On Micro-Regular Open Sets And Micro-Regular Continuous In Micro Topological Spaces

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Abstract- *Micro topology was introduced by S. Chandrasekar. The purpose of this paper is to define and study a new class of sets called Micro-Regular open sets in Micro topological Spaces. Basic properties of Micro-Regular open sets are analyzed. Also we introduced Micro-Regular interior and Micro-Regular closure and their properties are investigated. We also used this set to introduce the new type of continuous functions called Micro-Regular continuous function and its properties are investigated.*

Keywords- Micro-Regular open sets, Micro-Regular interior, Micro-Regular closure, Micro-Regular continuous.

Mathematics Subject Classification: 54A05, 54C05, 54B05, 54C10.

I. INTRODUCTION

In 1937, Stone [3] introduced the concept of regular open sets in Topological Spaces. Nano Topology introduced by Thivagar [4] in the year 2013. The concept of Micro Topology was introduced by S. Chandrasekar [6] in the year 2018. In this paper, we introduce Micro-Regular open sets, Micro-Regular continuous functions and some of their properties are investigated.

II. PRELIMINARIES

Definition 2.1 [6]: $(U, \tau_R(X))$ is a Nano topological space here $\mu_R(X) = \{N \cup (N' \cap \mu)\} : N, N' \in \tau_R(X)$ and called its Micro topology of $\tau_R(X)$ by μ where $\mu \notin \tau_R(X)$.

Definition 2.2 [6]: The Micro Topology μ_R(X) satisfies the following axioms.

(i) $U, \emptyset \in \mu_R$(X).

(ii) The union of the elements of any sub-collection of $\mu_R(X)$ is in $\mu_R(X)$.

(iii) The intersection of the elements of any finite sub collection of $\mu_R(X)$ is in $\mu_R(X)$.

Then $\mu_R(X)$ is called the Micro topology on U with respect to X. The triplet $(U, \tau_R(X), \mu_R(X)$ is called Micro topological spaces and the element of $\mu_R(X)$ are called Micro open sets and the complement of a Micro open set is called a Micro closed set.

Definition 2.3 [6]: The Micro interior of a set A is denoted by Mic-int(A) and is defined as Mic-int(A) = $\cup$\{B:B$\subseteq$A, B is a Micro open set\}. The Micro closure of a set A is denoted by Micro-cl(A) and is defined as Mic-cl(A) = $\cap$\{B:A$\subseteq$B, B is a Micro closed set\}.

Definition 2.4 [6, 7]: Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space and A $\subseteq$ U.
Then A is said to be:
(i) Micro-semi open if A $\subseteq$ Mic-cl(Mic-int(A)). (ii) Micr-pre open if A $\subseteq$ Mic-int(Mic-cl(A)). (iii) Micro-b open if A $\subseteq$ Mic-int(Mic-cl(A)) $\cup$ Mic-cl(Mic-int(A)).
(iv) Micro-α open if A $\subseteq$ Mic-int(Mic-cl(Mic-int(A))).
(v) Micro-semi-pre open if A $\subseteq$ Mic-cl(Mic-int(Mic-cl(A)))

Theorem 2.5 [7]: If A is Micro open in $(U, \tau_R(X), \mu_R(X))$, then it is Micro-α open in U.

Proof: Since A is Micro open in $(U, \tau_R(X), \mu_R(X)$, (Mic-int(A) = A. Then Mic-cl(Mic-int(A)) = Mic-cl(A). Since Mic-cl(A) $\supseteq$ A, Mic-cl(Mic-int(A) $\supseteq$ A. Thus Mic-int(Mic-cl(Mic-int(A))) $\supseteq$ Mic-int(A). Hence A $\subseteq$ Mic-int(Mic-cl(Mic-int(A))).

III. MICRO-REGULAR OPEN SETS

Definition 3.1: Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space and A $\subseteq$ U. Then A is said to be Micro-Regular open if A $=$ Mic-int(Mic-cl(A)).

Definition 3.2: Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space and A $\subseteq$ U. Then A is said to be Micro-Regular closed if its complement is Micro-Regular open in U. That is $A =$ Mic-cl(Mic-int(A)).

Example 3.3: Let U = {a,b,c,d} with U/R={{a}, {c},{b,d}} and X = {b,d}.Then the Nano topology $\tau_R(X)$ ={U,$\emptyset$,{b,d}}and μ = {b} Micro open sets are $\mu_R(X)$ = {U,$\emptyset$,{b},{b,d}}. Here Micro-Regular open sets is U,$\emptyset$.

Theorem 3.4: Every Micro-Regular open set is Micro open.

Proof: Let A be a Micro open set in $(U, \tau_R(X), \mu_R(X))$. Then A = Mic-int (Mic-cl(A)) Now, Mic-int(A) = Mic-int (Mic-int(Mic-cl(A)) = Mic-int(Mic-cl(A)) = A. Hence A is Micro-Regular open in U.

Remark 3.5: The converse of the above theorem need not be true as shown in the following example.

Example 3.6: Let U = {a,b,c,d} with U/R={{a},{b,c},{d}} and X = {a,c}.Then the Nano topology $\tau_R(X)$ = {U,$\emptyset$, {a},{a,b,c}{b,c}} and μ = {b}. Micro open sets are μ_R(X) = {U,$\emptyset$, {a},{b},{a,b},{b,c},{a,b,c}} and Micro closed sets is μ_R^c={U,$\emptyset$,{b,c,d},{a,c,d},{c,d},{a,d}}. The Micro-Regular open sets are U,$\emptyset$,{a} and {b,c}.Since {a,b,c} is Micro open but it is not Micro-Regular open in U.

Remark 3.7: The union of two Micro-Regular open sets need not be Micro-Regular open as shown in the following example.

Example 3.8: In example 3.6, we have {a} and {b,c} are Micro-Regular open but {a}$\cup${b,c} = {a,b,c} is not Micro-Regular open in U.

Remark 3.9: The intersection of two Micro-Regular closed sets need not be Micro-Regular closed as shown in the following example.

Example 3.10: In example 3.6, the Micro-Regular closed sets are U, $\emptyset$,{a,d}and{b,c,d} but {a,d} $\cap$ {b,c,d} = {d} is not Micro-Regular closed in U.

Theorem 3.11: Every Micro-Regular open set is Micro-Pre open.

Proof: Let A be a Micro-Regular open set in $(U, \tau_R(X), \mu_R(X))$. Then A = Mic-int (Mic-cl(A)). Hence A $\subseteq$ Mic-int(Mic-cl(A)). Thus A is Micro-Pre open.

Theorem 3.12: Every Micro-Regular open set is Micro-b open.

Proof: Let A be a Micro-Regular open set in $(U, \tau_R(X), \mu_R(X))$. Then A=Mic-int (Mic-cl(A)). Hence A $\subseteq$ Mic-int(Mic-cl(A)) $\cup$ Mic-cl(Mic-int(A)). Thus A is Micro-b open.

Theorem 3.13: Every Micro-Regular open set is Micro-α open.

Proof: Let A be a Micro-Regular open set in $(U, \tau_R(X), \mu_R(X))$. By theorem 2.5&3.4, A is Micro-α open.

Theorem 3.14: Every Micro-Regular open set is Micro-semi open.

Proof: Let A be a Micro-Regular open set in $(U, \tau_R(X), \mu_R(X))$. By theorem 3.13, we have A is Micro-α open. That is A $\subseteq$ Mic-int(Mic-cl(Mic-int(A))). This implies A $\subseteq$ Mic-cl(Mic-int(A)). Thus A is Micro-semi open.

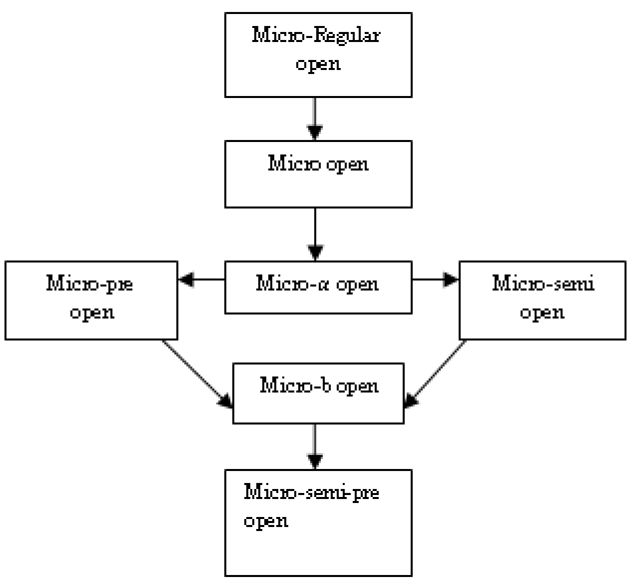
Theorem 3.15: Every Micro-Regular open set is Micro-semi-pre open.

Proof: Let A be a Micro-Regular open set in $(U, \tau_R(X), \mu_R(X))$. By theorem 3.11, we have A is Micro-pre open. That is A $\subseteq$ Mic-int(Mic-cl(A)). Hence A $\subseteq$ Mic-cl (Mic-int(Mic-cl(A))). Thus A is Micro-semi-pre open.

Remark 3.16: The converse of the above theorem need not true which can be seen from the following example.

Example 3.17: Let U = {a,b,c,d} with U/R={{a}, {b,c},{d}} and X = {a,c}.Then the nano topology $\tau_R(X)$ = {U,$\emptyset$, {a},{a,b,c}{b,c}} and then μ= {b}. Micro open sets are $\mu_R(X)$ = {U,$\emptyset$,{a},{b}, {a,b}, {b,c},{a,b,c}}. The Micro-Regular open sets are U,$\emptyset$, {a}and{b,c}. Now, U,$\emptyset$, {b},{a,b}, {a,b,c} and {a,b,d} are Micro-α open but it is not Micro-Regular open. Now, U,$\emptyset$,{b},{a,b}, {a,d},{b,d} and {b,c,d} are Micro-semi open but they are not Micro-Regular open. Here U,$\emptyset$,{b}, {a,b},{a,b,c}, {a,b,d} and {b,c,d} are Micro-Pre open but they are not Micro-Regular open. Now U,$\emptyset$,{b},{a,b}, {a,d},{a,b,c},{a,b,d} and {b,c,d} are Micro-b open but it is not Micro-Regular open. Here U,$\emptyset$,{b}, {a,b},{a,d},{b,d}, {a,b,c}, {a,b,d} and {b,c,d} are Micro-semi-pre open but it is not Micro-Regular open.

Remark 3.18: From the above theorems we have the following implication diagram.



Definition 3.19: The union of all Micro-Regular open sets in a Micro topological space $(U, \tau_R(X), \mu_R(X))$ contained in A is called Micro-Regular-interior of A and is denoted Mic-R-int(A) = $\cup$ {B:B $\subseteq$ A, B is a Micro-Regular open set}.

Definition 3.20: The intersection of all Micro-Regular closed sets in a Micro topological space $(U, \tau_R(X), \mu_R(X))$ containing A is called Micro-Regular-closure of A and is denoted by Mic-R-cl(A) = $\cap$ {B:A $\subseteq$ B, B is a Micro-Regular closed set}.

VI. MICRO-REGULAR CONTINUOUS FUNCTIONS

Definition 4.1: Let $(U, \tau_R(X), \mu_R(X))$ and $(V, \tau'_R(Y), \mu'_R(Y))$ be two Micro topological spaces. Then the mapping f: $(U, \tau_R(X), \mu_R(X)) \to (V, \tau'_R(Y), \mu'_R(Y))$ is said to be Micro-Regular continous if f^{-1}(B) is Micro-Regular open in U for every Micro open set B in V.

Example 4.2: Let U={1,2,3,4} with U/R = {{1}, {3}, {2,4}} and X = {1,2}, $\tau_R(X)$ = {U, $\emptyset$, {1}, {1,2,4}, {2,4}} , μ = {3}. $\mu_R(X)$ = {U, $\emptyset$, {1}, {3}, {1,3}, {2,4}, {2,3,4}, {1,2,4}} and Micro-Regular open sets are U, $\emptyset$,{1}, {3}, {1,3}, {2,4}, {2,3,4}, {1,2,4}. Let V = {a,b,c,d} with U/R = {{a}, {c}, {b,d}} and Y = {b,d}, $\tau'_R(Y)$={U, $\emptyset$, {b,d}} , μ={b}. $\mu'_R(Y)$ = {U, $\emptyset$, {b}, {b,d}}. Define f: $(U, \tau_R(X), \mu_R(X)) \to (V, \tau'_R(Y), \mu'_R(Y))$ by $f(1)$ = {b}, $f(3)$ = {d}, $f(2)$ = {a} and $f(4)$ ={c}. Then f^{-1}(b) = {1} and f^{-1}(b,d) = {1,3}. Hence f is Micro-Regular continuous.

Theorem 4.3: Every Micro-Regular continuous function is Micro continuous.

Proof:

Let f: $(U, \tau_R(X), \mu_R(X)) \to (V, \tau'_R(Y), \mu'_R(Y))$ be a Micro-Regular continuous function and B be a Micro open set in V. Since f is Micro-Regular continuous, f^{-1}(B) is Micro-Regular open in U. By theorem3.4, f^{-1}(B) is Micro open in U. Hence f is Micro continuous.

Theorem 4.4.: Every Micro-Regular continuous function is Micro-pre continuous.

Proof:

Let f: $(U, \tau_R(X), \mu_R(X)) \to (V, \tau'_R(Y), \mu'_R(Y))$ be a Micro-Regular continuous function and B be a Micro open set in V. Since f is Micro-Regular continuous, f^{-1}(B) is Micro-Regular open in U. By theorem3.11, f^{-1}(B) is Micro-pre open in U. Hence f is Micro-pre continuous.

Theorem 4.5: Every Micro-Regular continuous function is Micro-b continuous.

Proof:

Let f: $(U, \tau_R(X), \mu_{R(X)}) \rightarrow (V, \tau'_R(Y), \mu'_R(Y))$ be a Micro-Regular continuous function and B be a Micro open set in V. Since f is Micro-Regular continuous, f^{-1}(B) is Micro-Regular open in U. By theorem3.12, f^{-1}(B) is Micro-b open in U. Hence f is Micro-b continuous.

Theorem 4.6: Every Micro-Regular continuous function is Micro-semi continuous.

Proof:

Let f: $(U, \tau_R(X), \mu_{R(X)}) \rightarrow (V, \tau'_R(Y), \mu'_R(Y))$ be a Micro-Regular continuous function and B be a Micro open set in V. Since f is Micro-Regular continuous, f^{-1}(B) is Micro-Regular open in U. By theorem3.14, f^{-1}(B) is Micro-semi open in U. Hence f is Micro-semi continuous.

Theorem 4.7: Every Micro-Regular continuous function is Micro-α continuous.

Proof: Let f: $(U, \tau_R(X), \mu_{R(X)}) \rightarrow (V, \tau'_R(Y), \mu'_R(Y))$ be a Micro-Regular continuous function and B be a Micro open set in V. Since f is Micro-Regular continuous, f^{-1}(B) is Micro-Regular open in U. By theorem3.13, f^{-1}(B) is Micro-α open in U. Hence f is Micro-α continuous.

Theorem 4.8: Every Micro-Regular continuous function is Micro-semi-pre continuous.

Proof: Let f: $(U, \tau_R(X), \mu_{R(X)}) \rightarrow (V, \tau'_R(Y), \mu'_R(Y))$ be a Micro-Regular continuous function and B be a Micro open set in V. Since f is Micro-Regular continuous, f^{-1}(B) is Micro-Regular open in U. By theorem3.15, f^{-1}(B) is Micro-semi-pre open in U. Hence f is Micro-ssemi-pre continuous.

Remark 4.9: The converse of the above theorem need not true which can be seen from the following example.

Example 4.10: Let U = {a,b,c,d} with U/R={{a}, {b,c}, {d}} and X = {a,c}, $\tau_R(X)$ = {U, $\emptyset$, {a}, {a,b,c}, {b,c}}.Then μ = {b}. $\mu_R(X)$ = {U, $\emptyset$, {a}, {b}, {a,b}, {b,c}, {a,b,c}} and Micro-Regular open sets are U, $\emptyset$, {a} and {b,c}. Let V={1,2,3,4} with U/R = {{1}, {3}, {2,4}} and Y = {2,4}, $\tau'_R(Y)$={U, $\emptyset$, {2,4}}. Then μ = {2}. $\mu'_R(Y)$ = {U, $\emptyset$, {2}, {2,4}}. Define f: $(U, \tau_R(X), \mu_{R(X)}) \rightarrow (V, \tau'_R(Y), \mu'_R(Y))$ by $f(a)$ = {2}, $f(b)$ = {4}, $f(c)$= {1} and $f(d)$= {2}. Then f^{-1}(2) = {a} and f^{-1}(2,4) = {a,b}. Here f is Micro continuous, Micro-pre continuous, Micro-b continuous, Micro-semi continuous, Micro-α continuous and Micro-semi-pre continuous. But not Micro-Regular continuous. Since {a,b} is not Micro-Regular open in U.

Theorem 4.11: Composition of two Micro-Regular continuous function is Micro-Regular continuous.

Proof:

Let f: $(U, \tau_R(X), \mu_{R(X)}) \rightarrow (V, \tau'_R(Y), \mu'_R(Y))$ and g: $((V, \tau'_R(Y), \mu'_R(Y)) \rightarrow (W, \tau''_R(Y), \mu''_R(Y))$ be two Micro-Regular continuous function and B be a Micro open set in W. Since g is Micro-Regular continuous, g^{-1}(B) is Micro-Regular open in V. Since every Micro-Regular open set is Micro open, g^{-1}(B) is Micro open in V. Again since f is Micro-Regular continuous, $f^{-1}(g^{-1}$(B)) is Micro-Regular open in U. That is $(g \circ f)^{-1}$(B) is Micro-Regular open in U. Hence $g \circ f$ is Micro regular continuous.

Theorem 4.12: Composition of Micro continuous function and Micro-Regular continuous function is Micro-Regular continuous.

Proof:

Let f: $(U, \tau_R(X), \mu_{R(X)}) \rightarrow (V, \tau'_R(Y), \mu'_R(Y))$ be a Micro-Regular continuous function and

Let g: $(V, \tau'_R(Y), \mu'_R(Y)) \rightarrow (W, \tau''_R(Y), \mu''_R(Y))$ be Micro continuous function and B be a Micro open set in W. Since g is Micro continuous, g^{-1}(B) is Micro open in V. Since f is Micro-Regular continuous. $f^{-1}(g^{-1}$(B)) is

Micro-Regular open in U. That is $(g \circ f)^{-1}$(B) is Micro-Regular open in U. Hence $g \circ f$ is Micro regular continuous.

Theorem 4.13: A function f: $(U, \tau_R(X), \mu_R(X))$ $\rightarrow$ $(V, \tau'_R(Y), \mu'_R(Y))$ is Micro-Regular continuous if and only if the inverse image of every closed set in V is Micro-Regular closed.

Proof:

Let f: $(U,\ \tau_R(X), \mu_R(X))$ $\rightarrow$ $(V,\ \tau'_R(Y), \mu'_R(Y))$ be a Micro-Regular continuous function and B be a Micro closed set in U. Then V-B is Micro open in V. Since f is Micro-Regular continuous, f^{-1}(V-B) is Micro-Regular open in U. That is U-f^{-1}(B) is Micro-Regular open in U. Hence f^{-1}(B) is Micro-Regular closed in U. Conversely, Suppose G is a Micro open set in V. Then V-G is Micro closed in V. By hypothesis f^{-1}(V-G) is Micro-Regular closed in U. Thus U-f^{-1}(G) is Micro-Regular closed in U. Hence f^{-1}(G) is Micro-Regular open in U. Hence f is Micro-Regular continuous.

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