Pronic Product Cordial Labeling Of Star Related Graphs

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Abstract- *Let $G = (V,E)$ be a simple graph with p vertices and q edges. G is said to have Pronic Product cordial labeling if there is aninjective map $f: V(G) \to \{0,1,2,3,\ldots, Pr_p\}$ such that for every edge uv, the induced edge f^* is defined as,*

$$f^*(uv) = \begin{cases} 1 \; iff \; f(u).f(v) \text{ is a pronicnumber} \\ 0 \; iff \; f(u).f(v) \text{ is not a pronicnumber} \end{cases}$$

with the condition that, $|e_f(0) - e_f(1)| \le 1$ where, $e_f(0)$ is the number of edges with label 0 and $e_f(1)$ is the number of edges with label 1. If G admits Pronic product cordial labeling then G is called a Pronicproduct cordial graph.

In this paper we have proved the star $K_{1,n}$, Star related graphs Globe $Gl(n)$, Bi-Star $B_{n,n}$, Subdivided Star $< K_{1,n} : n >$ are Pronic product cordial graphs.

Keywords- Pronic number, Pronic Product cordial labeling, Pronic product cordial graph, Star graphs.

AMS Classification 05C78

Notation: Pr – Pronic Number

I. INTRODUCTION

A graph G is a finite non-empty set of objects called vertices together with a set of unordered pairs of distinct vertices of G called edges. The vertex set and the edge set of G are denoted by $V(G)$ and $E(G)$ respectively.

The concept of cordial labeling was introduced by **I.Cahit[1]**. It motivated us to define Pronic product cordial labeling.

II. PRELIMINARIES

Definition 2.1: (Sequence A002378 in the OEIS)

A **Pronic number**is a number which is the product of two consecutive integers, that is, a number of the form n. (n+1).

REMARK: Clearly, all pronicnumbers are even.

Definition 2.2: Let $G = (V,E)$ be a simple graph with p vertices and q edges. G is said to have **Pronic product cordial labeling** if there is an injective map $f: V(G) \to \{0,1,2,3,\ldots, Pr_p\}$ such that for every edge uv, the induced edge f^* is defined as,

$$f^*(uv) = \begin{cases} 1 \; iff \; f(u).f(v) \text{ is a pronicnumber} \\ 0 \; iff \; f(u).f(v) \text{ is not a pronicnumber} \end{cases}$$

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Definition 2.3: A **star** $K_{1,n}$ is a tree with one internal vertex and n edges.

Definition 2.4: Globe is defined as the two isolated vertices are joined by n path of length 2. It is denoted by $Gl(n)$.

Definition 2.5: The **Bi-Star** $B_{n,n}$ is a graph obtained from K_2 by identifying the centers of $K_{1,n}$ and $K_{1,n}$ at the end vertices of K_2 respectively.

Definition 2.6: A **Subdivided graph** is obtained by replacing every edge of G by P_3. It is denoted by $S\ (G)$

Definition 2.7: Subdivided star is a graph obtained as one point union of n paths of path length 2. It is denoted by $< K_{1,n}: n >$.

III. MAIN RESULTS

THEOREM 3.1: The star $K_{1,n}$ is a Pronic Product cordial graph.

Proof: Let $G = K_{1,n}$ be a graph where

$V(G) = \{v, u_i : 1 \le i \le n\}$ and

$E(G) = \{(v, u_i) : 1 \le i \le n\}$

Then $|V(G)| = n + 1$ and $|E(G)| = n$

Case(i): When n is even, $n = 2k$ (say)

Define $f: V(G) \to \{0,1,2,3, \ldots\ldots, Pr_{2k+1}\}$ as follows:

$f(v) = 1$

$$f(u_i) = \begin{cases} Pr_i & ,1 \le i \le k \\ Pr_i - 1 & ,\ k+1 \le i \le 2k \end{cases}$$

The induced edge labels are given below:

For $1 \le i \le k,$

$f(v)\,.\,f(u_i) = 1\,.\,Pr_i = Pr_i$

$f^*(vu_i) = 1$

For $k + 1 \le i \le 2k$,

$f(v)\,.\,f(u_i) = 1\,.\,[Pr_i - 1] = Pr_i - 1$

(Which is odd and not a Pronic number)

$f^*(vu_i) = 0.$

Case(ii): When n is odd, n = 2k + 1 (say)

Define $f: V(G) \to \{0, 1, 2, 3, \cdots, Pr_{2k+2}\}$ as follows:

$f(v) = 1$

$$f(u_i) = \begin{cases} Pr_i & ,1 \le i \le k+1 \\ Pr_i - 1 & ,\ k+2 \le i \le 2k+1 \end{cases}$$

The induced edge labels are given below:

For $1 \le i \le k+1,$

$f(v)\,.\,f(u_i) = 1\,.\,Pr_i = Pr_i$

$f^*(vu_i) = 1$

For $k+2 \le i \le 2k+1$,

$f(v)\,.\,f(u_i) = 1\,.\,[Pr_i - 1] = Pr_i - 1$

(Which is odd, not a pronic number)

$f^*(vu_i) = 0$

It is observed that,

$e_f(0) = k$ and $e_f(1) = k$, n - even.

$e_f(0) = k$ and $e_f(1) = k + 1$, n -odd.

Clearly, $|e_f(0) - e_f(1)| \le 1$.

Then f is a Pronic Product cordial labeling.

Hence the star $K_{1,n}$ is a Pronic Product cordial graph.

Example 3.2: Fig 1 and 2 represents the pronic product cordial labeling of $K_{1,4}$ and $K_{1,5}$ respectively

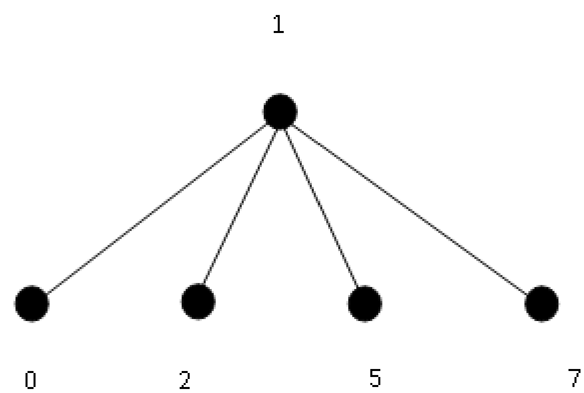

Fig 1: $K_{1,4}$

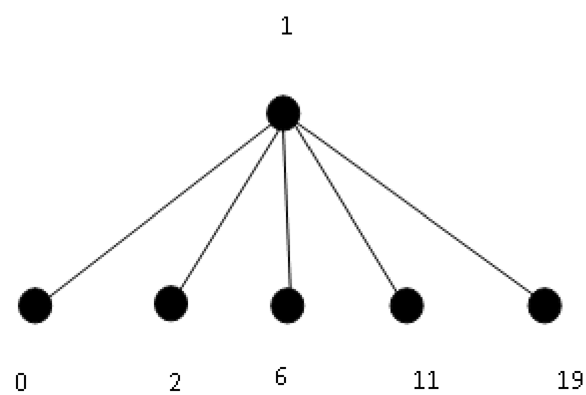

Fig.2: $K_{1,5}$

THEOREM 3.3: Globe $Gl\ (n)$ is a Pronic product cordial graph.

Proof: Let $G = Gl\ (n)$ be a graph where

$V(G) = \{u, v, w_i : 1 \le i \le n\}$ and

$E(G) = \{(u, w_i) : 1 \le i \le n\}$

$\cup \{(v, w_i) : 1 \le i \le n\}$.

Then $|V(G)| = n + 2$ and $|E(G)| = 2n$.

Define $f : (G) \rightarrow \{0,1,2,3, \ldots, Pr_{n+2}\}$ as follows:

$f(u) = 0$ and $f(v) = 1$

$f(w_i) = 2i + 1, 1 \le i \le n$

The induced edge labels are given below:

For $1 \le i \le n$,

$f(u).f(w_i) = 0.(2i+1) = 0 = Pr_1$

$f^*(uw_i) = 1.$

$f(v).f(w_i) = 1.(2i+1) = 2i+1$

(Which is odd, not a Pronic number)

$f^*(vw_i) = 0.$

It is observed that,

$e_{f(0)} = n$ and $e_{f(1)} = n$

Clearly, $|e_f(0) - e_f(1)| \le 1$.

Then f is a Pronic product cordial labeling.

Hence the Globe $Gl(n)$ is a Pronic product cordial graph.

Example 3.4: Fig 3 and 4 represents the Pronic product cordial labeling of $Gl(4)$ and $Gl(5)$ respectively

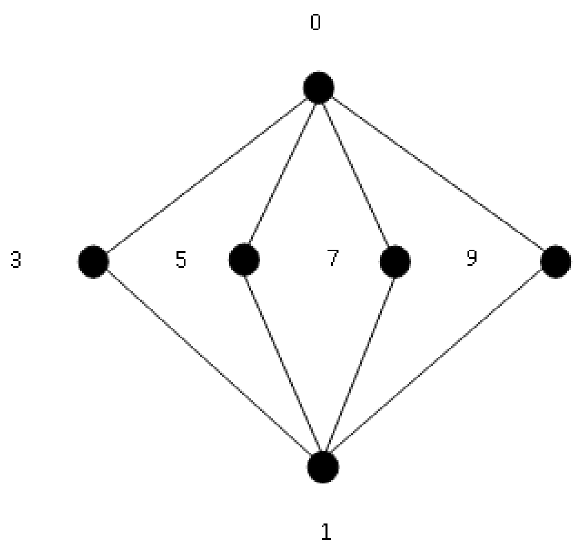

Fig 3: $Gl(4)$

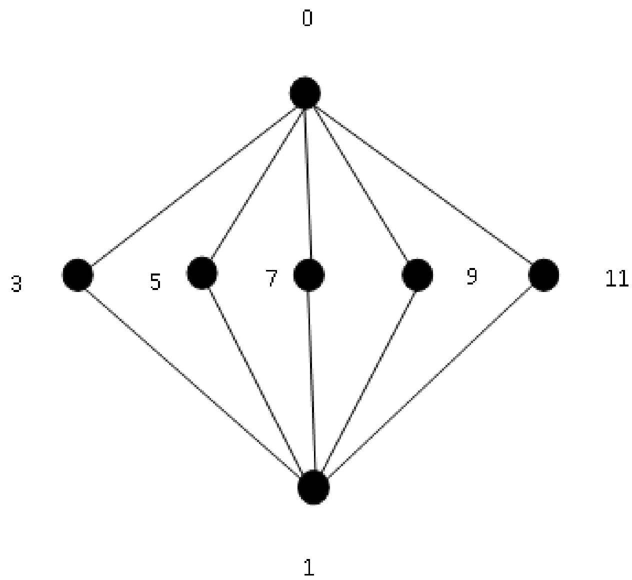

Fig 4: $Gl(5)$

THEOREM 3.5: Bi-Star $B_{n,n}$ is a Pronic product cordial graph.

Proof: Let $G = B_{n,n}$ be a graph where,

Let $V(G) = \{u, v, u_i, v_i : 1 \le i \le n\}$

Let $E(G) = \{(u,v)\}$

$\cup \{(u, u_i) : 1 \le i \le n\}$

$\cup \{(v, v_i) : 1 \le i \le n\}$

Then $|V(G)| = 2n + 2$ and $|E(G)| = 2n + 1$

Define $f : V(G) \rightarrow \{0,1,2,3,\ldots, Pr_{2n+2}\}$ as follows:

$f(u) = 0, f(v) = 1$

$f(u_i) = Pr_{i+1}, 1 \le i \le n$

$f(v_i) = Pr_{n+1+i} - 1, 1 \le i \le n$

The induced edge labels are given below:

$f(u).f(v) = 0.1 = 0 = Pr_1$

$f^*(uv) = 1$

For $1 \le i \le n$

$f(u).f(u_i) = 0.Pr_{i+1} = 0 = Pr_1$

$f^*(uu_i) = 1$

$f(v).f(v_i) = 1.[Pr_{n+1+i} - 1]$

$= Pr_{n+1+i} - 1$

(Which is odd, not a pronic number)

$f^*(vv_i) = 0$

It is observed that,

$e_{f(0)} = n$ and $e_{f(1)} = n + 1$

Clearly, $|e_f(0) - e_f(1)| \leq 1$.

Then f is a Pronic product cordial labeling.

Hence the Bi-Star $B_{n,n}$ is a Pronic product cordial graph.

Example 3.6: Fig 5 and 6 represents the Pronic product cordial labeling of $B_{4,4}$ and $B_{5,5}$ respectively.

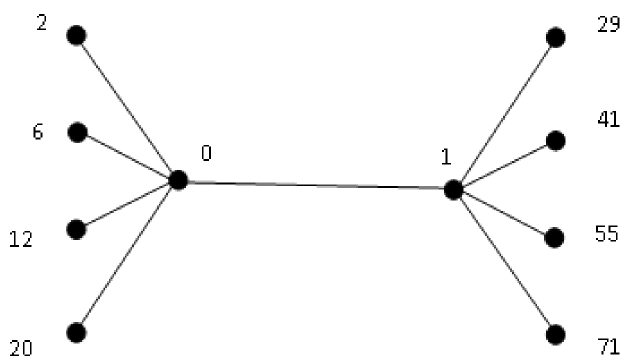

Fig 5: $B_{4,4}$

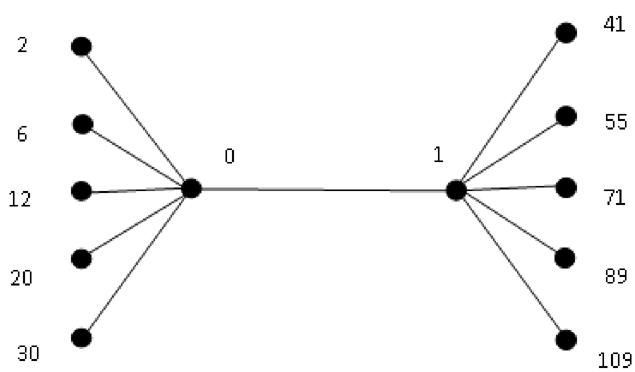

Fig 6: $B_{5,5}$

THEOREM 3.7: The subdivided star $< K_{1,n} : n >$ is a Pronic product cordial graph.

Proof: Let $G = < K_{1,n} : n >$ be a graph.

Let $V(G) = \{u, u_i, v_i : 1 \leq i \leq n\}$

Let $E(G) = \{(u, u_i) : 1 \leq i \leq n\}$

$\cup \{(u_i, v_i) : 1 \leq i \leq n\}$

Then $|V(G)| = 2n + 1$ and $|E(G)| = 2n$

Define $f: V(G) \to \{0,1,2,3, \ldots, Pr_{2n+1}\}$ as follows:

$f(u) = Pr_1 = 0$

$f(u_i) = 2i - 1 \quad , 1 \leq i \leq n$

$f(v_i) = 2(n + i) - 1, 1 \leq i \leq n$

The induced edge labels are given below:

For $1 \leq i \leq n$,

$f(u) . f(u_i) = 0 . (2i - 1) = 0 = Pr_1$

$f^*(uu_i) = 1$

$f(u_i).f(v_i) = (2i - 1) . [2(n + i) - 1]$ (Which is odd ,not a Pronic number)

$f^*(u_i v_i) = 0$

It is observed as,

$e_{f(0)} = n$ and $e_{f(1)} = n$

Clearly, $|e_f(0) - e_f(1)| \leq 1$.

Then f is a Pronic product cordial labeling.

Hence the subdivided star $< K_{1,n} : n >$ is a Pronic product cordial graph.

Example 3.8: Fig 7 and 8 represents the Pronic product cordial labeling of the Subdivided star $< K_{1,4} : 4 >$ and $< K_{1,5} : 5 >$ respectively.

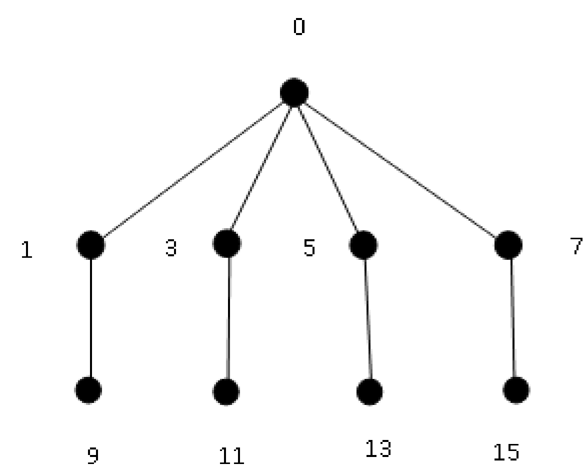

Fig 7: $< K_{1,4} : 4 >$

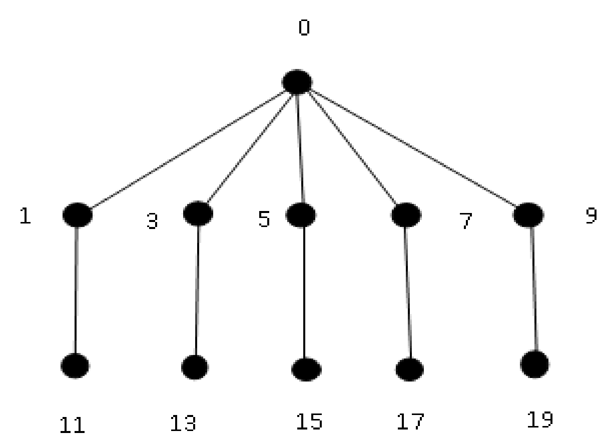

Fig 8: $< K_{1,5} : 5 >$

IV. CONCLUSION

We have introduced here a new ideaof Pronic product cordial labeling. This will add a new dimension to the research work in graph labeling on various numbers. Here we have proved that Starand a few Star related graphs are pronic product cordial graphs.

V. ACKNOWLEDGEMENT

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