Analysis And Design of Thin Concrete Roof Shells

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Abstract- *Shell structures are very interesting due to their impressive strength-to-weight ratios. They are able to span over large areas, while having an exceptionally less thickness. This is primarily due to their form based structural behavior. The geometry, that is their initial curvature, along with the boundary conditions and type of loading, dictates the way they transfer load or the way they fail (in case the load exceeds their load carrying capacity). Shells exhibit membrane like behavior which will be explained further in this thesis. The beauty of shells lies in the fact that a designer is able to design the shell as thin as possible, even in the presence of loads that disrupt its characteristic membrane behavior. The shell is able to confine this disturbance within regions which can be designed or optimized separately.*

I. INTRODUCTION

The period from about 1920 through 1970, the central time period of the modern movement in architecture, was markedby intense optimism about the future of construction, and adesire to place architecture and engineering on a shared and rational basis. The thin-shell concrete structure was developed as an engineering solution to economically achieve large spansfor industrial, commercial, and public structures, and was embraced by the architectural profession as a potent means of architectural expression. The optimism about this structural type was reflected in the large interest and popularity of the combined professions' writing and symposia on the topic, and in the proliferation of these structures throughout the world.

The properties mentioned above are highlighted by the various shell structures in existence today. Shells provide a means to obtain an aesthetical and structurally efficient design. They can take several shapes and forms, which lie at the mercy of the designer. Coupled with the fact that they take up less material for construction, shells became increasingly popular in the last seven decades. Since then there have been several hindrances in their growth, but these difficulties are a thing of the past. Development of advanced analysis methods, and new innovative developments in the construction field, have led to a resurgence in shell design. As a result, new designs for shells are gaining prominence, some of which were impossible in the past.

This project attempts to create the roof of a basketball arena, to have a capacity of 20000 spectators. The shell is proposed to be made of concrete.

Objectives of the project are:

- To gain knowledge about shell structures.
- To get predesign calculations for the basketball stadium.
- To design a safe shell structure.
- To analyze it to be structurally sound.
- Constructional efficiency.

1.1 Classical theory of thin shells

1.1.1 General

In order to design any structure in detail, it is necessary to have some specific set of guidelines based on scientific methods. As for any other structural element, the guidelines for designing shell structures is provided by the branch of mechanics called structural mechanics. Engineers are mainly concerned with the man-made structures. In order to construct these structures, they are highly dependent on developing conceptual models that rationalizes the phenomena of nature. The development of these models largely depends on the understanding of mathematics, conducting experiments, assumptions and approximations.

1.1.2 Background

The theory of thin shells is first formulated by L.E.H. Love in 1888 in his paper on thin elastic shell theory. Love developed the shell theory on the basis of Kirchhoff hypothesis for thin plate structures proposed in in the mid-1800s. Since then, there has been several shell theories developed with their own set of kinematic relations (strain – displacement relations). The central idea it that the deformation of shells due to loading is resisted by the membrane and bending effects, which can be separated. The theory of structures often deals with idealized forms of the physical structures. A beam is for example often represented as a line that possesses a certain mechanical property. Similarly, a shell is represented by a surface that possesses a certain mechanical property like stiffness and strength. In this way load effects can be calculated easily, however one has to be aware that for the design of local problems this idealization might not be adequate. Further development of the theory employs Hooke's law (elastic material), equilibrium and compatibility. Hooke's law relates strains with stresses, equilibrium relates stress resultants with external loading and compatibility relates strains with deformation/displacements. These three sets of equations together with appropriate boundary conditions make up the mathematical aspect of the problem. When dealing with dynamic loading the equilibrium equation is represented by the equation of motion. Compared to flat plates, the shell theory is more complicated due to the geometry of the shell. It is possible to argue that the problem of shell structures is dominated by the geometry of the surface of the shell.

1.1.3Assumptions

The classical theory of shells deals with shells that can be characterized as thin. A thin shell is a curved slab whose thickness h is small compared with its other dimensions and compared with its principal radii of curvature r_x and r_y. This can be quantified by the ratio, radii of curvature to thickness of the shell, R_t. It can be said that shells with the ratio greater than 20 can be characterized as thin shells. In comparison, an egg shell has a ratio of around 55 and an aluminium bear can has a ratio of around 325. In further development of the theory, we will mainly be dealing with uniform shells. The shells are uniform in the sense that the material properties do not vary through the thickness. Reinforced concrete (RC) is a composite material consisting of steel and concrete, nevertheless it is regarded as sufficiently uniform. This can be argued with the fact that the difference in Young's modulus between steel and concrete is not large enough.

Other assumptions include:

- Small deflections, the equilibrium equations refer to the original geometry.
- Linear elastic behavior.
- Shear deformation is neglected.
- Plane section remain plane after bending.
- The transverse normal stress is negligible.

The geometry of ashellisfully defined by its thickness and the form of its middle surface. The middle surface is defined as the surface that bisects the thickness of the plate. When analyzing the shell, an infinitely small element which is defined by two pairs of adjacent planes perpendicular to the middle surface is considered. These planes contain the principal radii of curvatures of the shell, r_x and r_y.

Further, the stresses and strains are denoted following their respective axes as σ_x, σ_y, $\tau_{xy} = \tau_{yx}$ and ε_x, ε_y, γ_{xy}.

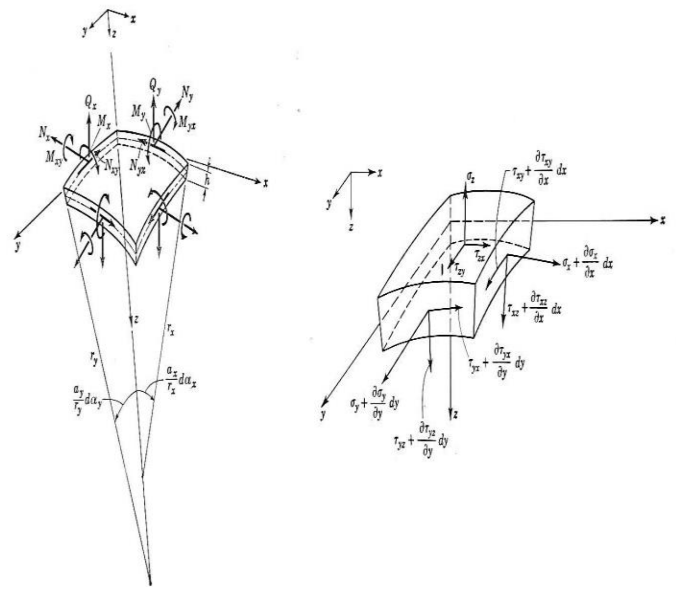


Figure 1: Notations

$$N_x = \int_{-h/2}^{h/2} \sigma_x \left(1 - \frac{z}{r_y}\right) dz \qquad N_y = \int_{-h/2}^{h/2} \sigma_y \left(1 - \frac{z}{r_x}\right) dz$$

$$N_{xy} = \int_{-h/2}^{h/2} \tau_{xy} \left(1 - \frac{z}{r_y}\right) dz \qquad N_{yx} = \int_{-h/2}^{h/2} \tau_{yx} \left(1 - \frac{z}{r_x}\right) dz$$

$$Q_x = \int_{-h/2}^{h/2} \tau_{xz} \left(1 - \frac{z}{r_y}\right) dz \qquad Q_y = \int_{-h/2}^{h/2} \tau_{yz} \left(1 - \frac{z}{r_x}\right) dz$$

$$M_x = \int_{-h/2}^{h/2} \sigma_x z \left(1 - \frac{z}{r_y}\right) dz \qquad M_x = \int_{-h/2}^{h/2} \sigma_y z \left(1 - \frac{z}{r_x}\right) dz$$

$$M_{xy} = -\int_{-h/2}^{h/2} \tau_{xy} z \left(1 - \frac{z}{r_y}\right) dz \qquad M_{yx} = \int_{-h/2}^{h/2} \tau_{yx} z \left(1 - \frac{z}{r_x}\right) dz$$

The expressions z/r_x and z/r_y comes from the trapezoidal shapes of the sides along planes x_{xz} and y_z. These expressions will however be neglected due to the thin shell approximations. As a result:

$$N_{xy} = N_{yx} \qquad \text{and} \qquad M_{xy} = -M_{yx}$$

In addition, σ_z, τ_{xz} and τ_{yz} are omitted due to the small thickness of the shell, and the same goes with the twisting moments about the z-axis. Thus, there will be a state of plane stress throughout the shell. Derivation of the differential equations for the most used concrete shell elements will be presented in the proceeding chapters.

1.1.4 Equilibrium

As for any other structural systems, the equilibrium conditions for the differential shell element must be met. The six equilibrium equations are:

$$\Sigma X = 0 \quad \Sigma Y = 0 \quad \Sigma Z = 0$$
$$\Sigma M_x = 0 \Sigma M_y = 0 \Sigma M_z = 0$$

Due to the simplification of thin shell element mentioned before, the equation $\Sigma M_z = 0$is omitted, thus five equations are remained. When setting up the equilibrium equations,the external loads on the shell element must also be included in the form of pressure components p_x, p_y, p_z. The equilibrium equations are given by:

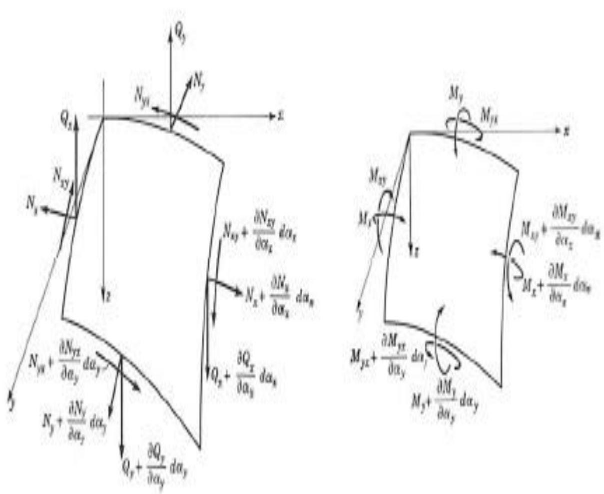


Figure 2: Forces on shell elements

$$\frac{\partial}{\partial \alpha_x}(N_x a_y) - N_y \frac{\partial a_y}{\partial \alpha_x} + N_{xy}\frac{\partial a_x}{\partial \alpha_y} + \frac{\partial}{\partial \alpha_y}(N_{yx} a_x) - Q_y \frac{a_x a_y}{r_{xy}} - Q_x \frac{a_x a_y}{r_x} + p_x a_x a_y = 0$$

$$\frac{\partial}{\partial \alpha_y}(N_y a_x) - N_x \frac{\partial a_x}{\partial \alpha_y} + N_{yx}\frac{\partial a_y}{\partial \alpha_x} + \frac{\partial}{\partial \alpha_x}(N_{xy} a_y) - Q_x \frac{a_x a_y}{r_{xy}} - Q_y \frac{a_x a_y}{r_y} + p_y a_x a_y = 0$$

$$\frac{\partial}{\partial \alpha_x}(Q_x a_y) + \frac{\partial}{\partial \alpha_x}(Q_y a_x) + N_x \frac{a_x a_y}{r_x} + N_{xy} \frac{a_x a_y}{r_{xy}} + N_{yx} \frac{a_x a_y}{r_{xy}} + N_y \frac{a_x a_y}{r_y} + p_z a_x a_y = 0$$

$$-\frac{\partial}{\partial \alpha_y}(M_y a_x) + M_x \frac{\partial a_x}{\partial \alpha_y} - M_{yx}\frac{\partial a_y}{\partial \alpha_x} + \frac{\partial}{\partial \alpha_x}(M_{xy} a_y) + Q_y a_x a_y = 0$$

$$-\frac{\partial}{\partial \alpha_x}(M_x a_y) + M_y \frac{\partial a_y}{\partial \alpha_x} + M_{xy}\frac{\partial a_x}{\partial \alpha_y} - \frac{\partial}{\partial \alpha_y}(M_{yx} a_x) + Q_x a_x a_y = 0$$

where α_x and α_y are curvilinear coordinates along the respective sides, and a_x and a_y arecalled Lame's parameters. The Lame's parameters are quantities which relate a change inarc length on the surface to the corresponding curvilinear coordinates.

1.1.5 Strains

The deformation of a shell element consists of strains both due to a change in curvature and axial deformation. The strains from the axial deformation are denoted as ε_1 andε_2 for the strains in x and y directions respectively, and the new radii of curvatures are denoted as r'_x and r'_y. The total expression for the strains, neglecting the small terms, is then given by

$$\varepsilon_x = \varepsilon_1 - z\left(\frac{1}{r'_x} - \frac{1}{r_x}\right)$$
$$\varepsilon_y = \varepsilon_2 - z\left(\frac{1}{r'_y} - \frac{1}{r_y}\right)$$
$$\gamma_{xy} = \gamma - 2z\chi_{xy}$$

where χ_{xy} represents the change in twisting curvature and γ is shear strain of the middle surface.

1.1.6 Stress-strain relations

The stress-strain relations are based on the Hook's law for flat plate element. In addition to the material being linearly elastic, it is also assumed that it is isotropic and homogeneous.

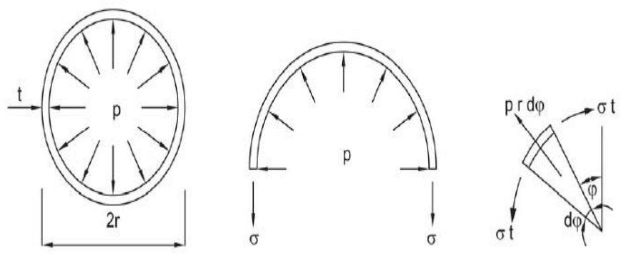
$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \frac{E}{1-\nu^2} & \frac{\nu E}{1-\nu^2} & 0 \\ \frac{\nu E}{1-\nu^2} & \frac{E}{1-\nu^2} & 0 \\ 0 & 0 & \frac{E}{2(1+\nu)} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}$$

1.1.7 Membrane theory

The membrane theory is based on the omission of the bending stresses. This reduces the equilibrium equations to only three unknowns, N_x,N_y and N_{xy}.Thus, Equation is reduced to just the first three equations with the three variables as unknowns. The problem is then statically determinate, and it can be easily solved for a given loading and geometry. For example, the resulting in-plane force for a cylindrical shell that is loaded with a constant pressure can be expressed as:

$$N_\varphi = \sigma t = \int_0^{\pi/2} pr\cos\varphi d\varphi = pr$$

Which is derived from the vertical equilibrium of the half circle in Figure.



Similarly, for a sphere like structure loaded with a constant pressure, it can be shown thatthe in-plane stresses are expressed as:

$$\sigma_x = \sigma_\theta = pr/2t$$

The bending effects from the boundary conditions in shells tend to damp quickly, thus a large portion of the shell surface is dominated by the membrane forces. Therefore, the membrane theory can sometimes provide a reasonable basis for design. However, the membrane theory can only be used upon the fulfilment of the following conditions:

•The displacements from membrane forces do not give rise to bending stresses.
• The loading is distributed smoothly over the surface of the shell.
• The boundaries can supply the forces and permit the displacements required by the membrane stress resultants.
• The stress is uniformly distributed through the thickness of the shell.

II. LITERATURE REVIEW

Hanibal Muruts, Ghebreselasie Yuting Situ From the perspective of structural engineering, shells due to their spatial curvature, possess a structurally efficient way of carrying loads acting perpendicular to their surfaces. However, the nature and geometry of shells makes them complicated to understand or predict their structural behaviour. The structural analysis of thin concrete shells can be conducted numerically using finite element analysis(FEA) or/and analytically on the basis of classical theory of thin shells. As finite element software are increasingly becoming primary tools for performing structural analysis, the knowledge of the analytical solution methods are becoming somehow less known among young structural engineers today. Hence, this paper aims to revisit the analytical analysis methods for concrete shell structures, and to investigate on how its results compare to that of the FEA. For a complete investigation of the structural analysis of thin concrete shells, the design and the accompanying verification by using nonlinear FEA is also briefly included. The study is limited to structural static analysis.

Vitória Vazquez Pereira As part of the research "Design and Performance of Ultra-Thin Concrete Shells", the proposed study seeks to explore the architectural dimension of an ultra-thin free-form shell structure. By applying spatio-functional analytic methods, its performance was evaluated in different real situations (natural and urban environments), studying how the visual fields, spatial movements and the shade produced, would be affected in the chosen site. Thus, it was sought to identify and validate with concrete hypothesis, different implantation layouts of the several possibilities introduced. Through the study of the historical evolution of shells and the existing technology and construction methods, it was intended to highlight the importance and advantages given of using this type of structures. Their natural form, associated to the development of new materials and prefabrication technology, transform this shell into a promising opportunity of future development. This work seeks to make part of that future, contributing for the understanding of the multifaceted spatio-functional dimension of these structures and their inseparable architectural component.

R.N. ter Maten Shell structures present immense structural and architectural potential. Generally speaking, shells are spatially curved surface structures which support external applied loads. The exceptional behavior of shell structures can be referred to as „form resistant structures". This implies a surface structure whose strength is derived from its shape, and which resists load by developing stresses in its own plane. Due to the initial curvature and low thickness to radius ratio a thin shell has a much smaller flexural rigidity than extensional rigidity. When subjected to an applied load it mainly produces in-plane actions, called membrane forces.

Antonio TomasPascual Martí Although concrete shells may adopt any form, it would be interesting to know to what extent changes in their shape may avoid the appearance of bending moments or reduce them. The use of optimization techniques may be effective in providing alternative geometric forms of shells that improve their mechanical behaviour, complying with the design conditions in an optimum way. In this paper, these techniques were used to find optimum geometrical designs having an aesthetic shape similar to the form initially designed for the structure. As an example, a shell based on Candela’s blueprints was optimized under a state of predominant gravitational loads. The results confirm that significant improvements in the structural behaviour of the shell may be achieved with only slight changes in its form.

Ha-Wong Song, Sang-Hyo Shim, Keun-Joo Byun and Koichi Maekawa In this paper, a finite element analysis technique is presented for the path-dependent nonlinear failure analysis of reinforced concrete shell structures. A so-called pressure node is added into a layered shell element utilizing in-plane constitutive models of reinforced concrete and layered formulation in the failure analysis. By controlling the volume of the shell structures using the pressure node, postpeak softening behavior after the ultimate load of the shell structures is obtained. Since the constitutive models cover loading, unloading, and reloading paths, the element is capable of predicting the behaviors of reinforced concrete shells under cyclic loading. For verification of the techniques in this paper, failure analyses of reinforced concrete slabs subjected to in-plane and out-of-plane loads and cyclic transverse loads are performed and numerical results are compared with experimental data. In addition, reinforced concrete dome structures designed with different reinforcement ratios are also analyzed to check the applicability of the technique in this paper. Results show that the techniques can be applied effectively to the failure analysis of various types of reinforced concrete shell structures.

Thomas E. Boothbyl and Barry T. Rossonz Thin-shell concrete structures were developed in the mid-twentieth century in response to the need for economy in large-span structures and in response to the design and aesthetic program of the modern movement in architecture. Although of European invention, these structures were widely employed in the United States for industrial and military structures, stadiums, auditoriums, and shopping centers. Because of changing building economics and changing tastes, significant thin-shell concrete structures have not been built in the United States since the mid-1970s. In spite of their relatively recent construction, many surviving thin-shell structures can be considered as historic according to the Criteria for Eligibility for the National Register of Historic Places. However, a lack of awareness of the significance of these structures has caused the recent removal of two important thin-shell concrete structures, the New Orleans Convention Center and "The Paraboloid," an entrance canopy for the May D&F store in downtown Denver. Others, such as Seattle's Kingdome Stadium, are clearly threatened. In this paper, we examine the historical and social context of thin-shell concrete structures, discuss the threats to the preservation of these structures, and outline a strategy of professional and public awareness and strategic repair.

K. S. Lay A four-noded quadrilateral pure shell element based on the thin-shell theory of Koiter (1966) has been developed. The element, having a variable number of nodal degrees of freedom with a maximum of 12, is formulated on the plane reference domain by a mapping of the curved shell middle surface from the threedimensional space. Any arbitrary global coordinate system can be used due to the implementation of tensorial coordinate transformation. Excellent behavior of the element is observed when tested against a set of severe benchmark tests. The benchmark tests demonstrate that the element is able to handle rigid-body motion without straining, inextensional modes of deformation, complex membrane strain states, and skewed meshes. The two-dimensional interpolation functions are formed from the tensor product of Lagrange and Hermitian one-dimensional interpolation functions, and the order of interpolation can be varied.

III. PROBLEM STATEMENTS

The project aims for the economical design of thin concrete shell roof of an indoor arena. For this various value of thickness (for various failure modes) and different span-rise ratios will be calculated, to find an optimum value of span-rise ratio and an optimum thickness, for various load combinations.

Based on the results of the findings a thin concrete shell roof is to be designed.

IV. METHODOLOGY

To fulfill the objectives, a specific design approach is defined. This is a crucial step in a project at hand, which if not done properly may result in delay of the project. The overall strategy of the project is mentioned below:

All theory related to shell design is acquired and studied. This includes study of different shell surfaces, their structural behavior and also the possible ways they can fail.

The entire history of concrete shell industry is studied, along with important associated people and structures. In the 1950s several shell designers rose to prominence. An attempt is made to learn about their ways and methods. Between these designers, they have an impressive list of shell structures that exist and function today. Some of these bear similar characteristics and aspects, compared to the expected shell design for this project.

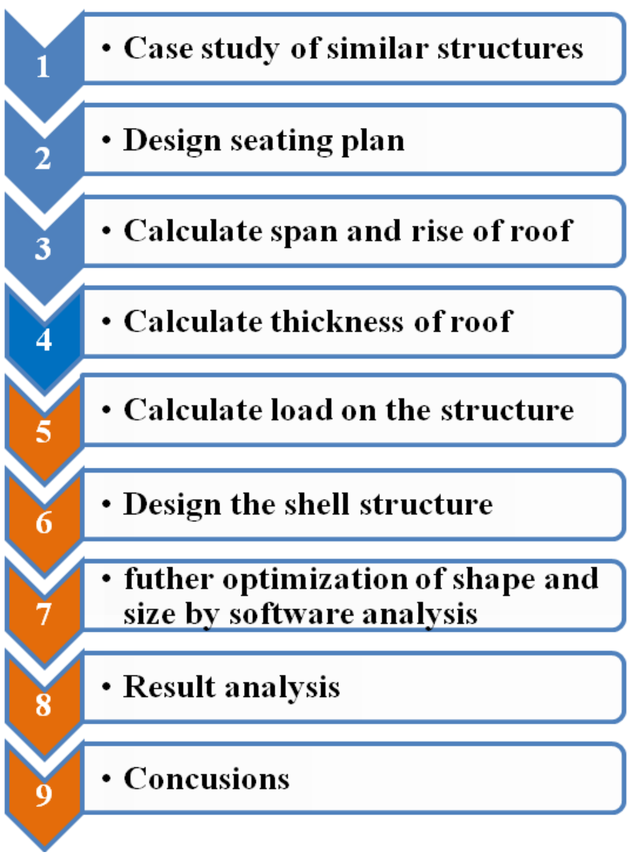
Functioning basketball arenas are studied to have a better understanding of loads expected on the roof. If there are any special loads that need to be taken into account, then they are found out in this step.

Similar structures are studied to understand how structural problems were dealt with.

An outline of the main design process is created. This procedure is strictly followed during the entire course of design.

Span and rise of the roof are calculated. Thickness of the roof is calculated based on various failure mode.

Finally, the behavior of the shell is studied and recorded, under various load combinations, to ensure proper functioning of the structure. Further optimization of shape and size is done by software analysis.

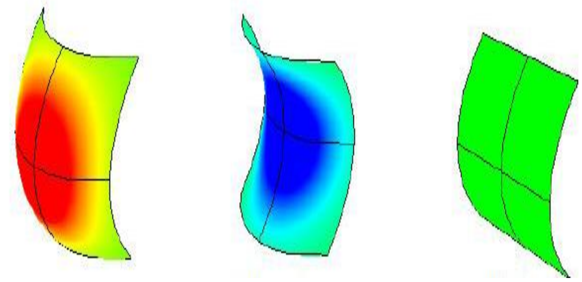


V. THEORETICAL CONTENTS

5.1 Classification of shell surfaces:

5.1.1 Gaussian Curvature:

Shell surfaces are usually classified based on their Gaussian curvatures. For a three-dimensional surface, the product of the maximum and minimum principal curvatures gives us the Gaussian curvature. They are orthogonal to each other and can be found out by intersecting infinite planes with the surface at any point. Based on the product of the principal curvatures we can further classify the Gaussian surfaces as discussed



a) Positive b) negative c) zero

Figure 3: Types of Gaussian curvatures

Synclastic: A Synclastic surface has a positive Gaussian curvature and is shown in figure 2-1 (a). Both the principal curvatures have the same sign. They generally exhibit in-plane meridional and circumferential stresses to carry loads. Spheres and elliptical paraboloids are common examples of this kind of surface.

Anticlastic: In this type of surface both the principal curvatures have different signs resulting in a negative gaussian curvature. Having opposite signed principal curvature enables these surface to act with a combination of compressive and tensile arch behavior under perpendicular loads. Hypars are good examples of anticlastic surfaces.

Monoclastic: If one of the principal curvatures is zero then it gives rise to monoclastic surfaces. They have zero gaussian curvature as seen in figure. Cylindrical shells are the most common examples of this type of surface.

5.1.2Developed and Undeveloped Surfaces

This basis of classification depends on whether the surface can be ’opened’ or deformed to obtain a plane form. Figure show examples of both.

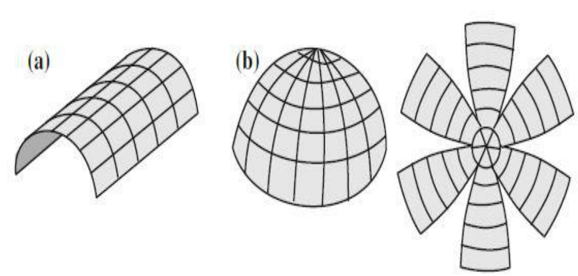


Figure 4: Examples of developable and undevelopable surfaces

Developable surfaces can be deformed and developed to obtain a plane surface, without any cutting or stretching.

Cylindrical roofs as shown in figure are an example of this type of surface. We can imagine that the roof can be easily deformed to obtain a plane. All monoclastic shells are developable surfaces.

Undevelopable surfaces, unlike their developable counterparts, cannot be deformed into their plane forms without alterations, which were mentioned before. All synclastic and anticlastic surfaces fall in this category. They have significant advantage over comparable developable surfaces in having more strength and stability. This stems from the fact that more external energy is required to cause any kind of deformation.

5.1.3 Generation of surfaces

Surfaces described above can be created using geometric or non-geometric techniques. The former uses mathematical functions while the latter uses natural processes like form-finding.

Geometrical Surface generation

They include surfaces of revolution, surfaces of translation, ruled surfaces and freeform surfaces as shown in figure. Surface of revolutions are created by rotating a meridional curve about the axis of revolution. Translational surface requires sliding of a constantly oriented generator curve over a directrix curve. Constructing ruled surface requires another method, where we slide two ends of a straight line on their own curve while keeping them parallel to an arbitrary direction or plane. Lastly, freeform surfaces can be generated using NURBS (Non-Uniform Rational B-Spline). They are different from the other geometrical methods as they cannot be described by fixed equations but can be used to make any possible shape.

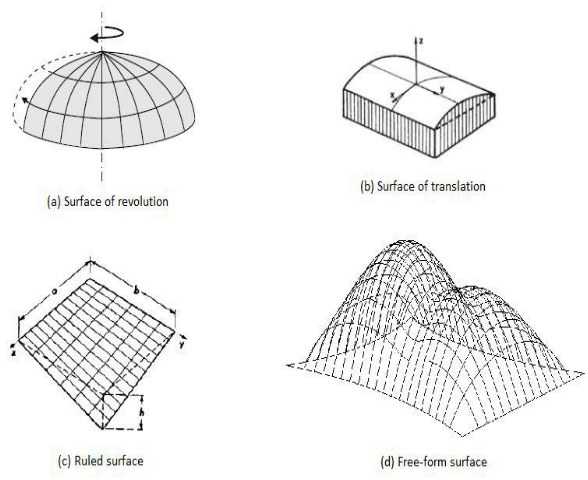


Fig5: Examples of geometrically generated surfaces

5.1.4 Non-Geometrical Surface generation

There are several physical and computational methods of form finding. They involve finding a shape which is at equilibrium with the forces that act upon it. Here we study the hanging model analysis which is the most native form physical form-finding. This method is extremely useful to obtain shapes for thin shells. Generally cloths or chain nets can be used for this analysis. These materials cannot absorb bending moments, so when they hang it is safe to assume they experience only tension due gravity. Keeping this in mind we can create hanging models as shown in figure. This shape represents the equilibrium shape of the cloth at that particular moment. If we were to freeze the model and invert it, logic dictates that the entire cloth is in compression. For a structural designer this is a very important aspect as the cloth acts like a membrane which, as discussed earlier, is very structurally efficient. Although, it should be known that presence of other dominant loads can cause problems.

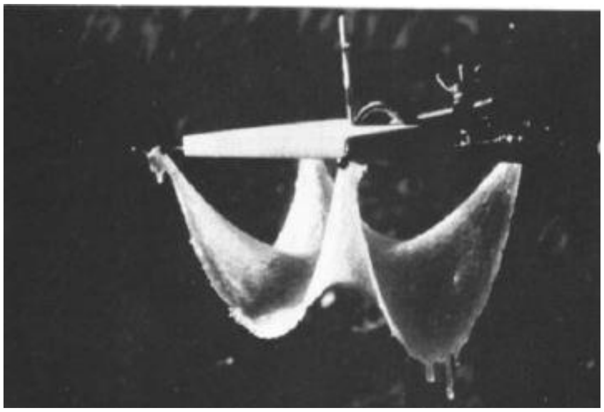


Fig6: Example of a hanging model using cloth

5.2 Structural behaviour of Shells

A good knowledge of the structural behaviour of shells is imperative for good shell design. It can vary hugely depending of kind of loads the shell is expected to carry. Shells are form based structures where the shape influences the shell's load carrying capacity. This structural geometry largely dictates the development of stresses in the shell elements. This has to be kept in mind to avoid unwanted deformations and structural failure.

5.2.1Membrane behavior

Membrane theory attributes to the membrane-like behavior of shells, which enables it to carry out-of-plane loads. They transfer these loads by generating in-plane membrane forces, a fact that separates them from plates. Shells, unlike membranes, stretch and contract without producing significant bending or changes in local curvature.

Ideally a designer would prefer to design the shell only for membrane forces, as this enables the shell to be thin and hence be more economic. This is not possible always due to presence of unfavorable loading and boundary conditions. These conditions are shown in figure 2-5. In such cases the membrane stress are not enough to reach a state of equilibrium giving rise to disturbed regions with additional stresses. Thus a bending theory is required for a complete analysis of the shell.

5.2.2 Bending behaviour

The bending behaviour of shell is very interesting. It only occurs at parts of the shell where the membrane stresses are insufficient to carry the applied loads. The bending moments developed in these regions only compensate for the inadequacy of membrane behaviour and do not carry any load. In figure 2-5 we can see, the presence of concentrated forces or geometry changes creates a disturbed region with bending moments. But these moments are confined to small region around the point of disturbance defined by its influence length' (li).

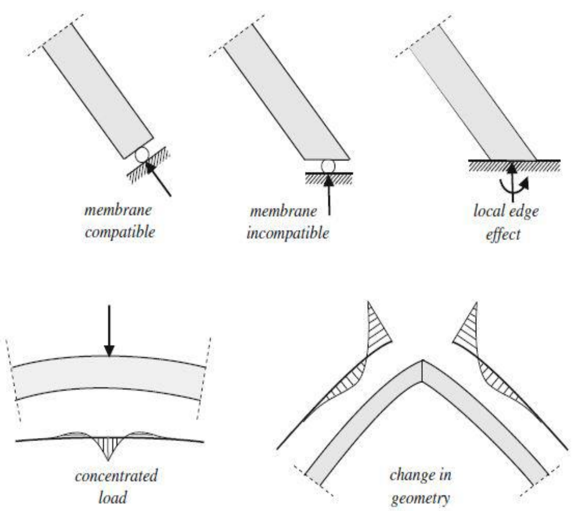


Fig7: Conditions for membrane and bending theory

This means the rest of the shell will still possesses a true membrane field. The influence length for a spherical dome is shown in figure. This behavior of shells is a boon to designers as it highlights the structural efficiency of shells. In a practical sense, this enables the designers to design a relatively thin shell, even in the presence of membrane incompatible conditions. Special optimizations or other structural solutions can be found to tackle these conditions, while expecting large parts of the shell to still behave like a true membrane.

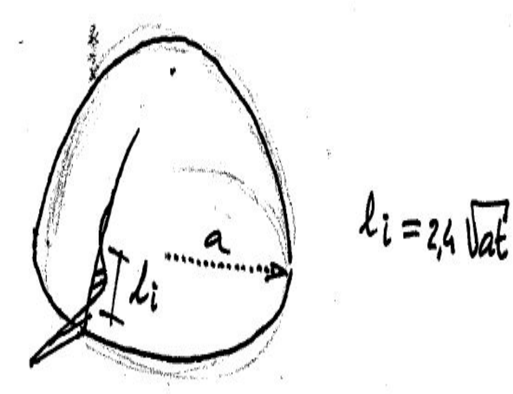


Fig8: Influence length of edge in for a spherical dome

5.3Structural Failure of Shells

It is important to know a structure might fail, because this knowledge helps the designer to design the structure accordingly. In this project, the aim is to design a thin shell which motivates us to study the various modes of shell failure. Shells can fail due to increasing deformations, failure of material or a combination of both. The former is called 'buckling instability' while the latter is referred to as 'strength failure'.

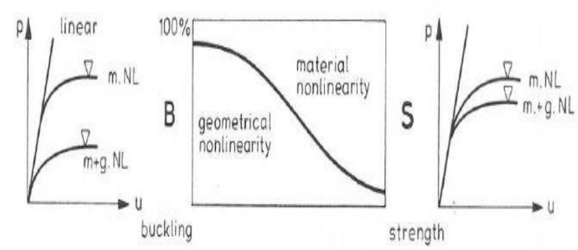


Fig9: Structural failure modes for shells

5.3.1Strength Failure

Strength failure occurs as a result of deterioration of material properties and is characterized by lower deformations. Tensile forces can cause the concrete to crack, or it might undergo compressive crushing which, in turn, will reduce the strength of concrete. In most cases of thin shells the stresses are not too high; hence this type of failure does not govern design, although this can lead to buckling failure. Usage of high strength concrete further decreases the possibility of this type of failure.

5.3.2Buckling Instability

Shells, for the most part, are expected to carry in-plane compressive forces. Initial imperfections in geometry can give rise to eccentricity of these forces, which threaten the stability of the structure. Usually one or more structural components can fail, at a lower stress than the design ultimate compressive value of the failing concrete member. Following this the structure undergoes a drastic loss in load carrying capacity finally leading to collapse. Nonlinearities in materials further add to this phenomenon. There are some more important points associated with shell buckling which are discussed below.

5.3.3Snap-back behaviour

This type of behaviour is typically seen in shell structures. When the shell is loaded it starts following a path of equilibrium. This part is shown by the linear line in figure. This path is followed till the bifurcation point is reached, after which the shell experiences sudden loss in its load carrying capacity. This is shown in figure as the curved line which represents snap-back. This behaviour is dramatic and catastrophic, as it happens suddenly and quickly without any warning. This curve represents the post buckling behaviour of the structure. The transition between the two paths of equilibrium can be smooth or sudden depending on the geometry of the shell. This phenomenon is further intensified with the presence of imperfections, hence the dotted line in figure is a closer assumption of the path of equilibrium a shell will follow in real life.

5.3.4Imperfection Sensitivity

Imperfection sensitivity of shells was first established when scientists found discrepancies, between the calculated critical load and the actual load that a shell could carry.

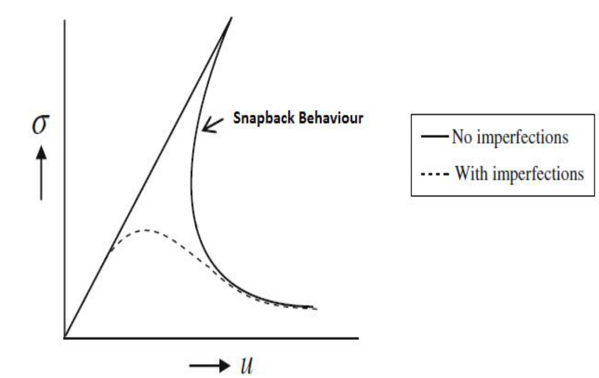


Fig10: Stress-Deflection curve showing snapback behaviour of a shell

Experiments were performed on axially loaded cylinders resulting in failure, much before reaching the critical load. Eventually it was realized that these cylinders were extremely sensitive to even a minute imperfection, which would otherwise be hidden from the naked eye. The effect of imperfections was first quantified by Professor Koiter during the Second World War.

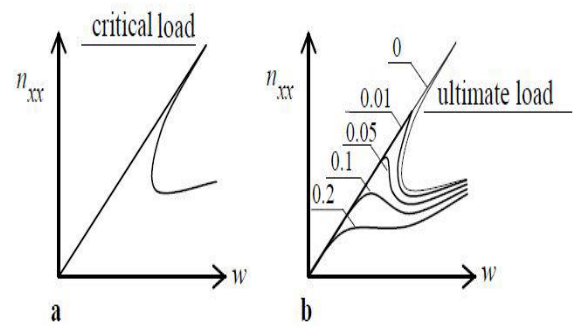


Fig11: Buckling of cylinders over different imperfection amplitudes

Figure shows us how increasing imperfection amplitudes can drastically reduce the load carrying capacity of the cylindrical shell. Although, not all shell are so sensitive to imperfections.

5.3.5Compound Buckling

Thin shells are especially sensitive to imperfections due to the occurrence of 'compound' or 'multi-mode' buckling. This happens due to interactions between the different buckling modes as all of them are associated with the same linear critical buckling load. Thin shells exhibit membrane dominant behaviour, hence have closely related buckling modes. This is due to the absence of shorter influence lengths of bending regions. As a result, thin shells are more susceptible to compound buckling than thick shells. Consequently, this is a major reason for snap back behaviour in thin shells.

Table 1: Imperfection sensitivity of some elementary shells

[1] Type of Shell	[2] T ype of Loading	[3] Impe rfection Sensitivity
[4] Open Cylinders	[5] R adial	[6] No
[7] Open Cylinders	[8] A xial	[9] Yes
[10] Open Cylinders	[11] T orsion	[12] No
[13] Hype rboloid	[14] A xial	[15] Yes
[16] Clos ed Cylinders	[17] U niform	[18] Yes
[19] Sphe res	[20] U niform	[21] Yes
[22] Dom es	[23] U niform	[24] Yes
[25] Hypa rs	[26] U niform	[27] No

5.3.6 Knock down factor

It was discussed earlier how experiments showed a large difference between the theoretical and actual critical load. This posed significant problems to shell designers as the various factors (a few discussed above) could not be separately incorporated in design. A solution to this problem was found by applying a factor, which accounted for all the factors causing snap back behaviour. This factor is called ’Knock down’ factor. The knock down factor for a particular design is the sum of the individual factors, for each negative effect on the load carrying capacity of the shell. These effects are shown for a spherical dome in figure 2-10. For this project the value of knock down factor (C) is chosen as 1/6, as is the norm in the absence of a stipulated value.

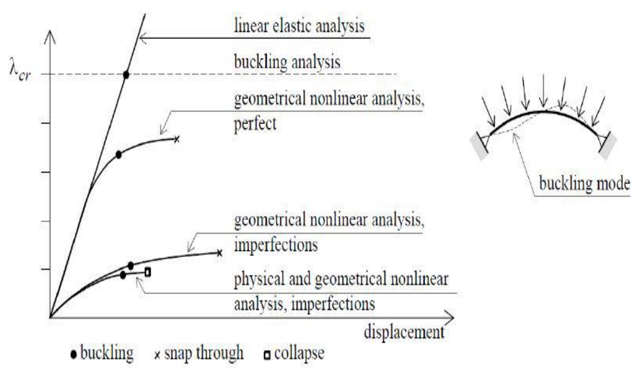


Fig11: Negative effects causing fall-back in load carrying capacity

VI. CONCLUSIONS

Gained knowledge.

Extensive amount of literature was studied. Various concepts of shells were cleared. Classification based on Gaussian Curvature, generation of surface, structural behavior and structural failures were studied carefully. Study of various prominent concrete shell designers and their method of approach was also studied for any upcoming problems.

Case study done.

Case studies of CNIT (‘Centres National des Industries et des Techniques’) which Is the longest spanning concrete shell structure situated in Paris and Palazzo dello sport, a sport palace in Rome were studied carefully.

Eliminated shapes.

It is decided to eliminate some elementary shell types to aid with the final choice of shape. On the basis of Gaussian Curvature, Shells are classified as Monoclastic, Synclastic and Anticlastic shells.

Found shape of shell.

After elimination of the rest a look at synclastic shells is taken. These shells are geometrically more stable than the anticlastic and monoclastic. This arrangement provide good stability to structure due to geometrical interlocking of shell element under the point load.

Predesign basketball arena details.

All the possible aspects for the design of arena were considered including basketball court dimension, formulation of cross section for seating levels and formulation of plan of seating level.

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