Rectilinear Crossing Number of Zero Divisor Graph

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Abstract- *In this paper, we evaluate the rectilinear crossing number of a zero divisor graph in a commutative ring. The rectilinear crossing number of G is denoted by Rcr(G). Then the definition becomes cr (G) ≤Rcr(G). Here we observed the study of theorems based on distinction between prime numbers and crossing numbers.*

Keywords- Rcr, cr, Inr

Definition

The Rectilinear crossing number of a graph with minimum number of edge crossing drawn in a plane satisfies the following conditions.

i. Edges are line segments.
ii. No three vertices are collinear.
iii. No three edges may intersect at a common vertex.

The rectilinear crossing number of G is denoted by Rcr(G).
It follows from the definition that, cr(G)≤Rcr(G).

Theorem

Inr(Z2p), where p is any prime, then Rcr(r(Z2p))=0.

Proof:

Using theorem, "In r(Z2p), where o is any prime >2 then

Cr (r(Z2p))=0

r(Z2p) is a planar graph and hence Rcr(r(Z2p))=0

Hence the proof.

Theorem

For any graph r(Z3p), where p is any prime p>3, then Rcr(r(Z3p))=0

Proof:

Using theorem "For any graph r(Z3p), where p is any prime > 3, Then

Cr (r(Z3p))=0."

r(Z3p) is a planner complete bipartite graph and hence

Rcr(r(Z3p))=0

Hence the proof.

Theorem

If p and q are distinct prime numbers with q>p then, Rcr(r(Zpq))=(p-1)(p-3)(q-1)(q-3)/16.

Proof:

The vertex set ofr(Zpq) is {p,2p,…,p(q-1),q,2q,…,(p-1)q}.
The proof is by induction on p and q.

Case (i)

Let p = 5.

Sub case (i)

When q = 7, the vertex set of r(Z5q) is {5,10, … ,5(q-1),q,3q,4q}.

Clearly |V(r(Z5q))|= q + 3.

Let u and v be any vertices in r(Z5q) with maximum and minimum degree , respectively.

Let u = q and v = 1q then 5q must divide uv then u and v are adjacent.

Let u = q and v = 2q Then 5q does not divide uv.

That is uv = 2q2=0.

Then the vertex set V can be partitioned into two parts V1 and V2, where V1={q, 2q, 3q, 4q} and V2={5, 10, … ,5(q-1)}.

Clearly, any vertices u and v in V1are non-adjacent and the same hold for V2.

Let u = q in V1and v = 10 are in V2then 5q divides uv = 10q

Finally we note that, every vertex in V1is adjacent to all the vertices in V2.

Moreover V(r(Z5q)) = V1 U V2and V1 U V2= ϕ.

If q = 7 > 5, then|V1| = 4 and |V2| =q – 1 = 6.

Let V1= {V1,V2,V3,V4} and V2 = {u1,u2,…..,uq-1}.

In a drawing D, we denote by RcrD (vi, ui) the number of crossings of lines, one terminating at vi, the other at ujand by RcrD (vi) the number of crossing lines which terminate at vi,

RcrD (vi) = $\sum_{j=1}^{6} RcrD(V_i, U_j)$

Clearly,RcrD(r(Z5q))= $\sum_{i=1}^{6} RcrD(V_i, U_j)$

Since, the rectilinear crossing number of r(Z5q) is either the sum of all rectilinear crossing number of the all vertices in v1 or the sum of all rectilinear crossing number of the vertices in v2.

Now, we consider the vertex set v1.

The proof is based on having vertices of v1 placed horizontally and v2 is placed randomly so that no vertices are collinear.

The first thing is so place $(\frac{q-1}{2})$ vertices on one side of the horizontal line and $(\frac{q}{2})$vertices on the other side.

Then we connect all the vertices on the horizontal line with all the vertices placed randomly by means of line segment such that no three on computing the rectilinear crossing number of r(Z5q), we get Rcr(7) = 0,Rcr(14) = 0, Rcr(21) = 6, Rcr(28) = 6.

Rcr(r(Z5p))= $\sum_{i=1}^{4}$ Rcr(vi)
= Rcr(7) + Rcr(14) + Rcr(21) + Rcr(28)
= 0 + 0 + 6 + 6
= 12
= 2 X 1 X 3
= $(\frac{5-1}{2})(\frac{5-3}{2})(\frac{7-1}{2})(\frac{7-3}{2})$
=$(\frac{p-1}{2})(\frac{p-3}{2})(\frac{q-1}{2})(\frac{q-3}{2})$

Subcase (ii)

When q = 11

The vertex set of r(Z5q) is {5,10, … ,5(q-1),q,2q,3q,4q}.

Similarly as in **sub case(i)**.

We get Rcr(11) = 0, Rcr(22) = 0,Rcr(33) = 20, Rcr(44) = 20.

Rcr(r(Z5q)) = Rcr(r(Z5q))
= $\sum_{j=1}^{4}$ Rcr(vi)
= Rcr(7) + Rcr(14) + Rcr(21) + Rcr(28)
= 0 + 0 + 20 + 20
= 40
= 2 X 1 X 5 X 4
= $(\frac{5-1}{2})(\frac{5-3}{2})(\frac{11-1}{2})(\frac{11-3}{2})$
= $(\frac{p-1}{2})(\frac{p-3}{2})(\frac{q-1}{2})(\frac{q-3}{2})$

Case (ii)

Let p = 7

Subcase (i)

The vertex set of r(Z7q) is {7,14, … ,7(q-1),q,2q,3q,4q,5q,6q}.

Let u = 7 and v = 9 in V(r(Z7 q)) then 7 q must divide uv, which implies that u and v are adjacent vertices in r(Z7q)

Let u = 7 and v = 14 then 7q does not divide uv.

Using theorem “for any graph r(Z5p), where p > 5 is any prime, then Cr (r(Z3p))= 2 $[\frac{p-1}{2}][\frac{p-2}{2}]$” partition the vertex set of r(Z7q) into two parts v1 and v2. Clearly no two vertices in v1 are adjacent and the same hold for v2.

Next, we calculate either the sum of the rectilinear crossing number of the vertices in v1 or the sum of the rectilinear crossing number of the vertices in v2.

Now place the six vertices {q, 2q, 3q, 4q, 5q, 6q} in v1 are placed horizontally and (q-1) vertices {7, 14 ….7(q-1)} in v2 are placed randomly in such a way that no three vertices are collinear.

Then, we connect all the vertices placed horizontally with all the vertices that are randomly placed.

Rcr(q) = 0,Rcr(2q) = 0, Rcr(3q) = 20, Rcr(4q) = 20,
Rcr(5q) = 40 and Rcr(6q) = 40.

Then Rcr(r(Z7q)) = $\sum_{i=1}^{6} \sum_{j=1}^{q-1}$ RcrD (vi,uj).

Where, vi $\in$v1 anduj$\in$v2.

Rcr(r(Z7q)) = sum of the rectilinear crossing number in v1 orv2.

= Rcr(11) + Rcr(22) + Rcr(33) + Rcr(44)+ Rcr(55)+ Rcr(66)

= 120

= 3 X 2 X 5 X 4

$= (\frac{7-1}{2})(\frac{7-3}{2})(\frac{11-1}{2})(\frac{11-3}{2})$

$=(\frac{p-1}{2})(\frac{p-3}{2})(\frac{q-1}{2})(\frac{q-3}{2})$

Subcase (ii)

When q = 13

The vertex set of r(Z7q) is {7,14, … ,7(q-1),q,2q,3q,4q,5q,6q}.

Now proceeding as in **sub case(i)**.

We get the rectilinear crossing as,

Rcr(r(Z5q)) = 180

= 2 X 1 X 5 X 4

$= (\frac{5-1}{2})(\frac{5-3}{2})(\frac{11-1}{2})(\frac{11-3}{2})$

Rcr(r(Zpq)) $= (\frac{p-1}{2})(\frac{p-3}{2})(\frac{q-1}{2})(\frac{q-3}{2})$

cr(r(Zpq)) = (p-1)(p-3)(q-1)(q-3)/16.

Hence the proof.