle (gA \wedge A)_{=}, (gA' \vee A')^{=} \rangle$

Example 4.3

Let $A = \begin{pmatrix} \langle 0.2,0.8 \rangle & \langle 0.5,0.5 \rangle \\ \langle 0.1,0.9 \rangle & \langle 0.8,0.2 \rangle \end{pmatrix}$ be a intuitionistic fuzzy matrix, the

i) Let us consider $k = 2, \alpha = 0.2$.

$\langle (A^{[k\oplus 1]})^{=}, ((A')^{[k\odot 1]})_{=} \rangle$

$= \begin{pmatrix} \langle (0.2)^{=}, (0.64)_{=} \rangle & \langle (0.5)^{=}, (0.25)_{=} \rangle \\ \langle (0.1)^{=}, (0.81)_{=} \rangle & \langle (0.8)^{=}, (0.04)_{=} \rangle \end{pmatrix}$

$\langle A^{0.2}, ((A')^{2})_{0.2} \rangle$

$= \begin{pmatrix} \langle 1,0.64 \rangle & \langle 1,0.25 \rangle \\ \langle 0,0.81 \rangle & \langle 1,0 \rangle \end{pmatrix}$

$\langle (A^{k} \vee A)^{=}, ((A')^{k} \wedge A)_{=} \rangle$

$= \begin{pmatrix} \langle (0.2)^{=}, (0.64)_{=} \rangle & \langle (0.5)^{=}, (0.25)_{=} \rangle \\ \langle (0.1)^{=}, (0.81)_{=} \rangle & \langle (0.8)^{=}, (0.04)_{=} \rangle \end{pmatrix}$

$\langle (A^{2} \vee A)^{0.2}, ((A')^{2} \wedge A)_{0.2} \rangle$

$= \begin{pmatrix} \langle 1,0.64 \rangle & \langle 1,0.25 \rangle \\ \langle 0,0.81 \rangle & \langle 1,0 \rangle \end{pmatrix}$

$\begin{pmatrix} \langle 1,0.64 \rangle & \langle 1,0.25 \rangle \\ \langle 0,0.81 \rangle & \langle 1,0 \rangle \end{pmatrix}$

$= \begin{pmatrix} \langle 1,0.64 \rangle & \langle 1,0.25 \rangle \\ \langle 0,0.81 \rangle & \langle 1,0 \rangle \end{pmatrix}$

Hence

$\langle (A^{[k\oplus 1]})^{=}, ((A')^{[k\odot 1]})_{=} \rangle = \langle (A^{k} \vee A)^{=}, ((A')^{k} \wedge A)_{=} \rangle$.

Similarly,

ii) $\begin{pmatrix} \langle 0,0.8 \rangle & \langle 1,0.5 \rangle \\ \langle 0,0.9 \rangle & \langle 1,0.2 \rangle \end{pmatrix} = \begin{pmatrix} \langle 0,0.8 \rangle & \langle 1,0.5 \rangle \\ \langle 0,0.9 \rangle & \langle 1,0.2 \rangle \end{pmatrix}$

Hence

$\langle (A^{[k\odot 1]})^{=}, ((A')^{[k\oplus 1]})_{=} \rangle = \langle (A^{k} \wedge A)^{=}, ((A')^{k} \vee A')_{=} \rangle$

iii) $\begin{pmatrix} \langle 0.2,1 \rangle & \langle 0.5,1 \rangle \\ \langle 0,1 \rangle & \langle 0.8,0 \rangle \end{pmatrix} = \begin{pmatrix} \langle 0.2,1 \rangle & \langle 0.5,1 \rangle \\ \langle 0,1 \rangle & \langle 0.8,0 \rangle \end{pmatrix}$

Hence

$\langle (A^{[k\oplus 1]})_{=}, ((A')^{[k\odot 1]})^{=} \rangle = \langle (A^{k} \vee A)_{=}, ((A')^{k} \wedge A')^{=} \rangle$

iv) $\begin{pmatrix} \langle 0,1 \rangle & \langle 0.25,1 \rangle \\ \langle 0,1 \rangle & \langle 0.64,1 \rangle \end{pmatrix} = \begin{pmatrix} \langle 0,1 \rangle & \langle 0.25,1 \rangle \\ \langle 0,1 \rangle & \langle 0.64,1 \rangle \end{pmatrix}$

Hence

$\langle (A^{[k\odot 1]})_{=}, ((A')^{[k\oplus 1]})^{=} \rangle = \langle (A^{k} \wedge A)_{=}, ((A')^{k} \vee A')^{=} \rangle$

v) $\begin{pmatrix} \langle 1,0 \rangle & \langle 1,0 \rangle \\ \langle 0,0 \rangle & \langle 1,0 \rangle \end{pmatrix} = \begin{pmatrix} \langle 1,0 \rangle & \langle 1,0 \rangle \\ \langle 0,0 \rangle & \langle 1,0 \rangle \end{pmatrix}$

Hence $\langle ((g\oplus 1)A)^{=}, ((g\odot 1)A')_{=}\rangle = \langle (gA\bar{\vee}A)^{=}, (gA'\wedge A')_{=}\rangle$

vi) $\begin{pmatrix} \langle 0,0.8\rangle & \langle 0,0.5\rangle \\ \langle 0,0.9\rangle & \langle 0,0.2\rangle \end{pmatrix} = \begin{pmatrix} \langle 0,0.8\rangle & \langle 0,0.5\rangle \\ \langle 0,0.9\rangle & \langle 0,0.2\rangle \end{pmatrix}$

Hence $\langle ((g\odot 1)A)^{=}, ((g\oplus 1)A')_{=}\rangle = \langle (gA\tilde{\wedge}A)^{=}, (gA'\vee A')_{=}\rangle$

vii) $\begin{pmatrix} \langle 0.2,0\rangle & \langle 0.5,0\rangle \\ \langle 0,0\rangle & \langle 0.8,0\rangle \end{pmatrix} = \begin{pmatrix} \langle 0.2,0\rangle & \langle 0.5,0\rangle \\ \langle 0,0\rangle & \langle 0.8,0\rangle \end{pmatrix}$

Hence $\langle ((g\oplus 1)A)_{=}, ((g\odot 1)A')^{=}\rangle = \langle (gA\vee A)_{=}, (gA'\wedge A')^{=}\rangle$

viii) $\begin{pmatrix} \langle 0,1\rangle & \langle 0,1\rangle \\ \langle 0,1\rangle & \langle 0,1\rangle \end{pmatrix} = \begin{pmatrix} \langle 0,1\rangle & \langle 0,1\rangle \\ \langle 0,1\rangle & \langle 0,1\rangle \end{pmatrix}$

Hence $\langle ((g\odot 1)A)_{=}, ((g\oplus 1)A')^{=}\rangle = \langle (gA\wedge A)_{=}, (gA'\vee A')^{=}\rangle$

V. RESULT ON COMPLEMENT OF INTUITIONISTIC FUZZY MATRIX

The complement of a intuitionistic fuzzy matrix is used to analysis the complement nature of any system. For example, if A represents the crowdness of a network at a particular time period of date then its complement A^c represents the clearness at the same time period of date. Using the following results we can study the complement nature of a system with the help of original intuitionistic fuzzy matrix. The operator complement obey the De Morgan's laws for the operator $\oplus$ and $\odot$. *The complement is the fuzzy value but it's not complement of intuitionistic fuzzy matrix.*

Theorem 5.1

For any intuitionistic fuzzy matrix A, then

i) $\langle A^{[k\oplus 1]}, (A')^{[k\odot 1]}\rangle^c = \langle (A^k \vee A), ((A')^k \wedge A')\rangle^c$
ii) $\langle A^{[k\odot 1]}, (A')^{[k\oplus 1]}\rangle^c = \langle (A^k \wedge A), ((A')^k \vee A')\rangle^c$
iii) $\langle (g\oplus 1)A, (g\odot 1)A'\rangle^c = \langle (gA\vee A), (gA'\wedge A')\rangle^c$
iv) $\langle (g\odot 1)A, (g\oplus 1)A'\rangle^c = \langle (gA\wedge A), (gA'\vee A')\rangle^c$

Proof :

i) $\langle A^{[k\oplus 1]}, (A')^{[k\odot 1]}\rangle^c = \langle (1-a_{ij}^{k\oplus 1}), (1-a_{ij}'^{\,k\odot 1})\rangle$ and $(A^k)^c = [1-a_{ij}^k]$. Therefore $\langle A^{[k\oplus 1]}, (A')^{[k\odot 1]}\rangle^c = \langle (A^k \vee A), ((A')^k \wedge A')\rangle^c$

ii) $\langle A^{[k\odot 1]}, (A')^{[k\oplus 1]}\rangle^c = \langle (1-a_{ij}^{k\odot 1}), (1-a_{ij}'^{\,k\oplus 1})\rangle$ and $(A^k)^c = [1-a_{ij}^k]$. Therefore $\langle A^{[k\odot 1]}, (A')^{[k\oplus 1]}\rangle^c = \langle (A^k \wedge A), ((A')^k \vee A')\rangle^c$

iii) $\langle (g\oplus 1)A, (g\odot 1)A'\rangle^c = \langle (1-[g\oplus 1]a_{ij}), (1-[g\odot 1]a_{ij}')\rangle$ and $(gA)^c = [1-ga_{ij}]$. Therefore $\langle (g\oplus 1)A, (g\odot 1)A'\rangle^c = \langle (gA\vee A), (gA'\wedge A')\rangle^c$

iv) $\langle (g\odot 1)A, (g\oplus 1)A'\rangle^c = \langle (1-[g\odot 1]a_{ij}), (1-[g\oplus 1]a_{ij}')\rangle$ and $(gA)^c = [1-ga_{ij}]$. Therefore $\langle (g\odot 1)A, (g\oplus 1)A'\rangle^c = \langle (gA\wedge A), (gA'\vee A')\rangle^c$

Example 5.1

Let $A = \begin{pmatrix} \langle 0.2,0.8\rangle & \langle 0.5,0.5\rangle \\ \langle 0.1,0.9\rangle & \langle 0.8,0.2\rangle \end{pmatrix}$ be a intuitionistic fuzzy matrix, the

i) Let us consider $k = 2$

$$\begin{pmatrix} \langle 0.2,0.64\rangle & \langle 0.5,0.25\rangle \\ \langle 0.1,0.81\rangle & \langle 0.8,0.04\rangle \end{pmatrix}^C = \begin{pmatrix} \langle 0.2,0.64\rangle & \langle 0.5,0.25\rangle \\ \langle 0.1,0.81\rangle & \langle 0.8,0.04\rangle \end{pmatrix}^C$$

$$\begin{pmatrix} \langle 0.8,0.36\rangle & \langle 0.5,0.75\rangle \\ \langle 0.9,0.19\rangle & \langle 0.2,0.96\rangle \end{pmatrix} = \begin{pmatrix} \langle 0.8,0.36\rangle & \langle 0.5,0.75\rangle \\ \langle 0.9,0.19\rangle & \langle 0.2,0.96\rangle \end{pmatrix}$$

Hence $\langle A^{[k\oplus 1]}, (A')^{[k\odot 1]}\rangle^C = \langle (A^k \vee A), ((A')^k \wedge A')\rangle^C$.

ii) Let us consider $k = 2$

$$\begin{pmatrix} \langle 0.04,0.8\rangle & \langle 0.25,0.5\rangle \\ \langle 0.01,0.9\rangle & \langle 0.64,0.2\rangle \end{pmatrix}^C = \begin{pmatrix} \langle 0.04,0.8\rangle & \langle 0.25,0.5\rangle \\ \langle 0.01,0.9\rangle & \langle 0.64,0.2\rangle \end{pmatrix}^C$$

$$\begin{pmatrix} \langle 0.96,0.2\rangle & \langle 0.75,0.5\rangle \\ \langle 0.99,0.1\rangle & \langle 0.36,0.8\rangle \end{pmatrix} = \begin{pmatrix} \langle 0.96,0.2\rangle & \langle 0.75,0.5\rangle \\ \langle 0.99,0.1\rangle & \langle 0.36,0.8\rangle \end{pmatrix}$$

Hence $\langle A^{[k\odot 1]}, (A')^{[k\oplus 1]}\rangle^C = \langle (A^k \wedge A), ((A')^k \vee A')\rangle^C$.

iii) Let us consider $k = 0.2$

$$\begin{pmatrix} \langle 0.2,0.16\rangle & \langle 0.5,0.1\rangle \\ \langle 0.1,0.18\rangle & \langle 0.8,0.04\rangle \end{pmatrix}^C = \begin{pmatrix} \langle 0.2,0.16\rangle & \langle 0.5,0.1\rangle \\ \langle 0.1,0.18\rangle & \langle 0.8,0.04\rangle \end{pmatrix}^C$$

$$\begin{pmatrix} \langle 0.8,0.84\rangle & \langle 0.5,0.9\rangle \\ \langle 0.9,0.82\rangle & \langle 0.2,0.96\rangle \end{pmatrix} = \begin{pmatrix} \langle 0.8,0.84\rangle & \langle 0.5,0.9\rangle \\ \langle 0.9,0.82\rangle & \langle 0.2,0.96\rangle \end{pmatrix}$$

Hence

$\langle (g \oplus 1)A, (g \odot 1)A' \rangle^C = \langle (gA \vee A), (gA' \wedge A') \rangle^C$.

iv) Let us consider $k = 0.2$

$$\begin{pmatrix} \langle 0.04,0.8\rangle & \langle 0.1,0.5\rangle \\ \langle 0.02,0.9\rangle & \langle 0.16,0.2\rangle \end{pmatrix}^C = \begin{pmatrix} \langle 0.04,0.8\rangle & \langle 0.1,0.5\rangle \\ \langle 0.02,0.9\rangle & \langle 0.16,0.2\rangle \end{pmatrix}^C$$

$$\begin{pmatrix} \langle 0.96,0.2\rangle & \langle 0.9,0.5\rangle \\ \langle 0.98,0.1\rangle & \langle 0.84,0.8\rangle \end{pmatrix} = \begin{pmatrix} \langle 0.96,0.2\rangle & \langle 0.9,0.5\rangle \\ \langle 0.98,0.1\rangle & \langle 0.84,0.8\rangle \end{pmatrix}$$

Hence

$\langle (g \odot 1)A, (g \oplus 1)A' \rangle^C = \langle (gA \wedge A), (gA' \vee A') \rangle^C$.

Theorem 5.2

For any intuitionistic fuzzy matrix A, then

i) $\langle (A^{[k\oplus 1]})^c, ((A')^{[k\odot 1]})^T \rangle = \langle (A^k \vee A)^c, ((A')^k \wedge A')^T \rangle$

ii) $\langle (A^{[k\odot 1]})^c, ((A')^{[k\oplus 1]})^T \rangle = \langle (A^k \wedge A)^c, ((A')^k \vee A')^T \rangle$

iii) $\langle (A^{[k\oplus 1]})^T, ((A')^{[k\odot 1]})^c \rangle = \langle (A^k \vee A)^T, ((A')^k \wedge A')^c \rangle$

iv) $\langle (A^{[k\odot 1]})^T, ((A')^{[k\oplus 1]})^c \rangle = \langle (A^k \wedge A)^T, ((A')^k \vee A')^c \rangle$

v) $\langle ((g \oplus 1)A)^c, ((g \odot 1)A')^T \rangle = \langle (gA \vee A)^c, (gA' \wedge A')^T \rangle$

vi) $\langle ((g \odot 1)A)^c, ((g \oplus 1)A')^T \rangle = \langle (gA \wedge A)^c, (gA' \vee A')^T \rangle$

vii) $\langle ((g \oplus 1)A)^T, ((g \odot 1)A')^c \rangle = \langle (gA \vee A)^T, (gA' \wedge A')^c \rangle$

viii) $\langle ((g \odot 1)A)^T, ((g \oplus 1)A')^c \rangle = \langle (gA \wedge A)^T, (gA' \vee A')^c \rangle$

Proof :

i) $\langle (A^{[k\oplus 1]})^c, ((A')^{[k\odot 1]})^T \rangle = \langle [1 - a_{ij}^{k\oplus 1}], [1 - a'^{(k\odot 1)}_{ji}] \rangle$.

Therefore

$\langle (A^{[k\oplus 1]})^c, ((A')^{[k\odot 1]})^T \rangle = \langle (A^k \vee A)^c, ((A')^k \wedge A')^T \rangle$.

Similarly,

ii) $\langle (A^{[k\odot 1]})^c, ((A')^{[k\oplus 1]})^T \rangle = \langle (A^k \wedge A)^c, ((A')^k \vee A')^T \rangle$

iii) $\langle (A^{[k\oplus 1]})^T, ((A')^{[k\odot 1]})^c \rangle = \langle (A^k \vee A)^T, ((A')^k \wedge A')^c \rangle$

iv) $\langle (A^{[k\odot 1]})^T, ((A')^{[k\oplus 1]})^c \rangle = \langle (A^k \wedge A)^T, ((A')^k \vee A')^c \rangle$

v) $\langle ((g \oplus 1)A)^c, ((g \odot 1)A')^T \rangle = \langle (gA \vee A)^c, (gA' \wedge A')^T \rangle$

vi) $\langle ((g \odot 1)A)^c, ((g \oplus 1)A')^T \rangle = \langle (gA \wedge A)^c, (gA' \vee A')^T \rangle$

vii) $\langle ((g \oplus 1)A)^T, ((g \odot 1)A')^c \rangle = \langle (gA \vee A)^T, (gA' \wedge A')^c \rangle$

viii) $\langle ((g \odot 1)A)^T, ((g \oplus 1)A')^c \rangle = \langle (gA \wedge A)^T, (gA' \vee A')^c \rangle$

Example 5.2

Let $A = \begin{pmatrix} \langle 0.2, 0.8\rangle & \langle 0.5,0.5\rangle \\ \langle 0.1,0.9\rangle & \langle 0.8,0.2\rangle \end{pmatrix}$ be a intuitionistic fuzzy matrix, the

i) Let us consider $k = 2$

$$\langle (A^{[k\odot 1]})^C, ((A')^{[k\oplus 1]})^T \rangle = \left\langle \begin{bmatrix} 0.2 & 0.5 \\ 0.1 & 0.8 \end{bmatrix}^C, \begin{bmatrix} 0.64 & 0.25 \\ 0.81 & 0.04 \end{bmatrix}^T \right\rangle$$

$$= \left\langle \begin{bmatrix} 0.8 & 0.5 \\ 0.9 & 0.2 \end{bmatrix}, \begin{bmatrix} 0.64 & 0.81 \\ 0.25 & 0.04 \end{bmatrix} \right\rangle$$

$$= \begin{pmatrix} \langle 0.8, 0.64\rangle & \langle 0.5,0.81\rangle \\ \langle 0.9,0.25\rangle & \langle 0.2,0.04\rangle \end{pmatrix}$$

$$\langle (A^k \wedge A)^C, ((A')^k \vee A')^T \rangle = \begin{pmatrix} \langle 0.8, 0.64\rangle & \langle 0.5,0.81\rangle \\ \langle 0.9,0.25\rangle & \langle 0.2,0.04\rangle \end{pmatrix}$$

$$\begin{pmatrix} \langle 0.8, 0.64\rangle & \langle 0.5,0.81\rangle \\ \langle 0.9,0.25\rangle & \langle 0.2,0.04\rangle \end{pmatrix} = \begin{pmatrix} \langle 0.8, 0.64\rangle & \langle 0.5,0.81\rangle \\ \langle 0.9,0.25\rangle & \langle 0.2,0.04\rangle \end{pmatrix}$$

Hence

$\langle (A^{[k\oplus 1]})^C, ((A')^{[k\odot 1]})^T \rangle = \langle (A^k \vee A)^C, ((A')^k \wedge A')^T \rangle$

Similarly,

ii) $\begin{pmatrix}(0.96,0.8) & (0.75,0.9)\\(0.99,0.5) & (0.36,0.2)\end{pmatrix} = \begin{pmatrix}(0.96,0.8) & (0.75,0.9)\\(0.99,0.5) & (0.36,0.2)\end{pmatrix}$

Hence $((A^{[k\odot 1]})^c, ((A')^{[k\oplus 1]})^T) = ((A^k \wedge A)^c, ((A')^k \vee A')^T)$

iii) $\begin{pmatrix}(0.2,0.36) & (0.1,0.75)\\(0.5,0.19) & (0.8,0.96)\end{pmatrix} = \begin{pmatrix}(0.2,0.36) & (0.1,0.75)\\(0.5,0.19) & (0.8,0.96)\end{pmatrix}$

Hence $((A^{[k\oplus 1]})^T, ((A')^{[k\odot 1]})^c) = ((A^k \vee A)^T, ((A')^k \wedge A')^c)$

iv) $\begin{pmatrix}(0.04,0.2) & (0.01,0.5)\\(0.25,0.1) & (0.64,0.8)\end{pmatrix} = \begin{pmatrix}(0.04,0.2) & (0.01,0.5)\\(0.25,0.1) & (0.64,0.8)\end{pmatrix}$

Hence $((A^{[k\odot 1]})^T, ((A')^{[k\oplus 1]})^c) = ((A^k \wedge A)^T, ((A')^k \vee A')^c)$

Let us consider $g = 0.2$

v) $\begin{pmatrix}(0.8,0.16) & (0.5,0.18)\\(0.9,0.1) & (0.2,0.04)\end{pmatrix} = \begin{pmatrix}(0.8,0.16) & (0.5,0.18)\\(0.9,0.1) & (0.2,0.04)\end{pmatrix}$

Hence $(((g \oplus 1)A)^c, ((g \odot 1)A')^T) = ((gA \vee A)^c, (gA' \wedge A')^T)$

vi) $\begin{pmatrix}(0.96,0.8) & (0.9,0.9)\\(0.98,0.5) & (0.16,0.2)\end{pmatrix} = \begin{pmatrix}(0.96,0.8) & (0.9,0.9)\\(0.98,0.5) & (0.16,0.2)\end{pmatrix}$

Hence $(((g \odot 1)A)^c, ((g \oplus 1)A')^T) = ((gA \wedge A)^c, (gA' \vee A')^T)$

vii) $\begin{pmatrix}(0.2,0.84) & (0.1,0.9)\\(0.5,0.82) & (0.8,0.96)\end{pmatrix} = \begin{pmatrix}(0.2,0.84) & (0.1,0.9)\\(0.5,0.82) & (0.8,0.96)\end{pmatrix}$

Hence $(((g \oplus 1)A)^T, ((g \odot 1)A')^c) = ((gA \vee A)^T, (gA' \wedge A')^c)$

viii) $\begin{pmatrix}(0.04,0.2) & (0.02,0.5)\\(0.1,0.1) & (0.16,0.8)\end{pmatrix} = \begin{pmatrix}(0.04,0.2) & (0.02,0.5)\\(0.1,0.1) & (0.16,0.8)\end{pmatrix}$

Hence $(((g \odot 1)A)^T, ((g \oplus 1)A')^c) = ((gA \wedge A)^T, (gA' \vee A')^c)$

VI. CONCLUSION

In this paper , we are discussed in use the upper and lower α – cut to find the intuitionistic fuzzy matrix A represents the crowdness status of the network N at any time period and date of year. If we consider the crowdness as two states.

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