Root Square Mean Labeling Of Some Cycle Related Graphs

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Abstract- *A graph G with p vertices and q edges is called root square mean graph if it is possible to label the vertices $x \epsilon V$ with distinct lables $f(x)$ from $1,2,\dots,q+1$ in such a way that when each edge $e = uv$ is labeled with*

$$f(e = uv) = \left\lceil \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rceil \text{ or } \left\lfloor \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rfloor,$$

then the resulting edge labels are distinct. In this case f is called a Root Square Mean labeling of G. In this paper we prove that some cycle related graphs such as path union of cycles, $k-$path union of cycles, path union of crowns and $k-$path union of crowns are root square mean graphs.

Keywords- Graph, Root Square Mean labeling, Path, Cycle, Crown.

I. INTRODUCTION

In this paper we consider the graphs which are simple, finite and undirected with p vertices and q edges. For a detailed survey of graph labeling we refer to Gallain [1]. For all other standard terminology and notations we follow Harary [2]. The concept of Root Square Mean labeling has been introduced by S.S.Sandhya, S.Somasundaram and S.Anusa in 2014 [4]. In this paper, we investigate the Root Square Mean labeling of some new connected graphs. Some new examples are presented and verified. We now give the definitions which are necessary for the present investigation.

Definition 1.1:

A graph $G = (V, E)$ with p vertices and q edges is said to be a Root Square Mean graph if it is possible to label the vertices $x \epsilon V$ with distinct labels $f(x)$ from $1,2,\dots,q+1$ in such a way that when each edge $e = uv$ is labeled with $f(e = uv) = \left\lceil \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rceil$ or $\left\lfloor \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rfloor$, then the resulting edge labels are distinct. In this case f is called a Root square Mean labeling of G.

Definition 1.2:

A walk in which $u_1 u_2 \cdots u_n$ are distinct is called a path. A path on n vertices is denoted by P_n.

Definition 1.3:

A closed path is called a cycle. A cycle on n vertices is denoted by C_n.

Definition 1.4:

The union of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is a graph $G = G_1 \cup G_2$ with vertex set $V = V_1 \cup V_2$ and the edge set $= E_1 \cup E_2$.

Definition 1.5:

The Corona of two graphs G_1 and G_2 is the graph $G = G_1 \odot G_2$ formed by taking one copy of G_1 and $|V(G_1)|$ copies of G_2 where the i^{th} vertex of G_1 is adjacent to every vertex in the i^{th} copy of G_2.

Definition 1.6:

Let $G_1, G_2, \dots, G_n$, $n \geq 2$ be n copies of a fixed graph G. The graph G obtained by adding an edge between G_i and G_{i+1} for $i = 1,2,\dots,n-1$ is called a path union of G.

Definition 1.7:

The $k-$path union of two cycles C_n is the graph obtained by joining two vertices from two copies of C_n by a path P_k of length $k-1$.

II. MAIN RESULTS

In this paper, we investigate the Root Square Mean labeling of some cycle related graphs.

Theorem 2.1

Path union of two cycles C_n is a Root Square mean graph.

Proof:

Let $u_1u_2 \dots u_n$ and $v_1v_2 \dots v_n$ be the vertices of two cycles in G.

Let $V(G) = \{u_1u_2 \dots u_n, v_1v_2 \dots v_n\}$

$E(G) = \{u_iu_{i+1} \,/\, 1 \le i \le n-1\} \cup \{v_iv_{i+1} \,/\, 1 \le i \le n-1\} \cup \{u_nu_1, v_nv_1, u_1v_1\}$

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Define a function $f: V(G) \to \{1,2,\dots,2n+1\}$ by

$f(u_i) = i+1$ for $1 \le i \le n-1$

$f(u_n) = 1$

$f(v_i) = n+2+i$ for $1 \le i \le n-1$

$f(v_n) = n+2$

Then the edge lables are distinct.

Hence f is a Root Square mean labeling of G.

Example 2.1.1:

The Root Square mean labeling of path union of two cycles C_6 is given below:

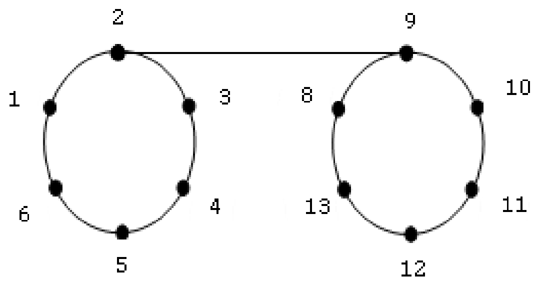

Figure 1

Theorem 2.2

Path union of three cycles C_n is a Root Square mean graph.

Proof:

Let $u_1u_2 \dots u_n$ and $v_1v_2 \dots v_n$ and $w_1w_2 \dots w_n$ be the vertices of three cycles in G.

Let $V(G) = \{u_1u_2 \dots u_n, v_1v_2 \dots v_n, w_1w_2 \dots w_n\}$

$E(G) = \{u_iu_{i+1} \,/\, 1 \le i \le n-1\} \cup \{v_iv_{i+1} \,/\, 1 \le i \le n-1\} \cup \{w_iw_{i+1} \,/\, 1 \le i \le n-1\} \cup \{u_nu_1, u_1v_1, v_nv_1, v_1w_1, w_nw_1\}$

Define a function $f: V(G) \to \{1,2,\dots,3n+3\}$ by

$f(u_i) = i+1$ for $1 \le i \le n-1$

$f(u_n) = 1$

$f(v_i) = n+2+i$ for $1 \le i \le n-1$

$f(v_n) = n+2$

$f(w_i) = 2n+4+i$ for $1 \le i \le n-1$

$f(w_n) = 2n+4$

Then the edge lables are distinct.

Hence f is a Root Square mean labeling of G.

Example 2.2.1:

The Root Square mean labeling of path union of three cycles C_5 is given below:

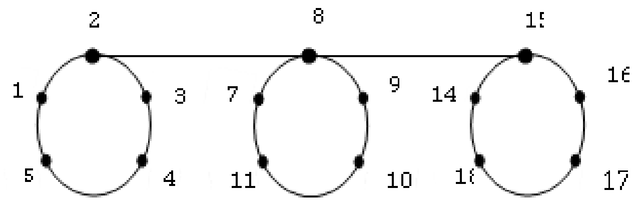

Figure 2

Theorem 2.3

The $k-$path union of two cycles C_n with path P_k is a Root Square mean graph.

Proof:

Let $u_1u_2 \dots u_n$ and $v_1v_2 \dots v_n$ be the vertices of two cycles C_n in G.

Let $u_1 = w_1w_2 \dots w_k = v_1$ be the vertices of the path P_k.

Define a function $f: V(G) \to \{1,2,\dots,2n+k\}$ by

$f(u_i) = i+1$ for $1 \le i \le n-1$

$f(u_n) = 1$

$f(v_i) = n+k+1+i$ for $1 \le i \le n-1$

$f(v_n) = n+k+1$

$f(w_i) = n+i$ for $2 \le i \le k-1$

Then the edge lables are distinct.

Hence f is a Root Square mean labeling of G.

Example 2.3.1:

The Root Square mean labeling of $k-$path union of C_5 is given below:

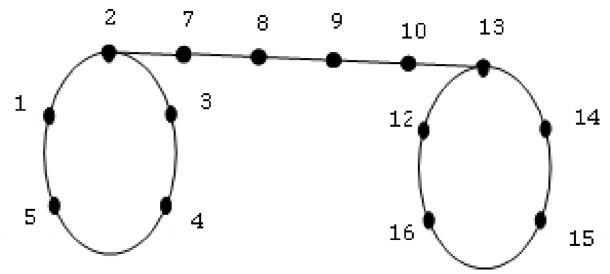

Figure 3

Theorem 2.4

Path union of two Crowns C_n^* is a Root Square mean graph.

Proof:

Let $u_1u_2 \dots u_n$ and $v_1v_2 \dots v_n$ be the vertices of two cycles C_n in G.
Let $u_1', u_2', \dots, u_n'$ be the pendent vertices attached at $u_1u_2 \dots u_n$ respectively and $v_1', v_2', \dots, v_n'$ be the pendent vertices attached at $v_1v_2 \dots v_n$ respectively.

Define a function $f: V(G) \to \{1, 2, \dots, 4n+1\}$ by

$f(u_i) = 2i + 1$ for $1 \le i \le n-1$

$f(u_n) = 1$

$f(v_i) = 2n + 2i + 1$ for $1 \le i \le n-1$

$f(v_n) = 2n + 1$

$f(u_i') = 2i + 2$ for $1 \le i \le n-1$

$f(u_n') = 2$

$f(v_i') = 2n + 2i + 2$ for $1 \le i \le n-1$

$f(v_n') = 2n + 2$

Then clearly the edge labels are distinct.
Hence f is a Root Square mean labeling of G.

Example 2.4.1:

The Root Square mean labeling of path union of two crowns C_4^* is given below:

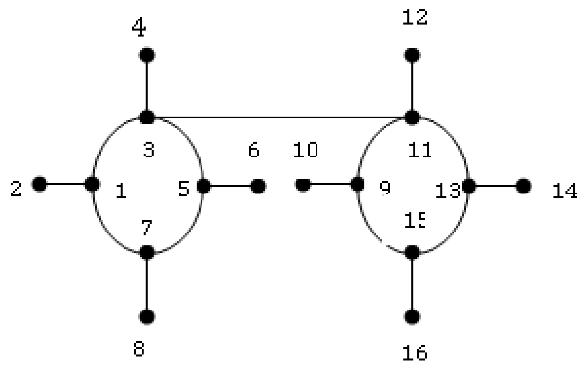

Figure 4

Theorem 2.5

Path union of three Crowns C_n^* is a Root Square mean graph.

Proof:

Let $u_1u_2 \dots u_n$, $v_1v_2 \dots v_n$ and $w_1, w_2, \dots, w_n$ be the vertices of three cycles C_n in G.
Let $u_1', u_2', \dots, u_n'$, $v_1', v_2', \dots, v_n'$ and $w_1', w_2', \dots, w_n'$ be the pendent vertices attached at $u_1, u_2, \dots, u_n$, $v_1, v_2, \dots, v_n$ and $w_1, w_2, \dots, w_n$ respectively.

Define a function $f: V(G) \to \{1, 2, \dots, 6n+2\}$ by

$f(u_i) = 2i + 1$ for $1 \le i \le n-1$

$f(u_n) = 1$

$f(v_i) = 2n + 2i + 1$ for $1 \le i \le n-1$

$f(v_n) = 2n + 1$

$f(w_i) = 4n + 2i + 1$ for $1 \le i \le n-1$

$f(w_n) = 4n + 1$

$f(u_i') = 2i + 2$ for $1 \le i \le n-1$

$f(u_n') = 2$

$f(v_i') = 2n + 2i + 2$ for $1 \le i \le n-1$

$f(v_n') = 2n + 2$

$f(w_i') = 4n + 2i + 2$ for $1 \le i \le n-1$

$f(w_n') = 4n + 2$

Then clearly the edge labels are distinct.
Hence f is a Root Square mean labeling of G.

Example 2.5.1:

The Root Square mean labeling of path union of three crowns C_4^* is given below:

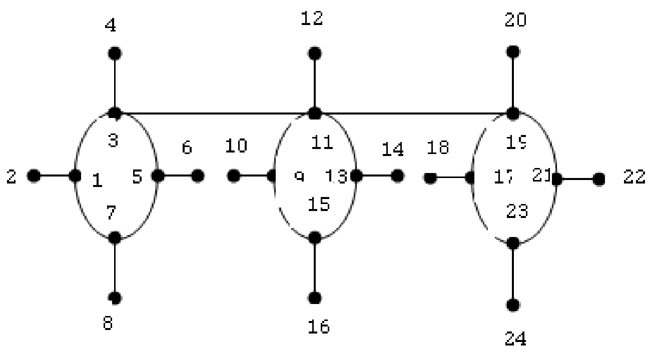

Figure 5

Theorem 2.6

The $k-$path union of two Crowns C_n^* with path P_k is a Root Square mean graph.

Proof:

Let $u_1u_2\cdots u_n$ and $v_1v_2\cdots v_n$ be the vertices of two cycles C_n in G.
Let $u_1 = w_1, w_2, \dots, w_k = v_1$ be the vertices of path P_k.
Let $u_1', u_2', \dots, u_n'$, and $v_1', v_2', \dots, v_n'$ be the pendent vertices attached at $u_1, u_2, \dots, u_n$ and $v_1, v_2, \dots, v_n$ respectively.

Define a function $f: V(G) \to \{1, 2, \dots, 4n + k\}$ by
$f(u_i) = 2i + 1$ for $1 \le i \le n - 1$
$f(u_n) = 1$
$f(v_i) = 2n + k + 2i - 1$ for $1 \le i \le n - 1$
$f(v_n) = 2n + k - 1$
$f(w_i) = 2n + i - 1$ for $2 \le i \le k - 1$
$f(u_i') = 2i + 2$ for $1 \le i \le n - 1$
$f(u_n') = 2$
$f(v_i') = 2n + k + 2i$ for $1 \le i \le n - 1$
$f(v_n') = 2n + k$
Then clearly the edge labels are distinct.
Hence f is a Root Square mean labeling of G.

Example 2.6.1:

The Root Square mean labeling of $k-$path union of C_4^* is given below:

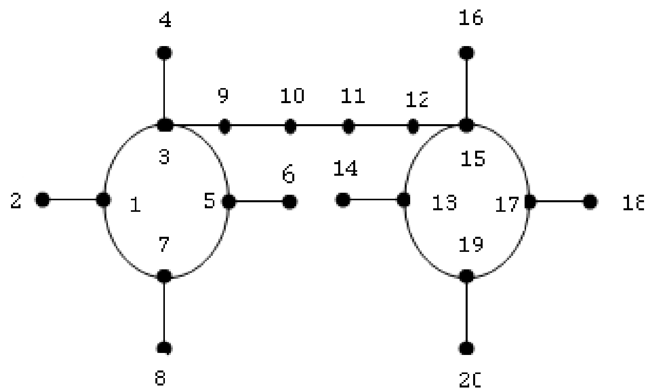

Figure 6

III. CONCLUSION

As all graphs are not Root Square Mean graphs, it is very interesting to investigate graphs which admitsRoot Square Mean labeling. In this paper we present six new results on Root Square mean labeling ofgraphs. Analogous work can be carried out forother graph families.

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