On Micro-Semi Alpha-Open Sets In Micro Topological Spaces

A.Parvathy[1], Dr. J. Rajakumari[2]
[1]Dept of MATHEMATICS
[2]Assistant Professor, Dept of MATHEMATICS
[1, 2] Aditanar College of Arts and Science, Tiruchendur– TamilNadu

Abstract- *The concept of Micro topology is the extension of Nano topology. Nano topology was introduced by LellisThivagar. The Micro topology was introduced by S.Chandrasekar by using the Nano topology. The aim of this paper is to introduce and study a new type of open sets namely Micro-Semi Alpha-open sets in Micro topological spaces, along with their several fundamental properties. We also used this set to introduce the Micro-Semi Alpha- interior and Micro- Semi alpha-closure and its properties are investigated.*

Keywords- Micro-Semi Alpha-open sets, Micro-Semi Alpha-closed sets, Micro-Semi Alpha-interior,Micro-Semi Alpha-closure.

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I. INTRODUCTION

In 2000,G.B.Navalagi [1] presented the idea of Semi-α-open sets in topological spaces.The concept of Nano topology was introduced by LellisThivagar [3] in the year 2013 which has defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it. In 2018, S.Chandrasekar [6] introduced the concept of Micro topology by using the Nano topology. In this paper we introduce Micro-Semi Alpha-open sets, Micro-Semi Alpha-closed sets, Micro-Semi Alpha-interior and Micro-Semi Alpha-closure.

Throughout this paper, ($\mathcal{U}$,$\tau_{\mathcal{R}}(X)$,$\mu_{\mathcal{R}}(X)$) (or simply $\mathcal{U}$) always mean a Micro topological space (or simply M.T.S.). The complement of a Micro-open set (briefly *Mic-O.S.*) is called a Micro-closed set (briefly *Mic-C.S.*) in ($\mathcal{U}$,$\tau_{\mathcal{R}}(X)$,$\mu_{\mathcal{R}}(X)$) . For a set $\mathcal{A}$ in a M.T.S. ($\mathcal{U}$,$\tau_{\mathcal{R}}(X)$,$\mu_{\mathcal{R}}(X)$) ,*Mic-cl*($\mathcal{A}$) and *Mic-int*($\mathcal{A}$) denote the Micro-closure of $\mathcal{A}$ and Micro-interior of $\mathcal{A}$ respectively.

II. PRELIMINARIES

Definition 2.1 [6, 7]: The subset $\mathcal{A}$ of a Micro topological space ($\mathcal{U}$,$\tau_{\mathcal{R}}(X)$,$\mu_{\mathcal{R}}(X)$) is said to be:

(i)A Micro-pre-open set (briefly *Mic-p-O.S.*) if $\mathcal{A} \subseteq$*Mic-int*(*Mic-cl*($\mathcal{A}$)).The complement of a *Mic-p-O.S.* is called a Micro-pre-closed set (briefly *Mic-p-C.S.*) in($\mathcal{U}$,$\tau_{\mathcal{R}}(X)$,$\mu_{\mathcal{R}}(X)$).The family of all *Mic-p-O.S.* (resp.*Mic-p-C.S.*)of$\mathcal{U}$ is denoted by *Mic-p.O*($\mathcal{U}$,*X*)(resp.*Mic-p.C* ($\mathcal{U}$,*X*)).

(ii) AMicro-semi-open set (briefly *Mic-s-O.S.*) if $\mathcal{A} \subseteq$ *Mic-cl*(*Mic-int*($\mathcal{A}$)).The complement of a *Mic-s-O.S.* is called a Micro-semi-closed set (briefly *Mic-s-C.S.*) in($\mathcal{U}$,$\tau_{\mathcal{R}}(X)$,$\mu_{\mathcal{R}}(X)$). The family of all *Mic-s-O.S.* (resp.*Mic-s-C.S.*) of $\mathcal{U}$ is denoted by *Mic-s.O* ($\mathcal{U}$,*X*)(resp.*Mic-s.C* ($\mathcal{U}$,*X*)).

(iii)AMicro-α-open set (briefly *Mic-*α*-O.S.*) if $\mathcal{A} \subseteq$ *Mic-int*(*Mic-cl*(*Mic-int*($\mathcal{A}$))). The complement of a *Mic-*α*-O.S.* is called a Micro-α-closed set (briefly *Mic-*α*-C.S.*) in($\mathcal{U}$,$\tau_{\mathcal{R}}(X)$,$\mu_{\mathcal{R}}(X)$). The family of all *Mic-*α*-O.S.* (resp.*Mic-*α*-C.S.*) of $\mathcal{U}$ is denoted by *Mic-*α*.O* ($\mathcal{U}$,*X*) (resp.*Mic-*α*.C*($\mathcal{U}$,*X*)).

Definition 2.2 [6, 7]:

(i) The Micro-pre-interior of a set $\mathcal{A}$ of a Micro topological space ($\mathcal{U}$,$\tau_{\mathcal{R}}(X)$,$\mu_{\mathcal{R}}(X)$) is the union of all *Mic-p-O.S.* contained in $\mathcal{A}$ and is denoted by *Mic-p-int*($\mathcal{A}$).

(ii)The Micro-semi-interior of a set $\mathcal{A}$ of a Micro topological space ($\mathcal{U}$,$\tau_{\mathcal{R}}(X)$,$\mu_{\mathcal{R}}(X)$) is the union of all *Mic-s-O.S.* contained in $\mathcal{A}$ and is denoted by *Mic-s-int*($\mathcal{A}$).

(iii)The Micro-α-interior of a set $\mathcal{A}$ of a Micro topological space ($\mathcal{U}$,$\tau_{\mathcal{R}}(X)$,$\mu_{\mathcal{R}}(X)$) is the union of all *Mic*-α-*O.S.* contained in $\mathcal{A}$ and is denoted by *Mic*-α-*int*($\mathcal{A}$).

Definition 2.3 [6, 7]:

(i)The Micro-pre-closure of a set $\mathcal{A}$ of a Micro topological space ($\mathcal{U}$,$\tau_{\mathcal{R}}(X)$,$\mu_{\mathcal{R}}(X)$) is the intersection of all *Mic-p-C.S.* that contain $\mathcal{A}$ and is denoted by *Mic-p-cl*($\mathcal{A}$).

(ii) The Micro-semi-closure of a set $\mathcal{A}$ of a Micro topological space ($\mathcal{U}$,$\tau_{\mathcal{R}}(X)$,$\mu_{\mathcal{R}}(X)$) is the intersection of all *Mic-s-C.S.* that contain $\mathcal{A}$ and is denoted by *Mic-s-cl*($\mathcal{A}$).

(iii) The Micro-α-closure of a set $\mathcal{A}$ of a Micro topological space ($\mathcal{U}$,$\tau_{\mathcal{R}}(X)$,$\mu_{\mathcal{R}}(X)$) is the intersection of all *Mic*-α-*C.S.* contain $\mathcal{A}$ and is denoted by *Mic*-α-*cl*($\mathcal{A}$).

Proposition 2.4 [7]: In a Micro topological space($\mathcal{U}$,$\tau_{\mathcal{R}}(X)$,$\mu_{\mathcal{R}}(X)$), then the following statements hold:

(i) Every *Mic-O.S.* (resp. *Mic-C.S.*) is a*Mic*-α-*O.S.* (resp.*Mic*-α-*C.S.*).

(ii) Every *Mic*-α-*O.S.* (resp.*Mic*-α-*C.S.*) is a *Mic-s-O.S.* (resp.*Mic-s-C.S.*).

(iii) Every *Mic*-α-*O.S.* (resp.*Mic*-α-*C.S.*) is a *Mic-p-O.S.* (resp.*Mic-p-C.S.*).

Proposition 2.5 [7]:A subset $\mathcal{A}$of a M. T.S.($\mathcal{U}$,$\tau_{\mathcal{R}}(X)$,$\mu_{\mathcal{R}}(X)$) is a*Mic*-α-*O.S.* if and only if $\mathcal{A}$ is a *Mic-s-O.S.* and *Mic-p-O.S.*.

Lemma 2.6:

(i) If $\mathcal{K}$ is a *Mic-O.S.*, then *Mic-s-cl*($\mathcal{K}$) = *Mic-int*(*Mic-cl*($\mathcal{K}$)).

(ii) If $\mathcal{A}$ is a subset of a M.T.S. ($\mathcal{U}$,$\tau_{\mathcal{R}}(X)$,$\mu_{\mathcal{R}}(X)$), then *Mic-s-int*(*Mic-csl*($\mathcal{A}$)) = *Mic-cl*(*Mic-int*(*Mic-cl*($\mathcal{A}$))).

III. MICRO-SEMI-α-OPEN SETS

Definition 3.1: A subset $\mathcal{A}$ of a M.T.S.($\mathcal{U}$,$\tau_{\mathcal{R}}(X)$,$\mu_{\mathcal{R}}(X)$) is called Micro-Semi-α-open set(briefly *Mic*-$\mathcal{S}_\alpha$-*O.S.*) if there exists a *Mic*-α-*O.S.* $\mathcal{P}$in$\mathcal{U}$ such that$\mathcal{P} \subseteq \mathcal{A} \subseteq$*Mic-cl*($\mathcal{P}$) or equivalently if $\mathcal{A} \subseteq$*Mic-cl*(*Mic*-α-*int*($\mathcal{A}$)). The family of all *Mic*-$\mathcal{S}_\alpha$-*O.S.* of $\mathcal{U}$ is denoted by *Mic*-$\mathcal{S}_\alpha O$ ($\mathcal{U}$, *X*).

Definition 3.2: The complement of *Mic*-$\mathcal{S}_\alpha$-*O.S.*is called a Micro-Semi-α-closed set (briefly *Mic*-$\mathcal{S}_\alpha$-*C.S.*). The family of all *Mic*-$\mathcal{S}_\alpha$-*C.S.* of $\mathcal{U}$ is denoted by *Mic*-$\mathcal{S}_\alpha C$ ($\mathcal{U}$, *X*).

Example 3.3: Let $\mathcal{U}$ = {*a*,*b*,*c*,*d*} with$\mathcal{U}/\mathcal{R}$ = {{*a*},{*c*},{*b*,*d*}} and *X* = {*a*,*b*} $\subseteq \mathcal{U}$, $\tau_{\mathcal{R}}(X)$ = {$\emptyset$,$\mathcal{U}$,{*a*},{*a*,*b*,*d*},{*b*,*d*}}.Then μ= {*b*}.*Mic-O* ($\mathcal{U}$, *X*) = $\mu_{\mathcal{R}}(X)$ = {$\emptyset$,$\mathcal{U}$,{*a*},{*b*},{*a*,*b*,*d*},{*a*,*b*},{*b*,*d*}}.*Mic-C* ($\mathcal{U}$, *X*) = {$\emptyset$,$\mathcal{U}$,{*c*},{*a*,*c*,*d*},{*b*,*c*,*d*},{*a*,*c*},{*c*,*d*}}. The family of all *Mic*-α-*O.S.* of $\mathcal{U}$ is:*Mic*-α *O*($\mathcal{U}$, *X*) = {$\emptyset$,$\mathcal{U}$,{*a*},{*b*},{*a*,*b*,*d*},{*a*,*b*},{*b*,*d*},{*a*,*b*,*c*}}.The family of all*Mic*-$\mathcal{S}_\alpha$-*O.S.* of$\mathcal{U}$ is: *Mic*-$\mathcal{S}_\alpha O$ ($\mathcal{U}$, *X*) = {$\emptyset$,$\mathcal{U}$,{*a*},{*b*},{*a*,*b*,*d*},{*a*,*b*},{*b*,*d*},{*b*,*c*},{*a*,*c*},{*a*,*b*,*c*},{*b*,*c*,*d*}}. The family of all *Mic*-α-*C.S.* of $\mathcal{U}$ is: *Mic*-α *C* ($\mathcal{U}$, *X*) = {$\emptyset$,$\mathcal{U}$,{*c*},{*d*},{*a*,*c*,*d*},{*b*,*c*,*d*},{*a*,*c*},{*c*,*d*}}. The family of all *Mic*-$\mathcal{S}_\alpha$-*C.S.* of $\mathcal{U}$ is: *Mic*-$\mathcal{S}_\alpha C$ ($\mathcal{U}$, *X*) = {$\emptyset$,$\mathcal{U}$,{*c*},{*d*},{*a*,*c*,*d*},{*b*,*c*,*d*},{*a*,*c*},{*c*,*d*},{*a*},{*a*,*d*},{*b*,*d*}}.

Remark 3.4: In a M.T.S. ($\mathcal{U}$,$\tau_{\mathcal{R}}(X)$,$\mu_{\mathcal{R}}(X)$),then the following statements hold:

(i) Every *Mic-O.S.*(resp. *Mic-C.S.*) is a *Mic*-$\mathcal{S}_\alpha$-*O.S.* (resp. *Mic*-$\mathcal{S}_\alpha$-*C.S.*).

(ii) Every *Mic*-α-*O.S.* (resp. *Mic*-α-*C.S.*) is a *Mic*-$\mathcal{S}_\alpha$-*O.S.* (resp. *Mic*-$\mathcal{S}_\alpha$-*C.S.*).

The converse of the above remark need not be true as shown in the following example.

Example 3.5: In example 3.3, we have

(i) the sets {*a*,*c*},{*b*,*c*},{*a*,*b*,*c*} and {*b*,*c*,*d*} are*Mic*-$\mathcal{S}_\alpha$-*O.S.* but not *Mic-O.S.*.

(ii) Also, the sets {*a*,*c*},{*b*,*c*} and {*b*,*c*,*d*} are *Mic*-$\mathcal{S}_\alpha$-*O.S.* but not *Mic*-α-*O.S.*.

Remark 3.6: If every *Mic-O.S.* is a*Mic-C.S.*, then the following conditions are hold:

(i) Every *Mic*-S_α-*O.S.*is a *Mic*-α-*O.S.*.

(ii) Every *Mic*-S_α-*O.S.* is a *Mic-p-O.S.*.

(iii) If every Mic-nowhere dense set is a *Mic-C.S.*, then every *Mic*-S_α-*O.S.* is a *Mic-O.S.*.

Theorem 3.7: Every *Mic-s-O.S.*and *Mic-p-O.S.* of any M.T.S.($\mathcal{U}$,$\tau_{\mathcal{R}}(X)$,$\mu_{\mathcal{R}}(X)$) is a *Mic*-S_α-*O.S.*.

Proof: This follows from proposition (2.5) and remark (3.4) (ii).

Theorem 3.8: For any subset $\mathcal{A}$ of a M.T.S. ($\mathcal{U}$,$\tau_{\mathcal{R}}(X)$,$\mu_{\mathcal{R}}(X)$), $\mathcal{A} \in$ *Mic*-α *O* ($\mathcal{U}$, *X*) iff there exists a *Mic-O.S.*$\mathcal{K}$such that $\mathcal{K} \subseteq \mathcal{A} \subseteq$*Mic-int*(*Mic-cl*($\mathcal{K}$)).

Proof: Let $\mathcal{A}$ be a *Mic*-α-*O.S.*.Then$\mathcal{A} \subseteq$*Mic-int*(*Mic-cl*(*Mic-int*($\mathcal{A}$))), so let $\mathcal{K}$ = *Mic-int*($\mathcal{A}$), we get *Mic-int*($\mathcal{A}$) $\subseteq \mathcal{A} \subseteq$*Mic-int*(*Mic-cl*(*Mic-int*($\mathcal{A}$))). Then there exists a *Mic-O.S.Mic-int*($\mathcal{A}$) such that $\mathcal{K} \subseteq \mathcal{A} \subseteq$*Mic-int*(*Mic-cl*($\mathcal{K}$)), where$\mathcal{K}$ = *Mic-int*($\mathcal{A}$).

Conversely, suppose that there is a *Mic-O.S.*$\mathcal{K}$such that $\mathcal{K} \subseteq \mathcal{A} \subseteq$*Mic-int*(*Mic-cl*($\mathcal{K}$)).Since *Mic-int*($\mathcal{A}$) is the largest *Mic-O.S.* contained in $\mathcal{A}$,we get $\mathcal{K} \subseteq$*Mic-int*($\mathcal{A}$). Then *Mic-cl*($\mathcal{K}$) $\subseteq$*Mic-int*(*Mic-cl*($\mathcal{A}$) implies that *Mic-int*(*Mic-cl*($\mathcal{K}$)) $\subseteq$*Mic-int*(*Mic-cl*(*Mic-int*($\mathcal{A}$))). Hence $\mathcal{A} \subseteq$*Mic-int*(*Mic-cl*(*Mic-int*($\mathcal{A}$))). Therefore, $\mathcal{A} \in$*Mic*-α *O* ($\mathcal{U}$, *X*).

Theorem 3.9: For any subset $\mathcal{A}$ of a M.T.S. ($\mathcal{U}$,$\tau_{\mathcal{R}}(X)$,$\mu_{\mathcal{R}}(X)$) . The following properties are equivalent:

(i) $\mathcal{A} \in$*Mic*-$S_\alpha O$ ($\mathcal{U}$, *X*).

(ii) There exists a *Mic-O.S.* say $\mathcal{K}$ such that $\mathcal{K} \subseteq \mathcal{A} \subseteq$*Mic-cl*(*Mic-int*(*Mic-cl*($\mathcal{K}$))).

(iii) $\mathcal{A} \subseteq$ *Mic-cl*(*Mic-int*(*Mic-cl*(*Mic-int*($\mathcal{A}$)))).

Proof: (i) $\Rightarrow$ **(ii)** Let $\mathcal{A} \in$*Mic*-$S_\alpha O$ ($\mathcal{U}$, *X*). Then there exists $\mathcal{P} \in$*Mic*-α *O* ($\mathcal{U}$, *X*), such that $\mathcal{P} \subseteq \mathcal{A} \subseteq$*Mic-cl*($\mathcal{P}$). Then there exists $\mathcal{K}$ *Mic-O.S.* such that $\mathcal{K} \subseteq \mathcal{P} \subseteq$ *Mic-int*(*Mic-cl*($\mathcal{K}$)) (by theorem 3.8). Therefore, *Mic-cl*($\mathcal{K}$) $\subseteq$*Mic-cl*($\mathcal{P}$) $\subseteq$*Mic-cl*(*Mic-int*(*Mic-cl*($\mathcal{K}$))), implies that *Mic-cl*($\mathcal{P}$) $\subseteq$*Mic-cl*(*Mic-int*(*Mic-cl*($\mathcal{K}$))). Hence$\mathcal{K} \subseteq \mathcal{P} \subseteq \mathcal{A} \subseteq$*Mic-cl*($\mathcal{P}$) $\subseteq$*Mic-cl*(*Mic-int*(*Mic-cl*($\mathcal{K}$))). Therefore, $\mathcal{K} \subseteq \mathcal{A} \subseteq$ *Mic-cl*(*Mic-int*(*Mic-cl*($\mathcal{K}$))), for some $\mathcal{K}$ *Mic-O.S.*. **(ii)** $\Rightarrow$ **(iii)** Suppose thatthere exists a *Mic-O.S.*$\mathcal{K}$ such that $\mathcal{K} \subseteq \mathcal{A} \subseteq$*Mic-cl*(*Mic-int*(*Mic-cl*($\mathcal{K}$))). We know that *Mic-int*($\mathcal{A}$) $\subseteq \mathcal{A}$. On the other hand, $\mathcal{K} \subseteq$*Mic-cl*($\mathcal{A}$) (since *Mic-int*($\mathcal{A}$) is the largest *Mic-O.S.* contained in $\mathcal{A}$). Then*Mic-cl*($\mathcal{K}$) $\subseteq$*Mic-cl*(*Mic-int*($\mathcal{A}$)), then *Mic-int*(*Mic-cl*($\mathcal{K}$)) $\subseteq$*Mic-int*(*Mic-cl*(*Mic-int*($\mathcal{A}$))), therefore *Mic-cl*(*Mic-int*(*Mic-cl*($\mathcal{K}$))) $\subseteq$*Mic-cl*(*Mic-int*(*Mic-cl*(*Mic-int*($\mathcal{A}$)))). Hence $\mathcal{A} \subseteq$*Mic-cl*(*Mic-int*(*Mic-cl*($\mathcal{K}$))) $\subseteq$*Mic-cl*(*Mic-int*(*Mic-cl*(*Mic-int*($\mathcal{A}$)))), thus$\mathcal{A} \subseteq$*Mic-cl*(*Mic-int*(*Mic-cl*(*Mic-int*($\mathcal{A}$)))). **(iii)** $\Rightarrow$ **(i)** Let $\mathcal{A} \subseteq$ *Mic-cl*(*Mic-int*(*Mic-cl*(*Mic-int*($\mathcal{A}$)))). To prove: $\mathcal{A} \in$*Mic*-$S_\alpha O$ ($\mathcal{U}$, *X*).Let $\mathcal{P}$ = *Mic-int*($\mathcal{A}$). Since *Mic-int*(*Mic-cl*(*Mic-int*($\mathcal{A}$))) $\subseteq$*Mic-cl*(*Mic-int*($\mathcal{A}$)), then *Mic-cl*(*Mic-int*(*Mic-cl*(*Mic-int*($\mathcal{A}$)))) $\subseteq$*Mic-cl*(*Mic-cl*(*Mic-int*($\mathcal{A}$))) = *Mic-cl*(*Mic-int*($\mathcal{A}$)). But $\mathcal{A} \subseteq$ *Mic-cl*(*Mic-int*(*Mic-cl*(*Mic-int*($\mathcal{A}$)))) (by hypothesis). Hence $\mathcal{A} \subseteq$ *Mic-cl*(*Mic-int*(*Mic-cl*(*Mic-int*($\mathcal{A}$)))) $\subseteq$*Mic-cl*(*Mic-int*($\mathcal{A}$)) $\Rightarrow$ $\mathcal{A} \subseteq$*Mic-cl*(*Mic-int*($\mathcal{A}$)). Hence there exists a *Mic-O.S.* say $\mathcal{P}$ such that $\mathcal{P} \subseteq \mathcal{A} \subseteq$*Mic-cl*($\mathcal{P}$). On the other hand, $\mathcal{P}$ is a *Mic*-α-*O.S.*.Hence $\mathcal{A} \in$*Mic*-$S_\alpha O$ ($\mathcal{U}$, *X*).

Corollary 3.10: For any subset $\mathcal{A}$ of a M.T.S. ($\mathcal{U}$,$\tau_{\mathcal{R}}(X)$,$\mu_{\mathcal{R}}(X)$) . The following properties are equivalent:

(i) $\mathcal{A} \in$*Mic*-$S_\alpha C$ ($\mathcal{U}$, *X*).

(ii) There exists a *Mic-C.S.*$\mathcal{F}$ such that *Mic-int*(*Mic-cl*(*Mic-int*($\mathcal{F}$))) $\subseteq \mathcal{A} \subseteq \mathcal{F}$.

(iii) *Mic-int*(*Mic-cl*(*Mic-int*(*Mic-cl*($\mathcal{A}$)))) $\subseteq \mathcal{A}$.

Proof: This follows directly from the theorem (3.9).

Proposition 3.11: The union of any family of *Mic*-α-*O.S.* is a *Mic*-α-*O.S.*.

Proof: Let $\{\mathcal{A}_i\}_{i\in\Lambda}$ be a family of *Mic*-α-*O.S.* of $\mathcal{U}$. To prove: $\cup_{i\in\Lambda}\mathcal{A}_i$ is a *Mic*-α-*O.S.*, that is, $\cup_{i\in\Lambda}\mathcal{A}_i \subseteq$ *Mic-int*(*Mic-cl*(*Mic-int*($\cup_{i\in\Lambda}\mathcal{A}_i$))). Then $\mathcal{A}_i \subseteq$ *Mic-int*(*Mic-cl*(*Mic-int*($\mathcal{A}_i$))), $\forall_i \in \Lambda$. Since $\cup_{i\in\Lambda}$*Mic-int*($\mathcal{A}_i$)$\subseteq$*Mic-int*($\cup_{i\in\Lambda}\mathcal{A}_i$) and $\cup_{i\in\Lambda}$*Mic-cl*($\mathcal{A}_i$)$\subseteq$*Mic-cl*($\cup_{i\in\Lambda}\mathcal{A}_i$) hold for any Micro topology, we have $\cup_{i\in\Lambda}\mathcal{A}_i \subseteq \cup_{i\in\Lambda}$*Mic-int*(*Mic-cl*(*Mic-int*($\mathcal{A}_i$)))$\subseteq$*Mic-int*($\cup_{i\in\Lambda}$*Mic-cl*(*Mic-int*($\mathcal{A}_i$)))$\subseteq$*Mic-int*(*Mic-cl*($\cup_{i\in\Lambda}$*Mic-int*($\mathcal{A}_i$)))$\subseteq$*Mic-int*(*Mic-cl*(*Mic-int*($\cup_{i\in\Lambda}\mathcal{A}_i$))).Hence $\cup_{i\in\Lambda}\mathcal{A}_i$ is a*Mic*-α-*O.S.*.

Theorem 3.12: The union of any family of *Mic*-S_α-*O.S.* is a *Mic*-S_α-*O.S.*.

Proof: Let $\{\mathcal{A}_i\}_{i\in\Lambda}$ be a family of *Mic*-S_α-*O.S.*of$\mathcal{U}$. To prove: $\cup_{i\in\Lambda}\mathcal{A}_i$ is a *Mic*-S_α-*O.S.*.Since $\mathcal{A}_i \in$ *Mic*-$S_\alpha O$ ($\mathcal{U}$, *X*), then there is a *Mic*-α-*O.S.*$\mathcal{B}_i$such that $\mathcal{B}_i \subseteq \mathcal{A}_i \subseteq$*Mic-cl*($\mathcal{B}_i$),$\forall_i \in \Lambda$. Hence $\cup_{i\in\Lambda}\mathcal{B}_i \subseteq \cup_{i\in\Lambda}\mathcal{A}_i \subseteq \cup_{i\in\Lambda}$*Mic-cl*($\mathcal{B}_i$) $\subseteq$*Mic-cl*($\cup_{i\in\Lambda}\mathcal{B}_i$). By proposition (3.11), $\cup_{i\in\Lambda}\mathcal{B}_i \in$*Mic*-$\alpha$ *O* ($\mathcal{U}$, *X*).. Hence $\cup_{i\in\Lambda}\mathcal{A}_i \in$ *Mic*-$S_\alpha C$ ($\mathcal{U}$, *X*).

Corollary 3.13: The intersection of any family of *Mic*-S_α-*C.S.* is a *Mic*-S_α-*C.S.*.

Proof: This follows directly from the theorem (3.12).

Remark 3.14: (i) The union of any two *Mic*-S_α-*C.S.* is not necessary *Mic*-S_α-*C.S.* (ii) The intersection of any two *Mic*-S_α-*O.S.* is not necessary*Mic*-S_α-*O.S.*.

Example 3.15: In example (3.3), we have(i) {*a*,*c*} and {*b*,*c*} are two *Mic*-S_α-*O.S.*, but {*a*,*c*}$\cap${*b*,*c*} = {*c*} is not a*Mic*-S_α-*O.S.*. (ii) {*a*} and {*b*,*d*} are *Mic*-S_α-*O.S.*, but {*a*}$\cup${*b*,*d*} = {*a*,*b*,*d*} is not a*Mic*-S_α-*O.S.*.

IV. MICRO-SEMI-α-INTERIOR AND MICRO-SEMI-α-CLOSURE

Definition 4.1: The union of all *Mic*-S_α-*O.S.* in a M.T.S. ($\mathcal{U}$,$\tau_{\mathcal{R}}(X)$,$\mu_{\mathcal{R}}(X)$) contained in $\mathcal{A}$ is called Micro-S_α-interior of $\mathcal{A}$ and is denoted by*Mic*-S_α-*int*($\mathcal{A}$), *Mic*-S_α-*int*($\mathcal{A}$) = $\cup\{\mathcal{B} : \mathcal{B} \subseteq \mathcal{A}, \mathcal{B}$ is a *Mic*-S_α-*O.S.*}.

Definition 4.2: The intersection of all *Mic*-S_α-*C.S.* in a M.T.S. ($\mathcal{U}$,$\tau_{\mathcal{R}}(X)$,$\mu_{\mathcal{R}}(X)$) containing $\mathcal{A}$ is called Micro-S_α-closure of $\mathcal{A}$ and is denoted by *Mic*-S_α-*cl*($\mathcal{A}$), *Mic*-S_α-*cl*($\mathcal{A}$) = $\cap\{\mathcal{B} : \mathcal{A} \subseteq \mathcal{B}, \mathcal{B}$ is a *Mic*-S_α-*C.S.*}.

Proposition 4.3: Let $\mathcal{A}$ be any set in a M.T.S. ($\mathcal{U}$,$\tau_{\mathcal{R}}(X)$,$\mu_{\mathcal{R}}(X)$), the following properties are true:(i) *Mic*-S_α-*int*($\mathcal{A}$) =$\mathcal{A}$ iff $\mathcal{A}$ is a *Mic*-S_α-*O.S.*. (ii) *Mic*-S_α-*cl*($\mathcal{A}$) =$\mathcal{A}$ iff $\mathcal{A}$ is a *Mic*-S_α-*C.S.*. (iii) *Mic*-S_α-*int*($\mathcal{A}$) is the largest *Mic*-S_α-*O.S.* contained in $\mathcal{A}$. (iv)*Mic*-S_α-*cl*($\mathcal{A}$) is the smallest *Mic*-S_α-*C.S.* containing $\mathcal{A}$.

Proof: (i), (ii), (iii) and (iv) are obvious.

Proposition 4.4: Let $\mathcal{A}$ be any set in a M.T.S. ($\mathcal{U}$,$\tau_{\mathcal{R}}(X)$,$\mu_{\mathcal{R}}(X)$), the following properties are true: (i) *Mic*-S_α-*int*($\mathcal{U} - \mathcal{A}$) =$\mathcal{U} -$ (*Mic*-S_α-*cl*($\mathcal{A}$)) (ii) *Mic*-S_α-*cl*($\mathcal{U} - \mathcal{A}$) =$\mathcal{U} -$(*Mic*-$S_\alpha$-*int*($\mathcal{A}$). **Proof:** (i) By definition (4.2), *Mic*-S_α-*cl*($\mathcal{A}$) = $\cap\{\mathcal{B} : \mathcal{A} \subseteq \mathcal{B}, \mathcal{B}$ is a *Mic*-S_α-*C.S.*}. Now, $\mathcal{U} -$ (*Mic*-S_α-*cl*($\mathcal{A}$)) = $\mathcal{U} - \cap\{\mathcal{B} : \mathcal{A} \subseteq \mathcal{B}, \mathcal{B}$ is a *Mic*-S_α-*C.S.*} = $\cup\{\mathcal{U} - \mathcal{B} : \mathcal{A} \subseteq \mathcal{B}, \mathcal{B}$ is a *Mic*-S_α-*C.S.*} = $\cup\{\mathcal{H} : \mathcal{H} \subseteq \mathcal{U} - \mathcal{A}, \mathcal{H}$ is a *Mic*-S_α-*O.S.*} = *Mic*-S_α-*int*($\mathcal{U} - \mathcal{A}$) . (ii) The proof is similar to (i).

Theorem 4.5: Let $\mathcal{A}$ and $\mathcal{B}$ be two sets in a M.T.S.($\mathcal{U}$,$\tau_{\mathcal{R}}(X)$,$\mu_{\mathcal{R}}(X)$), the following properties hold: (i)*Mic*-S_α-*int*($\emptyset$) =$\emptyset$, *Mic*-S_α-*int*($\mathcal{U}$) = $\mathcal{U}$. (ii) *Mic*-S_α-*int*($\mathcal{A}$) $\subseteq \mathcal{A}$. (iii) $\mathcal{A} \subseteq \mathcal{B} \Rightarrow$ *Mic*-S_α-*int*($\mathcal{A}$) $\subseteq$*Mic*-S_α-*int*($\mathcal{B}$). (iv) *Mic*-S_α-*int*($\mathcal{A}\cap\mathcal{B}$) $\subseteq$*Mic*-S_α-*int*($\mathcal{A}$) $\cap$ *Mic*-S_α-*int*($\mathcal{B}$). (v) *Mic*-S_α-*int*($\mathcal{A}$) $\cup$ *Mic*-S_α-

int($\mathcal{B}$) $\subseteq$*Mic*-S_α-*int*($\mathcal{A} \cup \mathcal{B}$). (vi) *Mic*-$S_\alpha$-*int*(*Mic*-$S_\alpha$-*int*($\mathcal{A}$)) =*Mic*-$S_\alpha$-*int*($\mathcal{A}$).

Proof: (i), (ii), (iii), (iv),(v) and (vi) are obvious.

Corollary 4.6: Let $\mathcal{A}$ and $\mathcal{B}$ be two sets in a M.T.S.($\mathcal{U}$,$\tau_{\mathcal{R}}(X)$,$\mu_{\mathcal{R}}(X)$), the following properties hold: (i) *Mic*-S_α-*cl*($\emptyset$) =$\emptyset$, *Mic*-S_α-*cl*($\mathcal{U}$) = $\mathcal{U}$(ii) $\mathcal{A} \subseteq$*Mic*-S_α-*cl*($\mathcal{A}$) . (iii) $\mathcal{A} \subseteq \mathcal{B} \Rightarrow$ *Mic*-S_α-*cl*($\mathcal{A}$) $\subseteq$*Mic*-S_α-*cl*($\mathcal{B}$). (iv) *Mic*-S_α-*cl*($\mathcal{A} \cap \mathcal{B}$) $\subseteq$*Mic*-S_α-*cl*($\mathcal{A}$) $\cap$ *Mic*-S_α-*cl*($\mathcal{B}$). (v) *Mic*-S_α-*cl*($\mathcal{A}$) $\cup$ *Mic*-S_α-*cl*($\mathcal{B}$) $\subseteq$*Mic*-S_α-*cl*($\mathcal{A} \cup \mathcal{B}$). (vi) *Mic*-$S_\alpha$-*cl*(*Mic*-$S_\alpha$-*cl*($\mathcal{A}$)) = *Mic*-$S_\alpha$-*cl*($\mathcal{A}$).

Proof: This proof follows from theorem (4.5).

Theorem 4.7: For any subset $\mathcal{A}$ of a M.T.S. ($\mathcal{U}$,$\tau_{\mathcal{R}}(X)$,$\mu_{\mathcal{R}}(X)$), then:
(i) *Mic*-*int*($\mathcal{A}$) $\subseteq$ *Mic*-α-*int*($\mathcal{A}$) $\subseteq$*Mic*-S_α-*int*($\mathcal{A}$) $\subseteq$*Mic*-S_α-*cl*($\mathcal{A}$) $\subseteq$ *Mic*-α-*cl*($\mathcal{A}$) $\subseteq$ *Mic*-*cl*($\mathcal{A}$) (ii) *Mic*-*int*(*Mic*-S_α-*int*($\mathcal{A}$) = *Mic*-S_α-*int*(*Mic*-*int*($\mathcal{A}$)) = *Mic*-*int*($\mathcal{A}$). (iii) *Mic*-α-*int*(*Mic*-S_α-*int*($\mathcal{A}$)) = *Mic*-S_α-*int*(*Mic*-α-*int*($\mathcal{A}$)) =*Mic*-α-*int*($\mathcal{A}$).(iv) *Mic*-*cl*(*Mic*-S_α-*cl*($\mathcal{A}$) = *Mic*-S_α-*cl*(*Mic*-*cl*($\mathcal{A}$)) = *Mic*-*cl*($\mathcal{A}$). (v) *Mic*-α-*cl*(*Mic*-S_α-*cl*($\mathcal{A}$)) = *Mic*-S_α-*cl*(*Mic*-α-*cl*($\mathcal{A}$)) =*Mic*-α-*cl*($\mathcal{A}$).(vi) *Mic*-*int*(*Mic*-*cl*($\mathcal{A}$)) $\subseteq$*Mic*-S_α-*int*(*Mic*-S_α-*cl*($\mathcal{A}$)).

Proof: (i) obvious.

(ii) Since *Mic*-*int*($\mathcal{A}$) is a *Mic*-*O.S.*, then *Mic*-*int*($\mathcal{A}$) is a *Mic*-S_α-*O.S.*. This implies that*Mic*-S_α-*int*(*Mic*-*int*($\mathcal{A}$)) = *Mic*-*int*($\mathcal{A}$) (by proposition (4.3)) $\Rightarrow$*Mic*-S_α-*int*(*Mic*-*int*($\mathcal{A}$)) = *Mic*-*int*($\mathcal{A}$). Since *Mic*-*int*($\mathcal{A}$) $\subseteq$ *Mic*-S_α-*int*($\mathcal{A}$)$\Rightarrow$ *Mic*-*int*(*Mic*-*int*($\mathcal{A}$)) $\subseteq$ *Mic*-*int*(*Mic*-S_α-*int*($\mathcal{A}$)) $\Rightarrow$ *Mic*-*int*($\mathcal{A}$) $\subseteq$ *Mic*-*int*(*Mic*-S_α-*int*($\mathcal{A}$)). Also *Mic*-S_α-*int*($\mathcal{A}$) $\subseteq \mathcal{A} \Rightarrow$ *Mic*-*int*(*Mic*-S_α-*int*($\mathcal{A}$)) $\subseteq$*Mic*-*int*($\mathcal{A}$). Hence *Mic*-*int*(*Mic*-S_α-*int*($\mathcal{A}$) =*Mic*-*int*($\mathcal{A}$). This proves (ii).

(iii) Since *Mic*-α-*int*($\mathcal{A}$) is a *Mic*-α-*O.S.*,then *Mic*-α-*int*($\mathcal{A}$) is a *Mic*-S_α-*O.S.*.$\Rightarrow$*Mic*-S_α-*int*(*Mic*-α-*int*($\mathcal{A}$)) = *Mic*-α-*int*($\mathcal{A}$) (by proposition (4.3)). Therefore *Mic*-S_α-*int*(*Mic*-α-*int*($\mathcal{A}$)) =*Mic*-α-*int*($\mathcal{A}$). Since *Mic*-α-*int*($\mathcal{A}$) $\subseteq$ *Mic*-S_α-*int*($\mathcal{A}$)$\Rightarrow$ *Mic*-α-*int*(*Mic*-α-*int*($\mathcal{A}$)) $\subseteq$ *Mic*-α-*int*(*Mic*-S_α-*int*($\mathcal{A}$)) $\Rightarrow$ *Mic*-α-*int*($\mathcal{A}$) $\subseteq$ *Mic*-α-*int*(*Mic*-S_α-*int*($\mathcal{A}$)). Also *Mic*-S_α-*int*($\mathcal{A}$) $\subseteq \mathcal{A} \Rightarrow$ *Mic*-α-*int*(*Mic*-S_α-*int*($\mathcal{A}$)) $\subseteq$*Mic*-α-*int*($\mathcal{A}$). Hence *Mic*-α-*int*(*Mic*-S_α-*int*($\mathcal{A}$) =*Mic*-α-*int*(($\mathcal{A}$). This proves (iii). (iv) and (v) follows from (ii) and (iii). (vi) Since *Mic*-S_α-*cl*($\mathcal{A}$) is a *Mic*-S_α-*C.S.*, then *Mic*-*int*(*Mic*-*cl*(*Mic*-*int*(*Mic*-*cl*(*Mic*-S_α-*cl*($\mathcal{A}$))))) $\subseteq$*Mic*-S_α-*cl*($\mathcal{A}$) (by corollary (3.10)). Therefore, *Mic*-*int*(*Mic*-*cl*($\mathcal{A}$)) $\subseteq$*Mic*-*int*(*Mic*-*cl*(*Mic*-*int*(*Mic*-*cl*($\mathcal{A}$)))) $\subseteq$*Mic*-S_α-*cl*($\mathcal{A}$) (by part (iv)). Hence*Mic*-S_α-*int*(*Mic*-*int*(*Mic*-*cl*($\mathcal{A}$))) $\subseteq$*Mic*-S_α-*int*(*Mic*-S_α-*cl*($\mathcal{A}$))$\Rightarrow$ *Mic*-*int*(*Mic*-*cl*($\mathcal{A}$)) $\subseteq$*Mic*-S_α-*int*(*Mic*-S_α-*cl*($\mathcal{A}$)) (by part (ii)).

Theorem 4.8: For any subset $\mathcal{A}$ of a M.T.S. ($\mathcal{U}$,$\tau_{\mathcal{R}}(X)$,$\mu_{\mathcal{R}}(X)$). The following properties are equivalent: (i) $\mathcal{A} \in$*Mic*-$S_{\alpha}O$ ($\mathcal{U}$, *X*). (ii) $\mathcal{K} \subseteq \mathcal{A} \subseteq$*Mic*-*cl*(*Mic*-*int*(*Mic*-*cl*($\mathcal{K}$))), for some *Mic*-*O.S.*, $\mathcal{K}$.

(iii) $\mathcal{K} \subseteq \mathcal{A} \subseteq$ *Mic*-*s*-*int*(*Mic*-*cl*($\mathcal{K}$)), for some *Mic*-*O.S.*, $\mathcal{K}$. (iv) $\mathcal{A} \subseteq$ *Mic*-*s*-*int*(*Mic*-*cl*(*Mic*-*int*($\mathcal{K}$))). **Proof:** **(i) $\Rightarrow$ (ii)** Let $\mathcal{A} \in$*Mic*-$S_{\alpha}O$ ($\mathcal{U}$, *X*), then $\mathcal{A} \subseteq$ *Mic*-*cl*(*Mic*-*int*(*Mic*-*cl*(*Mic*-*int*($\mathcal{A}$)))) and *Mic*-*int*($\mathcal{A}$)$\subseteq \mathcal{A}$. Hence $\mathcal{K} \subseteq \mathcal{A} \subseteq$*Mic*-*cl*(*Mic*-*int*(*Mic*-*cl*($\mathcal{K}$))), where $\mathcal{K}$ = *Mic*-*int*($\mathcal{A}$) . **(ii) $\Rightarrow$ (iii)** Suppose $\mathcal{K} \subseteq \mathcal{A} \subseteq$*Mic*-*cl*(*Mic*-*int*(*Mic*-*cl*($\mathcal{K}$))), for some *Mic*-S_α-*O.S.*, $\mathcal{K}$.But *Mic*-*s*-*int*(*Mic*-*cl*($\mathcal{A}$)) = *Mic*-*cl*(*Mic*-*int*(*Mic*-*cl*($\mathcal{A}$))) (by lemma (2.6)). Then $\mathcal{K} \subseteq \mathcal{A} \subseteq$ *Mic*-*s*-*int*(*Mic*-*cl*($\mathcal{K}$))), for some *Mic*-S_α-*O.S.*, $\mathcal{K}$. **(iii) $\Rightarrow$ (iv)** Suppose$\mathcal{K} \subseteq \mathcal{A} \subseteq$ *Mic*-*s*-*int*(*Mic*-*cl*($\mathcal{K}$)), for some *Mic*-S_α-*O.S.*, $\mathcal{K}$.Since $\mathcal{K}$ is a *Mic*-*O.S.* contained in $\mathcal{A}$. Then $\mathcal{K} \subseteq$*Mic*-*int*($\mathcal{A}$) $\Rightarrow$ *Mic*-*cl*($\mathcal{K}$) $\subseteq$*Mic*-*cl*(*Mic*-*int*($\mathcal{A}$))$\Rightarrow$*Mic*-*s*-*int*(*Mic*-*cl*($\mathcal{K}$)) $\subseteq$*Mic*-*s*-*int*(*Mic*-*cl*(*Mic*-*int*($\mathcal{A}$))). By hypothesis, we get $\mathcal{A} \subseteq$ *Mic*-*s*-*int*(*Mic*-*cl*(*Mic*-*int*($\mathcal{K}$))). **(iv) $\Rightarrow$ (i)** Let $\mathcal{A} \subseteq$ *Mic*-*s*-*int*(*Mic*-*cl*(*Mic*-*int*($\mathcal{K}$))). But *Mic*-*s*-*int*(*Mic*-*cl*(*Mic*-*int*($\mathcal{A}$))) = *Mic*-*cl*(*Mic*-*int*(*Mic*-*cl*(*Mic*-*int*($\mathcal{A}$)))) (by lemma (2.6)). Hence $\mathcal{A} \subseteq$ *Mic*-*cl*(*Mic*-*int*(*Mic*-*cl*(*Mic*-*int*($\mathcal{A}$)))) $\Rightarrow \mathcal{A} \in$*Mic*-$S_{\alpha}O$ ($\mathcal{U}$, *X*).

Corollary 4.9: For any subset $\mathcal{A}$ of a M.T.S. ($\mathcal{U}$,$\tau_{\mathcal{R}}$(X),$\mu_{\mathcal{R}}$(X)). The following properties are equivalent:
(i)$\mathcal{A} \in$*Mic*-$S_{\alpha}C$($\mathcal{U}$, *X*).
(ii) *Mic*-*int*(*Mic*-*cl*(*Mic*-*int*($\mathcal{K}$))) $\subseteq \mathcal{A} \subseteq \mathcal{K}$, for some *Mic*-*C.S.*, $\mathcal{K}$.
(iii)*Mic*-*s*-*cl*(*Mic*-*int*($\mathcal{K}$)))$\subseteq \mathcal{A} \subseteq \mathcal{K}$,for some *Mic*-*C.S.*, $\mathcal{K}$
(iv) *Mic*-*s*-*cl* (*Mic*-*int*(*Mic*-*cl*($\mathcal{K}$))) $\subseteq \mathcal{A}$.

Proof: This follows from theorem (4.8).

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