# Lower Separation Axioms Using Regular*-Open Sets

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***Abstract-*** *In this paper, we introduce the concepts of r*-$T_0$, r*-$T_1$ and r*-$T_2$ spaces using regular*-open sets and investigate some of their properties. We give characterizations for these spaces. We also study the relationships among themselves and with $T_i$ and r-$T_i$ spaces.*

***Keywords***- Regular open, regular* open, r* - Ti spaces

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## I. INTRODUCTION

Separation axioms on topological spaces are those to classify the classes of topological spaces. Maheswari and Prasad introduced the notion of semi-$T_i$ (i=0, 1, 2) spaces using semi-open sets in 1975. AskishKar and Bhattacharyya introduced the concepts of pre-$T_i$ (i=0, 1, 2) spaces. Balasubramanian et al. defined the concept r-$T_i$ using regular open sets. Quite recently S. Pious Missier et al. introduced a new class of nearly open set, namely regular*-open sets and studied some properties of these sets.

In this paper, we introduce r*-$T_i$ (i=0,1,2) spaces using regular*-open sets and investigate some of their basic properties. We also study the relationships among themselves and with known separation axioms $T_i$ and r-$T_i$ (i=0,1, 2).

## II. PRELIMINARIES

Throughout this paper (X, τ) will always denote a topological space on which no separation axioms are assumed, unless explicitly stated. If A is a subset of the space (X, τ), *cl*(A) and *int*(A) respectively denote the closure and the interior of A in X .

**Definition 2.1[4]:** A subset A of a topological space (X, τ) is called

(i) generalized closed (briefly g-closed) if *cl*(A)⊆U whenever A⊆U and U is open in X.
(ii) generalized open(briefly g-open) if X\A is g-closed in X.

**Definition 2.2[3]:** Let A be a subset of X. The generalized closure of A is defined as the intersection of all g-closed sets containing A and is denoted by cl*(A).

**Definition 2.3[3]:** Let A be a subset of X. The generalized interior of A is defined as the union of all g-open sets in X containing A and is denoted by int*(A).

**Definition 2.4[8]:** A subset A of a topological space (X, τ) is called regular open if A=int(cl(A)) and regular closed if A=cl(int(A)).

**Definition 2.5[6]:** A subset A of a topological space (X, τ) is called regular*-open if A=int(cl*(A)) and regular*-closed if A=cl(int*(A)).

**Definition 2.6[6]:** Let A be a subset of X. Then the regular*-closure of A is defined as the intersection of all regular*-closed sets containing A and is denoted by r*cl(A).

**Definition 2.7:** A space X is said to be $T_0$ (resp. r-$T_0$ [2]) if for every pair of distinct points x and y in X, there is an open (resp. regular-open) set in X containing one of x and y but not the other.

**Definition 2.8:** A space X is said to be $T_1$ (resp.r-$T_1$ [2]) if for every pair of distinct points x and y in X, there are open (resp. regular-open) sets U and V such that U contains x but not y and V contains y but not x.

**Definition 2.9:** A space X is said to be $T_2$ (resp. r-$T_2$ [2]) if for every pair of distinct points x and y in X, there are disjoint open (resp. regular-open) sets U and V in X containing x and y respectively.

**Lemma 2.10 [10]: A** topological space (X,τ) is $T_1$ if and only if {x} is closed for every x∈X.

**Definition 2.11[7]**: A function $f$ :X⟶Y is said to be

(i) regular*-continuous if $f^{-1}$(V) is regular*-open in X for every open set V in Y
(ii) regular*-open if *f*(V) is regular*-open in Y for every open set V in X.
(iii) regular*-closed if *f*(V) is regular*-closed in Y for every closed set V in X.
(iv) pre regular*-open if *f*(V) is regular*-open in Y for every regular*-open set V in X.

(v) regular*-irresolute if $f^{-1}$(V) is regular*-open in X for every regular*-open set V in Y.
(vi) strongly regular*-irresolute if $f^{-1}$(V) is open in X for every regular*-open set V in Y.

**Theorem 2.12[6]:** (i) Every regular open set is regular*-open.
(ii) Every regular*-open set is open.

### III. r*-$T_0$Spaces

In this section we introduce r*-$T_0$ spaces and investigate some of their basic properties.

**Definition 3.1:** A topological space X is said to be r*-$T_0$ if for any two distinct points x and y of X, there exists a Regular*-open set G such that ($x\in G$ and $y\notin G$) or ($y\in G$ and $x\notin G$).

**Theorem 3.2:** Every r*-$T_0$ space is $T_0$.

**Proof:** Let X be a r*-$T_0$ space. Let x and y be two distinct points in X. Since X is r*-$T_0$, there exists a regular*-open set U such that ($x\in U$ and $y \notin U$) or ($y\in U$ and $x\notin U$). Since every regular*-open set is open, we have U is an open set such that ($x\in U$ and $y \notin U$) or ($y\in U$ and $x\notin U$).

Hence X is $T_0$.

**Remark 3.3:** The converse of the above theorem is not true, as seen from the following example.

**Example 3.4:** Consider the space (X, $\tau$), where X= {a, b, c, d, e} and $\tau$={$\phi$, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}, {a, b, c, d}, X}. Clearly (X, $\tau$) is $T_0$. Here there is no regular open set containing one of the elements d and e but not other. So it is not r-$T_0$.

**Theorem 3.5:** Every r-$T_0$ space is r*-$T_0$.

**Proof:** Let X be a r*-$T_0$ space. Let x and y be two distinct points in X. Since X is r-$T_0$, there exists a regular-open set U such that $x\in U$ and $y\notin U$ (or) $y\in U$ and $x\notin U$. since every regular-open set is regular*-open, we have U is an regular*-open set such that $x\in U$ and $y\notin U$ (or) $y\in U$ and $x\notin U$. Hence X is r*-$T_0$.

**Remark 3.6:** The converse of the above theorem is not true, as seen from the following example.

**Example 3.7:** Consider the space (X, $\tau$), where X= {a, b, c, d} and $\tau$={$\phi$, {a}, {b}, {a, b}, {b, c}, {a, b, c}, X}. Clearly (X, $\tau$) is r*- $T_0$. Here there is no regular open set containing one of the elements b and c but not other. So it is not r-$T_0$.

**Theorem 3.8:** Let $f : X \rightarrow Y$ be a bijective function. The following results hold.

(i) If $f$ is regular*-open and X is $T_0$, then Y is r*-$T_0$.
(ii) If $f$ is pre regular*-open and X is r*-$T_0$, then Y is r*-$T_0$.
(iii) If $f$ is regular*-continuous and Y is $T_0$, then X is r*-$T_0$.
(iv) If $f$ is regular* irresolute and Y is r*-$T_0$, then X is r*-$T_0$.

**Proof:**

(i) Suppose $f$ is a regular*-open bijection and X is $T_0$. Let $y_1$, $y_2\in Y$ with $y_1\neq y_2$
Let $x_1=f^{-1}(y_1)$ and $x_2=f^{-1}(y_2)$. Since $f$ is one-to-one, $x_1\neq x_2$. Since X is $T_0$, there exist open set U such that $x_1\in U$ and $x_2\notin U$ (or) $x_2\in U$ and $x_1\notin U$. Again since $f$ is a bijection,
$y_1\in f$ (U) and $y_2\notin f$(U) (or) and $y_2\in f$ (U) but $y_1\notin f$(U). Since $f$ is regular*-open, $f$ (U) is regular*-open set in Y. Hence Y is r*-$T_0$.

(ii) Suppose $f$ is a pre regular*-open bijection and X is r*-$T_0$. Let $y_1$, $y_2\in Y$ with $y_1\neq y_2$. Let $x_1 = f^{-1}(y_1)$ and $x_2=f^{-1}(y_2)$. Since $f$ is one-to-one, $x_1\neq x_2$. Since X is r*-$T_0$, there exist regular* open sets U such that $x_1\in U$ and $x_2\notin U$ (or) $x_2\in U$ and $x_1\notin U$. Again since $f$ is a bijection, $y_1\in f$ (U) and $y_2\notin f$ (U) (or) $y_2\in f$ (U) and $y_1\notin f$ (U). Since $f$ is pre regular*-open, $f$ (U) is regular*-open set in Y. Hence Y is r*-$T_1$.

(iii) Suppose $f$ is a regular*-continuous bijection and Y is $T_0$. Let $x_1$, $x_2\in X$ with $x_1\neq x_2$. Let $y_1=f(x_1)$ and $y_2=f(x_2)$. Since $f$ is one-to-one, $y_1\neq y_2$. Since Y is $T_0$, there exist open sets U in Y such that $y_1\in U$ and $y_2\notin U$ (or) $y_2\in U$ and $y_1\notin U$. Again since $f$ is a bijection, $x_1\in f^{-1}$(U) and $x_2\notin f^{-1}$(U) (or) $x_2\in f^{-1}$(U) and $x_1\notin f^{-1}$(U). Since $f$ is regular*- continuous, $f^{-1}$(U) is a regular*-open set in X. Hence X is r*-$T_1$.

(vi) Suppose $f$ is a regular*-irresolute bijection and Y is r*-$T_0$. Let $x_1$, $x_2\in X$ with $x_1\neq x_2$. Let $y_1=f(x_1)$ and $y_2=f(x_2)$. Since $f$ is one-to-one, $y_1\neq y_2$. Since Y is r*-$T_0$, there exist regular*-open sets U in Y such that $y_1\in U$ and $y_2\notin U$ (or) $y_2\in U$ but $y_1\notin U$. Again since $f$ is a bijection, $x_1\in f^{-1}$(U) and $x_2\notin f^{-1}$(U) (or) $x_2\in f^{-1}$(U) and $x_1\notin f^{-1}$(U). Since $f$ is regular*-irresolute, $f^{-1}$(U) is a regular*-open set in X. Hence X is r*-$T_1$.

**Theorem 3.9:** In an r*-$T_0$ space X, the r*-closures of distinct points are distinct.

**Proof:** Let X be an r*-$T_0$ space. Let x and y be two distinct points in X. Then there exists a regular*-open set U such that

$x \in U$ but $y \notin U$ (or) $y \in U$ but $x \notin U$. If $x \in U$ and $y \notin U$, then U is a regular*-open set containing x that does not intersect {y}. It follows that $x \notin r^*cl(\{y\})$. But $x \in r^*cl(\{x\})$, so we get $r^*cl(\{x\}) \neq r^*cl(\{y\})$. The proof for the other case is similar.

## IV. r*-$T_1$ Spaces

In this section we introduce r*-$T_1$ spaces and investigate some of their basic properties.

**Definition 4.1:** A space X is said to be r*-$T_1$ if for every pair of distinct points x and y in X, there exist regular*-open sets U and V such that $x \in U$ but $y \notin U$ and $y \in V$ but $x \notin V$.

**Proposition 4.2:** Every r-$T_1$ space is r*-$T_1$.

**Proof:** Suppose X is a r-$T_1$ space. Let x and y be two distinct points in X. Since X is r-$T_1$, there exist regular-open sets U and V such that $x \in U$ but $y \notin U$ and $y \in V$ but $x \notin V$. Since every regular open set is regular*-open, we have U and V are regular*-open sets such tat $x \in U$ but $y \notin U$ and $y \in V$ but $x \notin V$. Hence X is r*-$T_1$.

**Proposition 4.3:** Every r*-$T_1$ space is $T_1$.

**Proof:** Suppose X is a r*-$T_1$ space. Let x and y be two distinct points in X. Since X is $r^*$-$T_1$, there exist regular*-open sets U and V such that $x \in U$ but $y \notin U$ and $y \in V$ but $x \notin V$. since every regular*- open set is open, we have U and V are open sets such that $x \in U$ but $y \notin U$ and $y \in V$ but $x \notin V$. Hence X is $T_1$.

**Theorem 4.4:** A topological space $(X,\tau)$ is r*-$T_1$, if and only if for every $x \in X$, $r^*cl\{x\}=\{x\}$.

**Proof:** Let $(X,\tau)$ be r*-$T_1$ and $x \in X$. Then for each $y \neq x$, there exist regular*-open sets G and H such that $x \in G$ but $y \notin G$ and $y \in H$ but $x \notin H$. This implies that $y \notin r^*cl\{x\}$, for every $y \in X$ and $y \neq x$. Thus $\{x\}= r^*cl\{x\}$.

Conversely, suppose $r^*cl\{x\}=\{x\}$ for every $x \in X$. Let x, y be two distinct points in X. Then $x \notin \{y\}= r^*cl\{y\}$implies that, there exists a regular*-closed set $B_1$such that $y \in B_1$, $x \notin B_1$. Therefore, $X \backslash B_1$ is a regular*-open set such that $x \in X \backslash B_1$ but $y \notin X \backslash B_1$. Also $y \notin \{x\}= r^*cl\{x\}$ implies that, there exists a regular*-closed set $B_2$ such that $x \in B_2$, $y \notin B_2$. It follows that, $X \backslash B_2$ is a regular*-open set such that $y \in X \backslash B_2$ but $x \notin X \backslash B_2$. Hence $(X,\tau)$ is r*-$T_1$.

**Theorem 4.5:** Let $f : X \rightarrow Y$ be a bijective function. The following results hold.

(i) If $f$ is regular*-open and X is $T_1$, then Y is r*-$T_1$.
(ii) If $f$ is pre regular*-open and X is r*-$T_1$, then Y is r*-$T_1$.
(iii) If $f$ is regular*-continuous and Y is $T_1$, then X is r*-$T_1$.
(iv) If $f$ is regular* irresolute and Y is r*-$T_1$, then X is r*-$T_1$.
(v) If $f$ is strongly regular* irresolute and Y is r*-$T_1$, then X is $T_1$.

**Proof:**

(i) Suppose $f : X \rightarrow Y$ is a regular*-open bijection and X is $T_1$. Let $y_1, y_2 \in Y$ with $y_1 \neq y_2$. Let $x_1=f^{-1}(y_1)$ and $x_2=f^{-1}(y_2)$. Since $f$ is one-to-one, $x_1 \neq x_2$. Since X is $T_1$, there exist open sets U and V such that $x_1 \in U$ but $x_2 \notin U$ and $x_2 \in V$ but $x_1 \notin V$. Again since $f$ is a bijection, $y_1 \in f(U)$ but $y_2 \notin f(U)$ and $y_2 \in f(V)$ but $y_1 \notin f(V)$. Since $f$ is regular*-open, $f(U)$ and $f(V)$ are regular*-open sets in Y. Hence Y is r*-$T_1$.

(ii) Suppose $f: : X \rightarrow Y$ is a pre regular*-open bijection and X is r*-$T_1$. Let $y_1$, $y_2 \in Y$ with $y_1 \neq y_2$. Let $x_1=f^{-1}(y_1)$ and $x_2=f^{-1}(y_2)$. Since $f$ is one-to-one, $x_1 \neq x_2$. Since X is r*-$T_1$, there exist regular* open sets U and V such that $x_1 \in U$ but $x_2 \notin U$ and $x_2 \in V$ but $x_1 \notin V$. Again since $f$ is a bijection, $y_1 \in f(U)$ but $y_2 \notin f(U)$ and $y_2 \in f(V)$ but $y_1 \notin f(V)$. Since $f$ is pre regular*-open, $f(U)$ and $f(V)$ are regular*-open sets in Y. Hence, Y is r*-$T_1$.

(iii) Suppose $f : X \rightarrow Y$ is a regular*-continuous bijection and Y is $T_1$. Let $x_1, x_2 \in X$ with $x_1 \neq x_2$. Let $y_1=f(x_1)$ and $y_2=f(x_2)$. Since $f$ is one-to-one, $y_1 \neq y_2$. Since Y is $T_1$, there exist open sets U and V in Y such that $y_1 \in U$ but $y_2 \notin U$ and $y_2 \in V$ but $y_1 \notin V$. Again since $f$ is a bijection, $x_1 \in f^{-1}(U)$ but $x_2 \notin f^{-1}(U)$ and $x_2 \in f^{-1}(V)$ but $x_1 \notin f^{-1}(V)$. Since $f$ is regular*- continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are regular*-open sets in X. Hence, X is r*-$T_1$.

(vi) Suppose $f : X \rightarrow Y$ is a regular*-irresolute bijection and Y is r*-$T_1$. Let $x_1, x_2 \in X$ with $x_1 \neq x_2$. Let $y_1=f(x_1)$ and $y_2=f(x_2)$. Since $f$ is one-to-one, $y_1 \neq y_2$. Since Y is r*-$T_1$, there exist regular*-open sets U and V in Y such that $y_1 \in U$ but $y_2 \notin U$ and $y_2 \in V$ but $y_1 \notin V$. Again since $f$ is a bijection, $x_1 \in f^{-1}(U)$ but $x_2 \notin f^{=1}(U)$ and $x_2 \in f^{-1}(V)$ but $x_1 \notin f^{-1}(V)$. Since $f$ is regular*- irresolute, $f^{-1}(U)$ and $f^{-1}(V)$ are regular*-open sets in X. Hence, X is r*-$T_1$.

(v) Suppose $f : X \rightarrow Y$ is a strongly regular*-irresolute bijection and Y is r*-$T_1$. Let $x_1, x_2 \in X$ with $x_1 \neq x_2$. Let $y_1=f(x_1)$ and $y_2=f(x_2)$. Since $f$ is one-to-one, $y_1 \neq y_2$. Since Y is r*-$T_1$, there exist regular*-open sets U and V in Y such that $y_1 \in U$ but $y_2 \notin U$ and $y_2 \in V$ but $y_1 \notin V$. Again since $f$ is a bijection, $x_1 \in f^{-1}(U)$ but $x_2 \notin f^{-1}(U)$ and $x_2 \in f^{-1}(V)$ but $x_1 \notin f^{-1}(V)$. Since $f$ is strongly regular*-irresolute, $f^{-1}(U)$ and $f^{-1}(V)$ are open sets in X. Hence, X is $T_1$.

## V. r*-$T_2$ Spaces

In this section we introduce r*-$T_2$ spaces and investigate some of their basic properties.

**Definition 5.1:** A space X is said to be r*-$T_2$ if for every pair of distinct points x and y in X, there are disjoint R*-open sets U and V in X containing x and y respectively.

**Theorem 5.2:** Every r-$T_2$ space is r*-$T_2$.

**Proof:** Let X be a r-$T_2$ space. Let x and y be two distinct points in X. Since X is r-$T_2$, there exist disjoint regular-open sets U and V such that $x \in U$ and $y \in V$. Since every regular-open set is regular*-open, we have U and V are disjoint regular*-open sets such that $x \in U$ and $y \in V$. Hence X is r*-$T_2$.

**Theorem 5.3:** Every r*-$T_2$ space is $T_2$.

**Proof:** Suppose X is a r*-$T_2$ space. Let x and y be two distinct points in X. Since X is r*-$T_2$, there exist disjoint regular*-open sets U and V such that $x \in U$ and $y \in V$. Since every regular*-open set is open, we have U and V are disjoint open sets such that $x \in U$ and $y \in V$. Hence X is $T_2$.

**Theorem 5.4:** Every r*-$T_2$ space is r*-$T_1$.

**Proof:** Let X be a r*-$T_2$ space. Let x and y be two distinct points in X. Since X is r*-$T_2$, there exist disjoint regular*-open sets U and V such that $x \in U$ and $y \in V$. Since U and V are disjoint, $x \in U$ but $y \notin U$ and $y \in V$ but $x \notin V$. Hence X is r*-$T_1$.

**Theorem 5.5:** If X is a r*-$T_2$ space and $x \in X$, then for each $y \neq x$ there exists a regular*-open set U such that $x \in U$ and $y \notin$ r*cl(U).

**Proof:** Suppose X is a r*-$T_2$ space. Then for each $y \neq x$ there exist disjoint regular*-open sets U and V such that $x \in U$ and $y \in V$. Since V is regular*-open, X\V is regular*-closed and $U \subseteq X \setminus V$. This implies that, r*cl(U)$\subseteq X \setminus V$. Since $y \notin X \setminus V$, $y \notin$ r*cl(U).

**Theorem 5.6:** Let $f : X \rightarrow Y$ be a bijective function. The following results hold.

(i) If $f$ is regular*-open and X is $T_2$, then Y is r*-$T_2$.
(ii) If $f$ is pre regular*-open and X is r*-$T_2$, then Y is r*-$T_2$.
(iii) If $f$ is regular*-continuous and Y is $T_2$, then X is r*-$T_2$.
(iv) If $f$ is regular* irresolute and Y is r*-$T_2$, then X is r*-$T_2$.
(v) If $f$ is strongly regular* irresolute and Y is r*-$T_2$, then X is $T_2$.

**Proof:**

(i) Suppose $f : X \rightarrow Y$ is a regular*-open bijection and X is $T_2$. Let $y_1, y_2 \in Y$ with $y_1 \neq y_2$. Let $x_1 = f^{-1}(y_1)$ and $x_2 = f^{-1}(y_2)$. Since $f$ is one-to-one, $x_1 \neq x_2$. Since X is $T_2$, there exist disjoint regular*-open sets U and V such that $x_1 \in U$ but $x_2 \notin U$ and $x_2 \in V$ but $x_1 \notin V$. Again since $f$ is a bijection, $y_1 \in f(U)$ but $y_2 \notin f(U)$ and $y_2 \in f(V)$ but $y_1 \notin f(V)$. Since $f$ is regular*-open, $f(U)$ and $f(V)$ are disjoint regular*-open sets in Y. Hence Y is r*-$T_2$.

(ii) Suppose $f: X \rightarrow Y$ is a pre regular*-open bijection and X is r*-$T_2$. Let $y_1, y_2 \in Y$ with $y_1 \neq y_2$. Let $x_1 = f^{-1}(y_1)$ and $x_2 = f^{-1}(y_2)$. Since $f$ is one-to-one, $x_1 \neq x_2$. Since X is r*-$T_2$, there exist disjoint regular* open sets U and V such that $x_1 \in U$ but $x_2 \notin U$ and $x_2 \in V$ but $x_1 \notin V$. Again since $f$ is a bijection, $y_1 \in f(U)$ but $y_2 \notin f(U)$ and $y_2 \in f(V)$ but $y_1 \notin f(V)$. Since $f$ is pre regular*-open, $f(U)$ and $f(V)$ are disjoint regular*-open sets in Y. Hence, Y is r*-$T_2$.

(iii) Suppose $f: X \rightarrow Y$ is a regular*-continuous bijection and Y is $T_2$. Let $x_1, x_2 \in X$ with $x_1 \neq x_2$. Let $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Since $f$ is one-to-one, $y_1 \neq y_2$. Since Y is $T_2$, there exist disjoint open sets U and V in Y such that $y_1 \in U$ but $y_2 \notin U$ and $y_2 \in V$ but $y_1 \notin V$. Again since $f$ is a bijection, $x_1 \in f^{-1}(U)$ but $x_2 \notin f^{-1}(U)$ and $x_2 \in f^{-1}(V)$ but $x_1 \notin f^{-1}(V)$. Since $f$ is regular*- continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint regular*-open sets in X. This shows that, X is r*-$T_2$.

(vi) Suppose $f : X \rightarrow Y$ is a regular*-irresolute bijection and Y is r*-$T_2$. Let $x_1, x_2 \in X$ with $x_1 \neq x_2$. Let $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Since $f$ is one-to-one, $y_1 \neq y_2$. Since Y is r*-$T_2$., there exist disjoint regular*-open sets U and V in Y such that $y_1 \in U$ but $y_2 \notin U$ and $y_2 \in V$ but $y_1 \notin V$. Again since $f$ is a bijection, $x_1 \in f^{-1}(U)$ but $x_2 \notin f^{-1}(U)$ and $x_2 \in f^{-1}(V)$ but $x_1 \notin f^{-1}(V)$. Since $f$ is regular*- irresolute, $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint regular*-open sets in X. Hence, X is r*-$T_1$.

(v) Suppose $f : X \rightarrow Y$ is a strongly regular*-irresolute bijection and Y is r*-$T_2$.. Let $x_1$, $x_2 \in X$ with $x_1 \neq x_2$. Let $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Since $f$ is one-to-one, $y_1 \neq y_2$. Since Y is r*-$T_2$., there exist disjoint regular*-open sets U and V in Y such that $y_1 \in U$ but $y_2 \notin U$ and $y_2 \in V$ but $y_1 \notin V$. Again since $f$ is a bijection, $x_1 \in f^{-1}(U)$ but $x_2 \notin f^{-1}(U)$ and $x_2 \in f^{-1}(V)$ but $x_1 \notin f^{-1}(V)$. Since $f$ is strongly regular*-irresolute, $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint open sets in X. Hence, X is $T_1$.

## REFERENCES


[1] AshishKar and Bhattacharyya, Some weak separation axioms, *Bull. Cal. Math.Soc.*82(1990), 415-422.

[2] Bala Subramanian, generalized separation axioms, *Scientia Magna*, 6(4)(2010), 1-14.

[3] Dunham, W., A new closure operator for Non-$T_1$ topologies, *Kyungpook Math. J*. 22, (1982), 55-60.

[4] Levine, N., Generalized closed sets topology, *Rend. Circ. Mat. Palermo* 19(2)(1970), 89-96.

[5] Maheswari, S. N., and Prasad, R., Some new separation axioms, *Annales de la Societe Scientifique de Bruxelles*, T. 89 III (1975), 395-402.

[6] S. Pious Missier and M. Annalakshmi, Between regular open and open sets, International journal of mathematical archive-7(5), 2016, 128-133.

[7] S. Pious Missier, M. Annalakshmi and G.Mahadevan, On regular*-open sets, Global journal of pure and applied mathematics,Volume 13, Number 9(2017), pp.5717-5726.

[8] Stone. M. H, Application of the theory of Boolean rings to general topology,Trans.Amer.Math.Soc., 41(1937), 374-481.

[9] M.K.Singal and A.Mathur, On nearly compact spaces, *Boll. Un.Math. Ital.* 4(2) (1969), 702-710.

[10] [Willard, S., General Topology, *Addison – Wesley Publishing Company*, *Inc*(1970).