Finding Shortest Path For An Octagonal Fuzzy Network

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Abstract- *In this paper, I have presented two algorithms to find the shortest path from a source node to a destination node of a network. We also solved a numerical example in which each edge assigns an octagonal fuzzy number.*

Keywords- Euclidean distance, network, octagonal fuzzy number, shortest path, $\alpha - c$ set.

I. INTRODUCTION

In a network, the most common problem and the main objective is to find the shortest path for a given network. This problem is circumvented with the model based on the paper 'fuzzy shortest path with α-cuts' presented by P.Sandhiya[1]. The fuzzy shortest path, which is used in many sources of applications such as route, communication, transportation etc.

In this paper, section 1 provides some elementary definitions need for this paper. In section 2, I proposed two algorithms for finding the shortest path for an octagonal fuzzy number network problem. In section3, a numerical example was solved. In section 4, some conclusions were discussed.

I. PRELIMINARIES

A. FUZZY SET

A fuzzy set A of a universal set X is defined by its membership function $\mu_A : X \to [0,1]$ which assigns real number $\mu_A(x)$ in the interval [0,1] to each element x$\in$X, where the value of $\mu_A(x)$ at x shows the grade of membership of x in A.

B. OCTAGONAL FUZZY NUMBER

Let A={h, a, r, s, i, n, k, d} be an octagonal fuzzy number whose membership function is given by

$$\mu_A(x) = \begin{cases} 0 & x < h \\ \frac{x-h}{a-h} & h \le x \le a \\ \frac{x-a}{r-a} & a \le x \le r \\ \frac{x-r}{s-r} & r \le x \le s \\ 1 & s \le x \le i \\ \frac{n-x}{n-i} & i \le x \le n \\ \frac{k-x}{k-n} & n \le x \le k \\ \frac{d-x}{d-k} & k \le x \le d \\ 0 & x > d \end{cases}$$

C. $\alpha -$CUT FOR OCTAGONAL FUZZY NUMBER:

Let A be the fuzzy set in X and any real number $\alpha \in [0,1]$, it is denoted by α_A= {x$\in$X/$\mu_A(x) \ge \alpha$} whose membership function is given by

$$\mu_A(x) = \begin{cases} \frac{x-h}{a-h} & h \le x \le a \\ \frac{x-a}{r-a} & a \le x \le r \\ \frac{x-r}{s-r} & r \le x \le s \\ \frac{n-x}{n-i} & i \le x \le n \\ \frac{k-x}{k-n} & n \le x \le k \\ \frac{d-x}{d-k} & k \le x \le d \end{cases}$$

Then the $\alpha - cut$ of A is $\alpha = \frac{x-h}{a-h}, \alpha = \frac{x-a}{r-a}$,
$\alpha = \frac{x-r}{s-r}, \alpha = \frac{n-x}{n-i}, \alpha = \frac{k-x}{k-n}$ & $\alpha = \frac{d-x}{d-k}$

Equating in terms of x, we get

α_A = $[(a-h)\alpha + h, (r-a)\alpha + a, (s-r)\alpha + r, n - (n-i)\alpha, k - (k-n)\alpha, d - (d-k)\alpha]$

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D. OPERATION ON OCTOGONAL FUZZY NUMBER:

If $\hat{A}$ = (h, a, r, s, i, n, k, d) and $\tilde{B}$ = (h', a', r', s', i', n', k', d') then

Addition $\tilde{A} \oplus \tilde{B}$= (h+ h', a+ a', r+ r', s+ s', i+ i', n+ n', k+ k', d+ d')

Subtraction $\tilde{A} \ominus \tilde{B}$= (h - h', a - a', r - r', s- s', i - i', n- n', k- k', d- d')

E. MINIMUM VALUE FOR α-CUTS:

Let $\alpha_A = [q, r, s, t, u, v]$ and $\alpha_B = [y, z, f, g, j, k]$ then the minimum value of α_A and α_B is given by

MV = min [α_A, α_B]

F. EUCLIDEAN DISTANCE:

Let A= [a, b, c, d, e, f] and B=[g, h, i, j, k, l] be two intervals, then the Euclidean distance D is defined as

$$D=\sqrt{(g-a)^2+((h-b)^2+(i-c)^2+(j-d)^2+(k-e)^2+(h-f)^2}$$

II. ALGORITHMS

METHOD 1:

STEP 1: FINDING ALL POSSIBLE PATHS

i. Find all the possible paths from a source node to a destination node.
ii. Assign the number of possible paths in an acyclic network.

STEP 2: COMPUTATION OF LENGTH OF PATHS

i. Set α value between [0, 1].
ii. Find the α- cut for every edge.
iii. By adding the α-cuts, find the length of all the possible paths.

STEP 3: COMPARISON OF PATHS

i. Assume $L_{min} = L_1$
ii. For i = 2 to n

MV = min {L_{min}, L_i}

D_1 = D {MV,L_{min}}

D_2 = D {MV,L_i}

If $D_1 < D_2$ then $L_{min} = L_{min}$

Otherwise $L_{min} = L_i$.

STEP 4: The shortest path is the corresponding path of L_{min}.

METHOD 2:

STEP 1: FINDING SHORTEST PATH

i. Find all the possible paths from a source node to a destination node.
ii. Assign the number of possible paths in an acyclic network.

STEP 2: COMPUTATION OF LENGTH OF PATHS

i. Set α value between [0, 1].
ii. Find the α- cut for every edge.

STEP 3: Assume $\overline{a_1} = (0,0,0,0,0,0)$ as a source node.

STEP 4: Find $\overline{a_j} = \min\{\overline{a_i} \oplus \overline{a_{ij}}\}$, where j$\neq$ 1, j=2,3,4,……,n and i={N/N=(1,2,3,…(j-1))}

STEP 5: Find P = i$\rightarrow j$which is the minimum value of $\overline{a_j}$. Find P for all the nodes, starting from a source node to destination node.

STEP 6: Discard the path which don't have link either directly (D) or indirectly (ID) with a source node and destination node.

STEP 7: Compare the remaining paths to get the fuzzy shortest path from a source node to a destination node.

NOTATIONS:

$\overline{a_i}$ = The membership function distance between node i and source node.

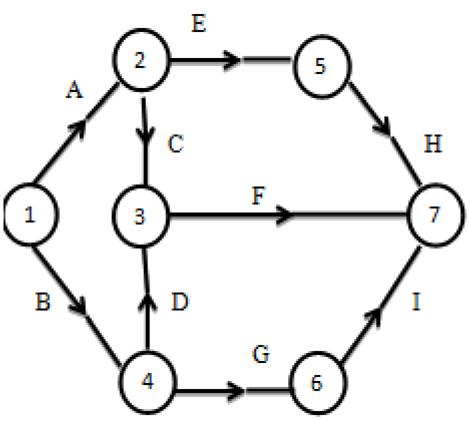
$\overline{a_{ij}}$ = The fuzzy distance of the octagonal network between the nodesiand j

P = path of corresponding node in $\overline{a_j}$.

N = Number of nodes in a network.

III. NUMERICAL EXAMPLE

Consider a fuzzy acyclic network whose edge weights are taken as the octagonal fuzzy number.



The fuzzy arc lengths are

A (1-2) = (0, 2, 3, 4, 5, 6, 7, 7)
B (1-4) = (4, 4, 5, 6, 7, 7, 8, 9)
C (2-3) = (3, 4, 4, 5, 6, 6, 7, 8)
D (4-3) = (0, 5, 5, 6, 7, 7, 9, 10)
E (2-5) = (6, 7, 8, 9, 10, 11, 12, 13)
F (3-7) = (10, 10, 12, 14, 15, 16, 18, 20)
G (4-6) = (8, 9, 10, 11, 12, 14, 15, 16)
H (5-7) = (13, 13, 15, 16, 17, 18, 19, 20)
I (6-7) = (12, 12, 14, 15, 15, 16, 18, 20)

METHOD 1:

STEP 1: FINDING ALL POSSIBLE PATHS

- Possible paths are

P1: $A \to E \to H$
P2: $A \to C \to F$
P3: $B \to D \to F$
P4: $B \to G \to I$

- The number of possible paths is 4.

STEP 2: COMPUTATION OF LENGTH OF PATHS

- Set α value between 0 and 1.

Let $\alpha = 0.5$

- $\alpha -$cuts for every edge

$\alpha_A = ($1, 2.5, 3.5, 5.5, 6.5, 7)
$\alpha_B = ($4, 4.5, 5.5, 7, 7.5, 8.5)
$\alpha_C = ($3.5, 4, 4.5, 6, 6.5, 7.5)
$\alpha_D = ($2, 5, 5.5, 7, 8, 9.5)
$\alpha_E = ($6.5, 7.5, 8.5, 10.5, 11.5, 12.5)
$\alpha_F = ($10, 11, 13, 15.5, 17, 19)
$\alpha_G = ($8.5, 9.5, 10.5, 13, 14.5, 15.5)
$\alpha_H = ($13, 14, 15.5, 17.5, 18.5, 19.5)
$\alpha_I = ($12, 13, 14.5, 15.5, 17, 19)

- Length of the possible path

L1: (20.5, 24, 27.5, 33.5, 36.5, 39)
L2: (14.5, 17.5, 21, 27, 30, 33.5)
L3: (16, 20.5, 24, 29.5, 32.5, 36)
L4: (24.5, 27, 34.5, 35.5, 37.5, 43)

STEP 3: COMPARISON OF PATHS

- Assume $L_{min} = (20.5, 24, 27.5, 33.5, 36.5, 39)$
- For i = 2

MV = min {L_{min} , L_2}
= (14.5, 17.5, 21, 27, 30, 33.5)
D_1 = D {MV,L_{min}}
= 15.3379
D_2 = D {MV,L_2}
= 0
$\therefore D_1 > D_2$,
$L_{min} = (14.5, 17.5, 21, 27, 30, 33.5)$.

- For i = 3

MV = min {L_{min} , L_3}
= (14.5, 17.5, 21, 27, 30, 33.5)
D_1 = D {MV,L_{min}}
= 0
D_2 = D {MV,L_3}
= 6.2449
$\therefore D_1 < D_2$,
$L_{min} = (14.5, 17.5, 21, 27, 30, 33.5)$.

- For i = 4

MV = min {L_{min} , L_4}
= (14.5, 17.5, 21, 27, 30, 33.5)
D_1 = D {MV,L_{min}}
= 0
D_2 = D {MV,L_3}
= 24.3156
$\therefore D_1 < D_2$,
$L_{min} = (14.5, 17.5, 21, 27, 30, 33.5)$.

STEP 4: The shortest path is P2: $A \to C \to F$.

METHOD 2:

STEP 1: FINDING ALL POSSIBLE PATHS

- Possible paths are

P1: $A \to E \to H$
P2: $A \to C \to F$

P3: $B \to D \to F$

P4: $B \to G \to I$

- The number of possible paths is 4.

STEP 2: COMPUTATION OF LENGTH OF PATHS

- Set α value between 0 and 1.

Let $\alpha = 0.5$

- $\alpha -$cuts for every edge

$\alpha_A = ($1, 2.5, 3.5, 5.5, 6.5, 7)

$\alpha_B = ($4, 4.5, 5.5, 7, 7.5, 8.5)

$\alpha_C = ($3.5, 4, 4.5, 6, 6.5, 7.5)

$\alpha_D = ($2, 5, 5.5, 7, 8, 9.5)

$\alpha_E = ($6.5, 7.5, 8.5, 10.5, 11.5, 12.5)

$\alpha_F = ($10, 11, 13, 15.5, 17, 19)

$\alpha_G = ($8.5, 9.5, 10.5, 13, 14.5, 15.5)

$\alpha_H = ($13, 14, 15.5, 17.5, 18.5, 19.5)

$\alpha_I = ($12, 13, 14.5, 15.5, 17, 19)

STEP 3:

Assume $\overline{a_1} = (0,0,0,0,0,0)$ as a source node.

STEP 4&5:

- Put $i = 1 \; and \; j = 2$

$\overline{a_2} = \min\{\overline{a_1} \oplus \overline{a_{12}}\}$

= (1, 2.5, 3.5, 5.5, 6.5, 7)

- $P: 1 \to 2$

- Put i$= 1,2 \; and \; j = 3$

$\overline{a_3} = \min\{\overline{a_1} \oplus \overline{a_{13}}, \overline{a_2} \oplus \overline{a_{23}}\}$

= (4.5, 6.5, 8, 11.5, 13, 14.5)

- $P: 2 \to 3$

- Put i$= 1,2,3 \; and \; j = 4$

$\overline{a_4} = \min\{\overline{a_1} \oplus \overline{a_{14}}, \overline{a_2} \oplus \overline{a_{24}}, \overline{a_3} \oplus \overline{a_{34}}\}$

= min {(4, 4.5, 5.5, 7, 7.5, 8.5),(6.5, 11.5, 13.5, 18.5, 21, 24)

=(4, 4.5, 5.5, 7, 7.5, 8.5)

- $P: 1 \to 4$

- Put i$= 1,2,3,4 \; and \; j = 5$

$\overline{a_5} = \min\{\overline{a_1} \oplus \overline{a_{15}}, \overline{a_2} \oplus \overline{a_{25}}, \overline{a_3} \oplus \overline{a_{35}}, \overline{a_4} \oplus \overline{a_{45}}\}$

= (7.5, 10, 12, 16, 18, 19.5)

- $P: 2 \to 5$

- Put i$= 1,2,3,4,5 \; and \; j = 6$

$\overline{a_6} = \min\{\overline{a_1} \oplus \overline{a_{16}}, \overline{a_2} \oplus \overline{a_{26}}, \overline{a_3} \oplus \overline{a_{36}}, \overline{a_4} \oplus \overline{a_{46}}, \overline{a_5} \oplus \overline{a_{56}}\}$

= (12.5, 14, 16, 20, 22, 24)

- $P: 4 \to 6$

- Put i$= 1,2,3,4,5,6 \; and \; j = 7$

$\overline{a_7} = \min\{\overline{a_1} \oplus \overline{a_{17}}, \overline{a_2} \oplus \overline{a_{27}}, \overline{a_3} \oplus \overline{a_{37}}, \overline{a_4} \oplus \overline{a_{47}}, \overline{a_5} \oplus \overline{a_{57}}, \overline{a_6} \oplus \overline{a_{67}}\}$

= min {(14.5, 17.5, 21, 27, 30, 33.5),
(20.5, 24, 27.5, 32.5, 36.5, 39),
(24.5, 27, 30.5, 35.5, 39, 43)}

= (14.5, 17.5, 21, 27, 30, 33.5)

- $P: 3 \to 7$

STEP 6:

j	Path	D/ID Link with source node	D/ID Link with destination node	D/ID Link with source & destination node
2	$1 \to 2$	Yes(DL)	Yes(IDL)	YES
3	$2 \to 3$	Yes(IDL)	Yes(IDL)	YES
4	$1 \to 4$	Yes(DL)	NO	DISCARD
5	$2 \to 5$	Yes(IDL)	NO	DISCARD
6	$4 \to 6$	Yes(IDL)	NO	DISCARD
7	$3 \to 7$	Yes(IDL)	Yes(DL)	YES

Here D stands for direct link and
IDL stands for indirect link.

STEP 7:

The optimized fuzzy shortest path is $1 \to 2 \to 3 \to 7$.

(i.e.) P2: $A \to C \to F$

IV. CONCLUSION

In this paper, I presented two algorithms with the numerical example for finding the shortest path using $\alpha - cut$ set and Euclidean distance in which each edge is assigned to an octagonal fuzzy number. The fuzzy shortest path problem is solved for each node to a destination node and hence the optimized shortest path is obtained.

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