Gaussian Diophantine Quadruples Involving Gnomonic Numbers With Property D(4)

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Abstract- *This paper concerns with the problem of constructing Gaussian Diophantine quadruples involving Gnomonic numbers such that the product of any two members of the set added by 4 is a square of a Gaussian integer.*

Keywords- Gnomonic number, Diophantine quadruples, Gaussian integer.

Notation:

$\text{Gno}_n = \text{Gnomonic number of rank n.}$

I. INTRODUCTION

Numbers are as unbounded as human comprehension is kept, so number theory and its varying subfields will continue exciting the brains of mathematicians for an exceptionally significant time-frame [1-4]. While individual equations present a sort of conundrum and have been considered since the beginning, the importance of general speculations of Diophantine equations was an accomplishment of the twentieth century [5-9].

Numerous mathematicians considered the problem of the existence of Diophantine quadruples with the property D(n) for any whole number n, besides for any direct polynomial in n. In this particular circumstance, one may allude for a extensive review of different problems on Gaussian dio quadruples [10-12].

In this communication, we construct Gaussian Diophantine quadruples (a,b,c,d) involving gnomonic numbers such that the product of any two members of the set added by 4 is a square of a gaussian integer.

II. BASIC DEFINITION

A set of m Gaussian integer is called a complex Diophantine m - tuple with property D(z) if the product of any two distinct elements increased by z is a square of a Gaussian integer.

III. METHOD OF ANALYSIS

Let $a = \text{Gno}_{\text{p+iq+2}} = 2p + 3 + 2iq$ & $b = \text{Gno}_{\text{p+iq+4}} = 2p + 7 + 2iq$ be two Gaussian integers, such that $ab + 4$ is a square of a Gaussian integer say α^2.

Let c be any Gaussian integer such that

$$ac + 4 = \beta^2 \quad (1)$$

$$bc + 4 = \gamma^2 \quad (2)$$

Setting $\beta = a + \alpha$ and $\gamma = b + \alpha$ and subtracting (1) from (2), we get

$$c = a + b + 2\alpha = 8p + 20 + 8iq = \text{Gno}_{\text{4n+10}} + 1$$

The fourth tuple is given by,

$$d = a + b + c + \frac{2}{n}[abc + \alpha\beta\gamma]$$
$$= a + b + c + \frac{2}{4}[abc + \alpha\beta\gamma]$$
$$= 32p^3 + 240p^2 - 96pq^2 + 592p - 240q^2 + 480 + i(-32q^3 + 480pq + 96p^2q + 592q)$$
$$\therefore d = \text{Gno}_{16(p+iq)^3 + 120(p+iq)^2 + 286(p+iq) + 240} + 1$$

Thus, we observe that (a,b,c,d) is a Gaussian diophantine quadruple with property D(4).

Remarkable Observation:

Proceeding in the same manner, for other choices of gnomonic numbers of different ranks, we obtain Gaussian Diophantine quadruples with the property D(4) are given in the following table 1.

Table 1

a	b	c	d
Gno_{p+iq}	Gno_{p+iq+2}	$Gno_{4p+4iq+2}+1$	$Gno_{16(p+iq)^3+24(p+iq)^2+8(p+iq)}+1$
Gno_{2p+2iq}	$Gno_{2p+2iq+2}$	$Gno_{8p+8iq+2}+1$	$Gno_{128(p+iq)^3+96(p+iq)^2+16(p+iq)}+1$
$Gno_{2p+2iq+3}$	$Gno_{2p+2iq+5}$	$Gno_{8p+8iq+14}+1$	$Gno_{128(p+iq)^3+672(p+iq)^2+1168(p+iq)+672}+1$
Gno_{2p+2iq}	$Gno_{2p+2iq-2}$	$Gno_{8p+8iq-6}+1$	$Gno_{128(p+iq)^3-288(p+iq)^2+208(p+iq)-48}+1$
$Gno_{3p+3iq+1}$	$Gno_{3p+3iq+3}$	$Gno_{12p+12iq+6}+1$	$Gno_{432(p+iq)^3+648(p+iq)^2+312(p+iq)+48}+1$
$Gno_{3p+3iq+4}$	$Gno_{3p+3iq+6}$	$Gno_{12p+12iq+18}+1$	$Gno_{432(p+iq)^3+1944(p+iq)^2+2904(p+iq)+1440}+1$
$Gno_{3p+3iq-1}$	$Gno_{3p+3iq+1}$	$Gno_{12p+12iq-2}+1$	$Gno_{432(p+iq)^3-216(p+iq)^2+24(p+iq)}+1$

IV. CONCLUSION

In this paper, we construct the Gaussian Diophantine quadruples involving Gnomonic numbers with the property D(4). One may search for other Gaussian Diophantine quadruples made up of different numbers with suitable properties.

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