Determination of Stress Intensity Factors For Semi Elliptical Crack of Two Cracked Rotor

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Abstract- *To examine the behavior of cracked rotor with two cracks, a mathematical model is developed for a simple rotor with two cracks. By solving the mathematical model , stress intensity factors are determined. The problem is solved by varying the crack ellipticity ratio (ie major to minor axis lengths) and the crack plane angles. The crack ellitpcity ratios considered are 0.25 , 0.5, 0.75 and 1 for different crack plane angles ranging from 0 to 40 degrees. To validate the results, the same model is solved using Ansys work bench and stress intensity factors are determined. The results obtained are compared and the effect of ellitpicity ratio and crack plane angle on stress intensity factors is determined.*

Keywords*-* Cracked rotor , Stress Intensity factor, semi elliptical crack, slant crack

I. INTRODUCTION

Flaws in the components of a structure can influence the dynamic behavior of the whole structure. It is well known from the literature that one form of damage that can lead to catastrophic failure if undetected is fatigue cracking of the structure elements. The recognition of the vibration effects of cracks is important in practice since vibration monitoring has revealed a great potential for investigation of cracks in the last three decades. Usually the physical dimensions, boundary conditions, the material properties of the structure play important role for the determination of its dynamic response. Their vibrations cause changes in dynamic characteristics of structures. In addition to this presence of a crack in structures modifies its dynamic behavior.

A.K. Darpe, K. Gupta, A. Chawla, et.al. [1] studied the effect of the presence of the single transverse crack on the response of the rotor The results of the study are useful in diagnosing fatigue cracks in real rotors, which invariably have some asymmetry. A.K. Darpe, K. Gupta, A. Chawla, [2] studied the coupling between longitudinal, lateral and torsional vibrations together for a rotating cracked shaft. Coupled torsional–longitudinal vibrations for a cracked rotor that has not been reported earlier and coupled torsional–bending vibrations with a breathing crack model have been studied. An attempt has been made to reveal crack specific signatures by using additional external excitations . Qing He , Huichun Peng, PengchengZhai, YaxinZhen [3] et.al. have taken the angular acceleration into the consideration for the modeling of equations of coupling vibration in rotational operation. The effects of angular acceleration on the amplitude of both lateral and torsion vibration of the breathing cracked rotor are studied. Yan liLin , Fule Chu et.al.[4] analysed steady responses and indicated that the combined frequencies of the rotating speed and the torsional excitation in the transversal response and the frequency of the torsional excitation in the longitudinal response can be used to detect the slant crack on the shaft of the rotor system. Rugerri Toni Liong , Carsten Proppe [5] et.al. proposed a method for the evaluation of the stiffness losses in the cross-section that contains the crack. This method is based on a cohesive zone model (CZM) instead of linear elastic fracture mechanics (LEFM).

The following aspects of the crack greatly influence the dynamic response of the structure.

(i) The position of crack
(ii) The depth of crack
(iii) The orientation of crack
(iv) The number of cracks

II. PROBLEM DESCRIPTION AND ANALYSIS PROCEDURE

A rotor having Diameter D is considered for analysis. Two surface cracks of depth a, are assumed along the length of the rotor. The cracks are assumed to have a straight edges. The objective is to determine the stress intensity factor when the cracks are making angle with the transverse plane of the rotor.

A mathematical model is developed and it is solved by writing mat lab code along with the simulink model.The analysis is performed using Ansys work bench and the results are compared.

III. MATHEMATICAL MODEL FOR TWO CRACKED ROTOR

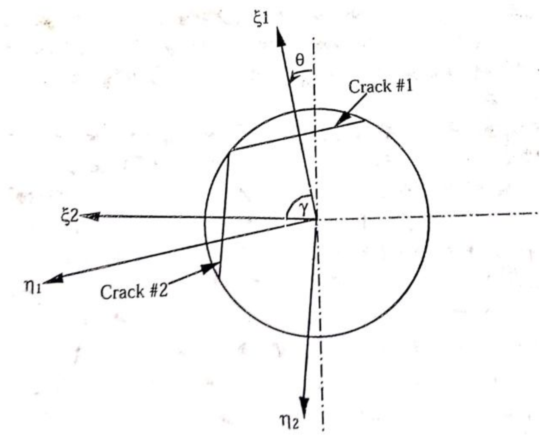

Fig. 1 co-ordinate system

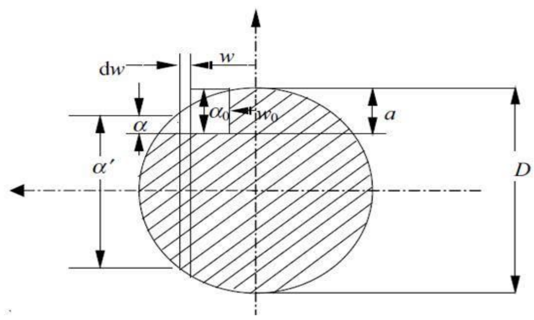

Fig.2 Details of a crack cross section

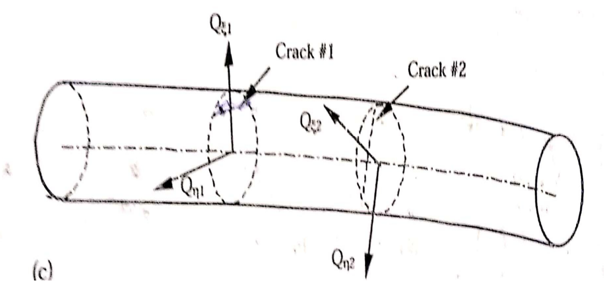

Fig.3 Forces acting on the rotor at the crack cross-section.

The stress intensity factor (K_1^I) on the crack

$$K_1^I = K_{Q_{\xi_1}}^I + K_{Q_{\eta_1}}^I$$

q1F and F which depend on crack parameters

$$F = \sqrt{\frac{2\alpha'}{\pi\alpha}\tan\left(\frac{\pi\alpha}{2\alpha'}\right)}\frac{0.923 + 0.199\left[1 - \sin\left(\frac{\pi\alpha}{2\alpha'}\right)\right]^4}{\cos\left(\frac{\pi\alpha}{2\alpha'}\right)}$$

$$F' = \sqrt{\frac{2\alpha'}{\pi\alpha}\tan\left(\frac{\pi\alpha}{2\alpha'}\right)}\frac{0.752 + 2.02\left(\frac{\alpha}{\alpha'}\right) + 0.37\left[1 - \sin\left(\frac{\pi\alpha}{2\alpha'}\right)\right]^3}{\cos\left(\frac{\pi\alpha}{2\alpha'}\right)}$$

The stress intensity factor (k1) for crack 1

$$K_I^1 = \frac{Q_{\xi_1} L\alpha^1\sqrt{\pi\alpha F}}{8I} + \frac{Q_{\eta_1} Lw\sqrt{\pi\alpha F^1}}{4I}$$

The SIF on the edge of crack 2

$$K_2^1 = \frac{Q_{\xi_2} L\alpha^1\sqrt{\pi\alpha F}}{8I} + \frac{Q_{\eta_2} Lw\sqrt{\pi\alpha F^1}}{4I}$$

IV. ANALYSIS USING ANSYS

Initially the model is drawn in solid works and it is imported to Ansys workbench.

The following procedure is used to create local coordinate systems

After selecting of editing, the sketch will be displayed in that click on co-ordinate system-insert-co-ordinate system choose the face option click on the face of the sketch.

After that set the face where ever we required change the co-ordinate system into RZ and given the angle as 90 degrees take RX and rotate the shaft into -90 degrees

Next click on offset 'Y' and given the values as 0.01m similarly click or offset 'X' and given the values 0.1m -press Enter.

Again take the second co-ordinate system and Next click on offset 'Y' and given the values as 0.01m similarly click or offset 'X' and given the values 0.1m -press Enter.

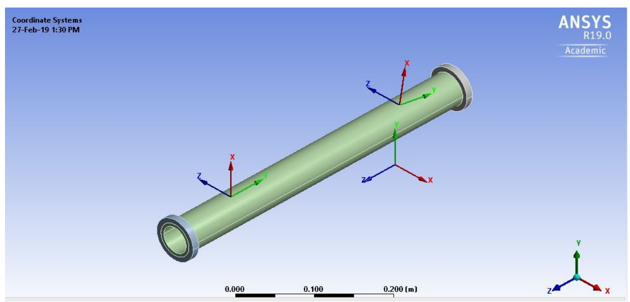

Fig 4 Cracked Shaft Model

Then the model is discredited using the mesh options.

Click on mesh -insert-method press ctrl+select the all parts of then sketch-apply change the method into tetrahedrons and go to mesh-geometry mesh.

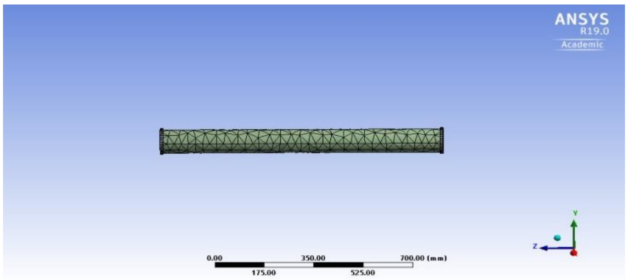

Fig 5 Meshed Model of Shaft

The following procedure is used to create semi elliptical cracks

select the fracture - insert - semi elliptical crack in this option we modify the geometry, co-ordinates system, major radius, minor radius and large counter radius.

Click the first co-ordinate system on geometry select the part of diagram and then click - apply. next click the co-ordinates system what we select already give the dimension of major radius as 0.003m - enter, minor radius as -0.001m -enter, large counter radius -0.001m-enter.

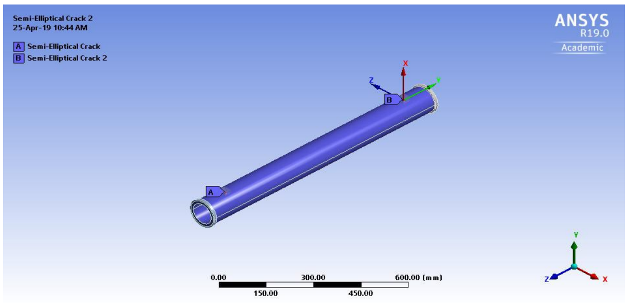

Fig 6 Cracked Shaft Model

Click the second co-ordinate system on geometry select the part of diagram and then click - apply. next click the co-ordinates system what we select already give the dimension of major radius as 0.003m - enter, minor radius as -0.0015m -enter, large counter radius -0.001m-enter.

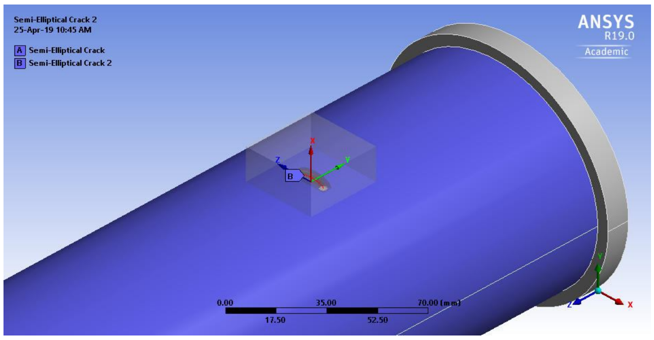

Fig 7 Cracked Shaft Model

The boundary conditions are applied and the problem is solved to determine the stress intensity factors.

click on static structural-insert-fixed support-click on fixed support of the shaft-apply go back static structural-insert-rotational velocity-define the -component in that change the vector into component select the global co-ordinate system for change the system into our required co-ordinate system for rotation of Z-component given the values as 5 rad/sec go back to static structural –solve.

click on solution -insert-total deformation after once again click on solution - insert-fracture tool select crack -semi elliptical crack again go to fracture tool-insert-SIFT RESULTS -select -SIFS(K2), again solution -equivalent all results.

V. RESULTS

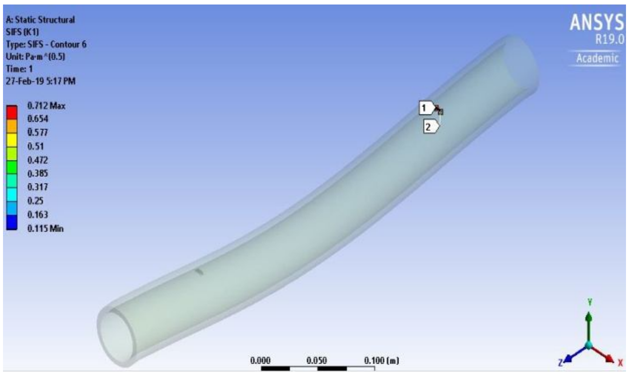

Fig:-8 Stress intensity factor for crack 1 at 10^0 and crack 2 at 10^0 with ellipticity ratio 0.25

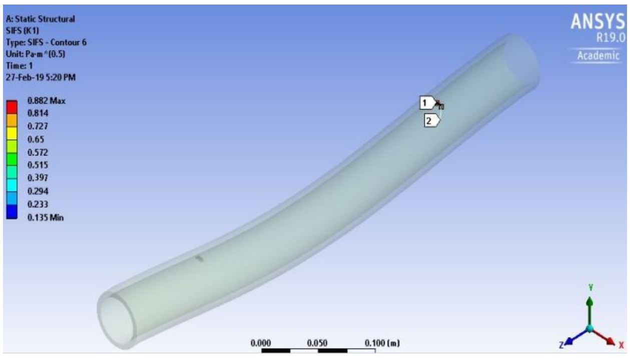

Fig:-9 Stress intensity factor for crack 1 at 10^0 and crack 2 at 10^0 with ellipticity ratio 0.5

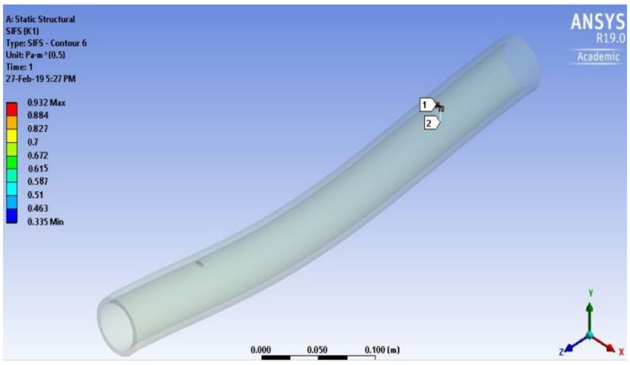

Fig:-10 Stress intensity factor for crack 1 at 10^0 and crack 2 at 10^0 with ellipticity ratio 0.75

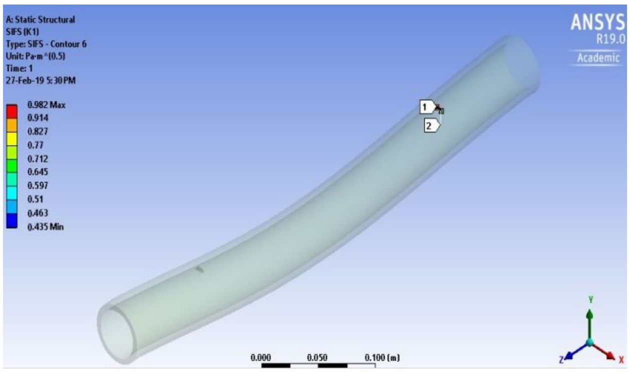

Fig:-11 Stress intensity factor for crack 1 at 10^0 and crack 2 at 10^0 with ellipticity ratio 1.

TABLE 1. VARIATION OF STRESS INTENSITY FACTOR

Plane angle for Crack 1	Plane angle for crack 2	Crack Ellipticity Ratio			
		0.25	0.5	0.75	1
0°	0°	0.918	1.183	1.442	1.738
	10°	0.824	0.904	1.482	1.180
	20°	0.695	0.750	0.958	0.912
	30°	0.495	0.612	0.865	0.857
	40°	0.182	0.432	0.732	0.728
10°	0°	0.851	1.028	1.321	1.529
	10°	0.721	0.813	0.947	0.997
	20°	0.557	0.712	0.857	0.921
	30°	0.392	0.590	0.682	0.789
	40°	0.152	0.382	0.512	0.678
20°	0°	0.812	0.981	1.280	1.420
	10°	0.687	0.872	0.912	0.990
	20°	0.539	0.691	0.741	0.810
	30°	0.371	0.589	0.610	0.690
	40°	0.129	0.316	0.458	0.571
30°	0°	0.791	0.912	1.192	1.358
	10°	0.698	0.758	0.893	0.987
	20°	0.487	0.599	0.710	0.890
	30°	0.378	0.487	0.550	0.698
	40°	0.112	0.258	0.412	0.518
40°	0°	0.772	0.821	1.082	1.217
	10°	0.698	0.721	0.891	0.989
	20°	0.451	0.591	0.657	0.751
	30°	0.298	0.389	0.487	0.578
	40°	0.091	0.212	0.381	0.411

TABLE2. COMPARARISION OF SIFS WITH ANSYS

Plane angle for Crack 1	Plane angle for crack 2	Crack Ellipticity Ratio			
		Ansys	Mathematical Model	Ansys	Mathematical Model
		0.25		0.5	
0°	0°	0.929	0.918	1.192	1.183
10°	10°	0.712	0.721	0.822	0.813
20°	20°	0.522	0.539	0.682	0.691
30°	30°	0.382	0.378	0.472	0.487
40°	40°	0.0887	0.091	0.2047	0.212

Plane angle for Crack 1	Plane angle for crack 2	Crack Ellipticity Ratio			
		Ansys	Mathematical Model	Ansys	Mathematical Model
		0.75		1	
0°	0°	1.452	1.442	1.722	1.738
10°	10°	0.932	0.947	0.982	0.997
20°	20°	0.732	0.741	0.802	0.810
30°	30°	0.542	0.550	0.682	0.698
40°	40°	0.3747	0.381	0.4047	0.411

VI. CONCLUSION

A simple rotor shaft with two semi elliptical cracks is considered for analysis.. The mathematical model is developed and solved by changing the crack ellipticity from 0.25 and 1 to determine the stress intensity factors. The crack plane angle for both the cracks is also varied from 0^0 to 40^0 . The same problem is solved using Ansys. It is observed that as the crack ellipticity increases , the stress intensity factors gets increased. The increase in crack plane angle decreases the stress intensity factors.

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