On Micro-B-Open Sets And Micro-B-Continuous In Micro Topological Spaces

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Abstract- *Every year different type of topological spaces are introduced by many topologist. Micro topology is a simple extension of Nano topology. Micro topology provides wide range of interesting results and applications. But some time we want extend some open sets in Micro topology. In this paper we introduce Micro-b-open sets and Micro-b-continuous in Micro topological spaces. Also we investigate some of their properties.*

Keywords- Micro-b-open sets, Micro-b-interior, Micro-b-closure, Micro-b-continuous.

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I. INTRODUCTION

In 1996, D. Andrijevic [1] introduced and studied a class of generalized open sets in a topological space called b-open sets. This class of sets contained in the class of β open sets and contains all semi-open sets and all pre-open sets. In 2013, notion of nano topology was introduced by Lellis Thivagar [4] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it. In 2018, Micro topology was introduced by S. Chandrasekar [7]. In this paper, we introduce a new class of sets on Micro topological spaces called Micro-b-open sets and the relation of this sets with existing sets.

II. PRELIMINARIES

Definition 2.1[7]: $(U,\tau_R(X))$ is a Nano topological space here $\mu_R(X) = \{N \cup (N' \cap \mu)\}: N, N' \in \tau_R(X)$ and is called it Micro topology of $\tau_R(X)$ by μ where $\mu \notin \tau_R(X)$.

Definition 2.2[7]: The Micro topology $\mu_R(X)$ satisfies the following axioms:

(i) $U, \emptyset \in \mu_R(X)$.

(ii) The union of the elements of any sub-collection of $\mu_R(X)$ is in $\mu_R(X)$.

(iii) The intersection of the elements of any finite sub-collection of $\mu_R(X)$ is in $\mu_R(X)$.

Then $\mu_R(X)$ is called the Micro topology on U with respect to X. The triplet $(U,\tau_R(X),\mu_R(X))$ is called Micro topological spaces and the element of $\mu_R(X)$ are called Micro open sets and the complement of a Micro open set is called a Micro closed set.

Definition 2.3[7, 8]: Let $(U,\tau_R(X),\mu_R(X))$ be a Micro topological space and $A \subseteq U$.

Then A is said to be:

(i) Micro-semi-open if $A \subseteq Mic-cl[Mic-int(A)]$ and Micro-semi-closed if $Mic-int[Mic-cl(A)] \subseteq A$.

(ii) Micro-pre-open if $A \subseteq Mic-int[Mic-cl(A)]$ and Micro-pre-closed if $Mic-cl[Mic-int(A)] \subseteq A$.

(iii) Micro-α-open if $A \subseteq Mic-int[Mic-cl(Mic-int(A))$ and Micro-α-closed if $Mic-cl[Mic-int(Mic-cl(A)] \subseteq A$.

(iv) Micro-regular-open if $A = Mic-int[Mic-cl(A)]$ and Micro-regular-closed if $Mic-cl[Mic-int(Mic-cl(A)] = A$.

(v) Micro-semi-pre-open if $A \subseteq Mic-cl\ [Mic-int(Mic-cl(A)]$ and Micro-semi-pre-closed if $Mic-int[Mic-cl(A)] \subseteq A$.

$NSO(U,X), NPO(U,X), NRO(U,X), NSPO(U,X)$, and $N\alpha O(U,X)$ denote the families of all Micro-semi-open, Micro-pre-open, Micro-regular-open, Micro-semi-pre-open and Micro-α-open subsets of U respectively.

III. MICRO-b-OPEN SET

Definition 3.1: Let (U,τ_R(X),μ_R(X)) be Micro topological space and $A \subseteq U$. Then A is said to be Micro-b-open (briefly Micro-b-open set) if $A \subseteq Mi-cl(Mic-int(A)) \cup Mic-int(Mic-cl(A))$. The complement of Micro-b-open set is called a Micro-b-closed set. (briefly Micro-b-closed).

Example 3.2: Let $U=$ {1,2,3,4} with U/R={{1},{3},{2,4}} and X={1,2}.Then the nano topology τ_R(X)={U,$\emptyset$,{1},{1,2,4},{2,4}} and μ={3}. Micro-b-open sets are μ_R(X)={U,$\emptyset$,{1}, {2},{3},{4},{1,2},{1,4},{1,3},{2,4},{1,2,4}, {2,3,4},{1,3,4},{1,2,3},{2,3},{3,4}}.

Theorem 3.3: Every Micro open set is Micro-b-open.

Proof: Let A be Micro open in (U,τ_R(X),μ_R(X)). Since $A=Mic-int(A)$ and $A \subseteq Mic-cl(A),\ Mic-int(A) \subseteq Mic-int(Mic-cl(A))$ and $Mic-int(A) \subseteq Mic-cl(Mic-int(A))$. This implies $Mic-int(A) \subseteq Mic-cl(Mic-int(A)) \cup Mic-int(Mic-cl(A)).Hence\ A \subseteq Mic-int(A) \subseteq Mic-cl(Mic-int(A)) \cup Mic-int(Mic-cl(A))$ and A is Micro-b-open in (U,τ_R(X),μ_R(X)).

Remark 3.4: The converse of the above theorem need not be true as shown in the following example.

Example 3.5: Let U={a,b,c,d} with U/R={{a},{c},{b,d}} and X={a,b}. Then the nano topology τ_R(X)={U,$\emptyset$,{a},{b,d},{a,b,d}} and μ={b}. μ_R(X)={$\cup$,$\emptyset$,{a},{b},{a,b}{b,d}, {a,b,d}}. Mic-b-O (X) ={{a},{b},{a,b},{a,c}, {b,d},{b,c},{a,b,c},{a,b,d}, {b,c,d}}. Here {{a,c},{b,c},{a,b,c},{b,c,d}}Micro-b-open but it is not Micro open.

Theorem 3.6: Every Micro-semi-open set is Micro-b-open.

Proof: Let A be Micro-semi-open in (U,τ_R(X),μ_R(X)). Then $A \subseteq Mic-cl(Mic-int(A))$. Hence $A \subseteq Mic-cl(Mic-int(A)) \cup Mic-int(Mic-cl(A))$ and A is Micro-b-open in (U,τ_R(X),μ_R(X)).

Remark 3.7: The converse of the above theorem need not be true as shown in the following example.

Example 3.8: Let $U=${1,2,3,4}with U/R={{1},{3},{2,4}}and X={1,2}.Then the nano topology τ_R(X)={U,$\emptyset$,{1},{2,4},{1,2,4}}and μ={3}. μ_R(X)={$\cup$,$\emptyset$,{1},{3},{1,3}{2,4},{1,2,4}, {2,3,4}}. Mic-b-O (X)$=\{U$,$\emptyset$,{1},{3},{1,3}, {2,4},{1,2,4},{2,3,4},{2},{4},{1,2}, {1,4}, {1,3,4},{1,2,3},{2,3},{3,4}}. Here {{2},{4}, {1,2},{2,3},{1,4},{3,4},{1,3,4},{1,2,3}} is Micro-b-open but it is not Micro-semi-open.

Theorem 3.9: Every Micro-pre-open set is Micro-b-open.

Proof: Let A be Micro-pre-open in (U,τ_R(X),μ_R(X)). Then $A \subseteq Mic-int(Mic-cl(A))$. Hence $Mic-cl(A) \subseteq Mic-cl(Mic-int(A)) \cup Mic-int(Mic-cl(A))$ and A is Micro-b-open in (U,τ_R(X),μ_R(X)).

Remark 3.10: The converse of the above theorem need not be true as shown in the following example.

Example 3.11: Let U= {1,2,3,4}with U/R={{1},{3},{2,4}} and X = {1,2}.Then the nano topology τ_R(X) = {U,$\emptyset$,{1},{2,4},{1,2,4}} and μ={2}. μ_R(X)={$\cup$,$\emptyset$,{1},{2},{1,3},{2,4}, {1,2},{1,2,4}}. Mic-b-O (X)={U,$\emptyset$, {1},{2},{3}, {4},{1,3}, {2,4},{1,2},{1,4},{2,3},{3,4},{1,2,3}, {1,2,4}, {2,3,4},{1,3,4}. Here{{1,3},{2,3},{2,3,4}} Micro-b-open but it is not Micro-pre-open.

Theorem 3.12: Every Micro-regular-open set is Micro-b-open.

Proof: Let A be Micro-regular-open in (U,τ_R(X),μ_R(X)). Then $A=Mic-int(Mic-cl(A))$. Since

$A \subseteq Mic - cl(A), Mic - int(A) \subseteq Mic - cl(Mic - int(A))$.

This implies $A \subseteq Mic - cl\ (Mic - int(A))$. Therefore $A \subseteq Mic - cl(Mic - int(A)) \cup Mic - int\ (Mic - cl(A))$
and A is Micro-b-open in (U,τ_R(X),μ_R(X)).

Remark 3.13: The converse of the above theorem need not be true as shown in the following example.

Example 3.14: Let U = {a,b,c,d} with U/R= {{a},{c},{b,d}} and X={a,b}. Then the nano topology τ_R(X) = {U,$\emptyset$,{a},{b,d},{a,b,d}} and μ={b}. μ_R(X)={$\cup$,$\emptyset$,{a},{b},{a,b}{b,d}, {a,b,d}}. Mic-b-O (X) = {{a},{b},{a,b}{a,c}, {b,d},{b,c},{a,b,c},{a,b,d},{b,c,d}}. Here {{b}, {a,b}{a,c},{b,c},{a,b,c},{a,b,d}, {b,c,d}} Micro-b-open but it is not Micro- regular-open.

Theorem 3.15: Every Micro-b-open set is Micro-semi-pre-open.

Proof: Let A be Micro-b-open in (U, τ_R(X),μ_R(X)). Then $A \subseteq Mic - cl\big(Mic - int(A)\big) \cup Mic - int\big(Mic - cl(A)\big)$. This implies $A \subseteq Mic - cl\big(Mic - int(A)\big)$ and $A \subseteq Mic - int\big(Mic - cl(A)\big)$. *This implies* $Mic - cl(A) \subseteq Mic - cl(Mic - int(Mic - cl(A))$.
Hence
$A \subseteq Mic - cl\big(Mic - int(A)\big) \cup Mic - int\big(Mic - cl(A)\big) \subseteq Mic - cl\,(Mic - int\,(Mic - cl(A)$
and A is semi-pre-open in (U, τ_R(X), μ_R(X)).

Remark 3.16: The converse of the above theorem need not be true as shown in the following example.

Example 3.17: Let U= {a,b,c,d} with U/R={{a},{d},{b,c}} and X = {a,c}. Then the nano topology τ_R(X) = {U,$\emptyset$,{a},{b,c},{a,b,c}} and μ={b}. μ_R(X)={$\cup$,$\emptyset$,{a},{b},{a,b},{b,c}, {a,b,c}}. Mic-b-O(X)={U,$\emptyset$,{a},{b},{c},{a,b}, {a,c},{a,d},{b,c},{b,d},{c,d},{a,b,c},{a,b,d}, {a,c,d},{b,c,d}}. Here {{b,d},{b,c}}Micro-semi-pre-open but it is not Micro-b-open.

Theorem 3.18: Every Micro-α-open set is Micro b-open.

Proof : Let A be Micro-α-open in (U,τ_R(X),μ_R(X)). Then $A \subseteq Mic - int(Mic - cl(Mic - int(A))$. This implies $A \subseteq Mic - int(Mic - cl(M$ic-$int(A)) \subseteq Mic - cl(Mic - int(A)) \subseteq Mic - cl(Mic - int(A))$.

Hence
$A \subseteq Mic - cl(Mic - int(A)) \cup Mic - int(Mic - cl(A))$
and A is Micro-b-open in (U,τ_R(X),μ_R(X)).

Remark 3.19: The converse of the above theorem need not be true as shown in the following example.

Example 3.20: Let U = {a,b,c,d} with U/R ={{a},{c},{b,d}}and X={a,b}.Then the nano topology τ_R(X) = {U,$\emptyset$,{a},{b,d},{a,b,d}} and μ={b}. μ_R(X)={$\cup$,$\emptyset$,{a},{b},{a,b},{b,d}, {a,b,d}. Mic-b-O(X)={a},{b},{a,b}{a,c},{b,d}, {b,c},{a,b,c},{a,b,d},{b,c,d}.Here {{a,c},{b,c}, {b,c,d}} is Micro-b-open but it is not Micro-α-open.

Theorem 3.21: Arbitrary union of two Micro-b-open sets in (U, τ_R(X),μ_R(X)) is a Micro-b-open sets in (U,τ_R(X),μ_R(X)).

Proof: Let A and B be two Micro-b-open sets. Then $A \subseteq Mic - cl(Mic - int(A)) \cup Mic - int(Mic - cl(A))$
and
$B \subseteq Mic - cl(Mic - int(B)) \cup Mic - int(Mic - cl(B))$.

Then
$A \cup B \subseteq [Mic - cl(Mic - int(A)) \cup M\ Mic - int(Mic - cl(A))] \cup [Mic - cl(Mic - int(B)) \cup Mic - int(Mic - cl(B))] \subseteq Mic - cl(Mic - int\big(Mic - cl(A)\big) \cup Mic - int\big(Mic - cl(B)\big) \subseteq Mic - cl\Big(Mic - int\big(Mic - cl(A \cup B)\big)\Big)$.

Hence

$A \cup B \subseteq Mic-cl(Mic-int(Mic-cl(A \cup B)))$.

Therefore $A \cup B$ is Micro-b-open.

Remark 3.22: Finite intersection of two Micro-b-open sets not a Micro-b-open.

Example 3.23: Let $U =$ {a,b,c,d} with U/R ={{a},{c},{b,d}} and X={a,b}. Then the nano topology τ_R(X) = {U,$\emptyset$,{a},{b,d},{a,b,d}} and μ={b}. μ_R(X)={U,$\emptyset$,{a},{b},{a,b},{b,d}, {a,b,d}}. Mic-b-O(X) = {U,$\emptyset$,{a},{b},{a,b},{a,c}, {b,d},{b,c},{a,b,c},{a,b,d},{b,c,d}}. Here {{a,c},{b,c}} are Micro-b-open sets but their intersection {c} is not Micro-b-open set.

Definition 3.24: The union of all Micro-b-open set in a Micro topological space (U,τ_R(X),μ_R(X)) contained in A is called Micro-b-interior of A and is denoted by $Mic-int(A)$, $Micro-int(A) = \cup\{B: B \subseteq A, B$ is a Micro-b-open set}.

Definition 3.25: The intersection of all Micro-b-closed set in a Micro topological space (U,τ_R(X),μ_R(X)) containing in A is called Micro-b-closure of A and is denoted by $Mic-cl(A)$, $Micro-cl(A)\ ) = \cap\{B: B \subseteq A, B$ is a Micro-b-closed set}.

Remark 3.26: It is clear that $Mic-int(A)$ is Micro-b-open set and $Mic-cl(A)$ is a Micro-b-closed set.

Theorem 3.27:

1) $X-(Mic-int_b(A)) = Mic-cl_b(X-A)$;
2) $X-(Mic-cl_b(A)) = Mic-int_b(X-A)$;
3) $A \subseteq Mic-cl_b(A)$ and $A = Mic{-}cl_b(A)$ iff A is a Micro-b-closed set;
4) $Mic-int_b(A) \subseteq A$ and $A = Mic-int_b(A)$ iff A is a Micro-b-open set;

Proof: 1) Let $x \in X-(Mic-int_b(A))$. Then $x \notin Mic-int_b(A)$. Then there is no b-open set U contained in A such that $x \notin U$. Hence $x \in U^C$ for all closed set containing A^C. Therefore $x \epsilon Mic-cl_b$(X$-$A). Hence $X-(Mic-int_b(A)) \subseteq Mic-cl_b(X-A))$.

Conversely, Let $x \epsilon Mic-cl_b$ (X$-$A). Then for every b-closed sets V containing (X$-A$) such that $x \in V$. Then there is no b-open set V^C contained in A such that $x \notin V^C$. Hence $x \notin Mic-int_b(A)$. Thus $x \epsilon X-(Mic-int_b(A))$. Therefore $Mic-cl_b(X-A) \subseteq X-(Mic-int_b(A)$. Hence $X-(Mic-int_b(A)) = Mic-cl_b(X-A)$.

2) Let $x \in X-(Mic{-}cl_b(A))$. Then $x \notin Mic-cl_b(A)$. Therefore, there exists an b-closed set U containing A such that $x \notin U$. Hence U^C is an open set containing x such that $U^C \subseteq A^C$. Therefore $x \in (Mic-int_b(A^C))$. This implies $x \in Mic-int_b(X-A)$. Hence $(X-A)(Mic-cl_b(A)) \subseteq Mic-int_b(X-A)$.

Conversely, Let $x \in Mic-int_b(X-A)$. Then there exists an b-open set U containg x such that $U \subseteq A^C$. Hence there exists an b-closed set U^C containing A such that $x \notin U^C$. Therefore $x \notin Mic-cl_b(A)$. Hence $x \in X-(Mic-int_b(A))$. Therefore $Mic-int_b(X-A) \subseteq X-(Mic-int_b(A))$. Hence $-(Mic-cl_b(A)) = Mic-int_b(X-A)$.

Also 3) and 4) are obvious.

IV. MICRO-b-CONTINUOUS

Definition 4.1: Let $(U, \tau'_R(X), \mu'_R(X))$ and $(V, \tau'_R(Y), \mu'_R(Y))$ be two Micro topological spaces. A function $f: (U, \tau'_R(X), \mu'_R(X)) \to (V, \tau'_R(Y), \mu'_R(Y))$ is called a Micro-b-continuous if $f^{-1}(H)$ is Micro-b-open in U for every Micro-open set H in V.

Example 4.2: Let U = {a,b,c,d} with U/R= {{a},{c},{b,d}}and X={a,b}. Then the nano topology τ_R(X) = {U,$\emptyset$,{a},{b,d},{a,b,d}} and μ={b}. μ_R(X)={U,$\emptyset$,{a},{b},{a,b}{b,d}, {a,b, d}}. Mic-b-O (X) = {{a},{b},{a,b},{a,c}, {b,d},{b,c},{a,b,c},{a,b,d},{b,c,d}}.

Let V = {a,b,c,d} with V/R={{a},{d},{b,c}} and Y={a,c}. Then the nano topology τ_R'(Y) = { $U,\emptyset$, {a},{b,c},{a,b,c}}and μ={b}. μ_R'(Y) ={{a},{b},{a,b},{b,c},{a,b,c}}. Define $f : U \rightarrow V$ by f(a)=b, f(b)=a, f(c)=c, f(d)=d f^{-1}(b)=a, f^{-1}(a)=b, f^{-1}(a,b)=(a,b), f^{-1}(b,c)=(a,c), f^{-1}(a,b,c)=(a,b,c). Therefore for every Micro-open set H in V, $f^{-1}(H)$ is Micro-b-open in U. Hence f is Micro-b-continuous.

The following theorem characterizes Micro-b continuous functions in terms of Micro-b-closed sets.

Theorem 4.3:A function $f: (U,\tau_R'(X),\mu_R'(X)) \rightarrow (V,\tau_R'(Y),\mu_R'(Y))$ is Micro-b-continuous iff the inverse image of every Micro-b-closed set in U is Micro closed in V.

Proof:

Let $f: (U,\tau_R'(X),\mu_R'(X)) \rightarrow (V,\tau_R'(Y),\mu_R'(Y))$ be Micro-b-continuous and F be Micro-b-closed in U. That is U-F is Micro-b-open in U. Since f is Micro continuous, $f^{-1}(F)$ is Micro closed in V. Thus the inverse image of every Micro-b-closed set in U is Micro closed in V. Conversely , Suppose the inverse image of every Micro-b-closed set in U is Micro closed in V. Let G be Micro-b-open in U. Then $U-G$ is Micro-b-closed in U. $f^{-1}(U-G)$ is Micro-b-closed in V. That is $V-f^{-1}(G)$ is Micro closed in V. Therefore $f^{-1}(G)$ is Micro open in V. Hence f be Micro-b-continuous in V.

The following theorem, we estabilish a characterization of Micro-b-continuous functions in terms of Micro-b-closure.

Theorem 4.4. A function $f: (U,\tau_R(X),\mu_R(X)) \rightarrow (V,\tau_R'(Y),\mu_R'(Y))$ is Micro-b-continuous iff $f(Mic-cl_b(A)) \subseteq Mic-cl_b(f(A))$ for every subset A of V.

Proof:

Let $f: (U,\tau_R'(X),\mu_R'(X)) \rightarrow (V,\tau_R'(Y),\mu_R'(Y))$ be Micro-b-continuous and $A \subseteq V$. Then $f(A) \subseteq U$. $Mic-cl_b f(A))$ is Micro-b-closed in U. Since f is Micro-b-continuous, $f^{-1}(Mic-cl_b(f(A)))$ is Micro closed in V. Since $f(A) \subseteq Mic-(f(A))$, $A \subseteq f^{-1}(Mic-cl_b(f(A)))$.

Thus $f^{-1}(Mic-cl_b(f(A)))$ is Micro-b-closed set containing A. Therefore $Mic-cl_b(A) \subseteq f^{-}(Mic-cl_b(f(A)))$. That is $f(Mic-cl_b(A)) \subseteq Mic-cl_b(f(A))$. Conversely , Let $f(Mic-cl_b(A)) \subseteq Mic-cl_b(f(A))$ for every subset of A of V. If F is Micro-b-closed in U then $f^{-1}(F) \subseteq V$, $f(Mic-cl_b(f^{-1}-(F)) \subseteq Mic-cl_b(f(f^{-1}-(F)) \subseteq Mic-cl_b(F)$.

That is $Mic-cl_b(f^{-1}(F) \subseteq f^{-1}(Mic-cl_b(F)) = f^{-1}(F)$

. Therefore $f^{-1}(F)$ is Micro-b-closed in V for every Micro-b-closed set F in U. Hence f is Micro-b-continuous.

Theorem 4.5. A function $f : (U,\tau_R(X),\mu_R(X)) \rightarrow (V,\tau_R'(Y),\mu_R'(Y))$ be two Micro topological space. Then $f: U \rightarrow V$ is Micro-b-continuous function iff $f(Mic-cl_b(A)) \subseteq Mic-cl_b(f(A))$.

Proof:

Suppose $f: (U,\tau_R'(X),\mu_R'(X)) \rightarrow (V,\tau_R'(Y),\mu_R'(Y))$ is Micro-b-continuous and $Mic_cl_b(f(A))$is Micro-b-closed in V. Then $f^{-1}(Mic\_cl_b(f(A)))$ is Micro-b-closed in U. Consequently, Mic-

$$cl_b\left(f^{-1}\left(Mic-cl_b(f(A))\right)\right) = f^{-1}\left(Mic-cl_b(f(A))\right).$$

Since $f(A) \subseteq Mic-cl_b(f(A))$, we have $A \subseteq f^{-1}(Mic-cl_b(f(A)))$. Therefore,

$$Mic-cl_b(A) \subseteq Mic-cl_b\left(f^{-1}\left(Mic-cl_b(f(A))\right)\right) = f^{-1}\left(Mic-cl_b(f(A))\right).$$

Hence, $f(Mic-cl_b(A)) \subseteq Mic-cl_b(f(A))$ ….(1)

Conversely, suppose $f(Mic-cl_b(A)) \subseteq Mic-cl_b(f(A))$. Let F be Mic-b-closed set in V, $Mic-cl_b(F) = F$. By hypothesis,

$f\left(Mic-cl_b\left(f^{-1}(F)\right)\right)\subseteq$ $Mic-cl_b\left(f\left(f^{-1}(F)\right)\right)\subseteq Mic-cl_b(F)$. By equation (1) $f\left(Mic-cl_b\left(f^{-1}(F)\right)\right)\subseteq Mic-cl_b(F)=F.$ Therefore, $\left(Mic-cl_b\left(f^{-1}(F)\right)\right)f^{-1}(F).$ Always, $f^{-1}(F)\subseteq\left(Mic-cl_b\left(f^{-1}(F)\right)\right).$ Hence $f^{-1}(F)$ is Micro-b-closed in U and f is Micro-b-continuous.

In the following theorem, we characterize Micro-b-continuous functions in terms of inverse image of Micro closure.

Theorem 4.6. A function $f:(U,\tau_R(X),\mu_R(X))\to(V,\tau_{R'}(Y),\mu_{R'}(Y))$ is Micro-b-continuous if and only if $Mic-cl_b(f^{-1}(B))\subseteq f^{-1}(Mic-cl_b(B))$ for every subset B of U.

Proof:

Let $f:(U,\tau'_R(X),\mu'_R(X))\to(V,\tau'_R(Y),\mu'_R(Y))$ be a Micro-b-continuous and $B\in U,$ ($\text{Mic}-cl_b(B))$ is Micro-b-closed in U and hence $f^{-1}(\text{Mic}-cl_b(B))$ is Micro closed in V.

Therefore, $Mic-cl_b\left(f^{-1}\text{Mic}-cl_b(B)\right)=f^{-1}(\text{Mic}-cl_b(B))$. Since $B\subseteq Mic-cl_b(B)$, $f^{-1}(B)\subseteq f^{-1}(Mic-cl_b(B))$. That implies $Mic-cl_b\subseteq Mic-cl_b\left(f^{-1}\left(Mic-cl_b(B)\right)\right)=(f^{-1}\left(Mic-cl_b(B)\right)$. That is $Mic-cl_b\left(f^{-1}(B)\right)\subseteq(f^{-1}\left(Mic-cl_b(B)\right)$ for every $B\in V$.

Conversely, Let B be Micro-b-closed in V. Then $Mic-cl_b(B)=B$. By assumption $Mic-cl_b\left(f^{-1}(B)\right)=f^{-1}(\text{Mic}-cl_b(B)=f^{-1}(B)$. Thus $Mic-cl_b\left(f^{-1}(B)\right)\subseteq f^{-1}(B)$. But $f^{-1}(B)\subseteq Mic-cl_b f^{-1}(B))$. Therefore, $Mic-cl_b f^{-1}(B)=f^{-1}(B)$. Therefore, $f^{-1}(B)$ is Micro closed in V for every Micro-b-closed set B in $U.$ Hence f is Micro-b-continuous on V.

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