Brahmagupta's Mathematical Works

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Brahmagupta holds a unique position in the history of ancient Indian mathematician. His works in the fields of astronomy and mathematics (included gravity theory, negative numbers, use of zero, quadratic equations and square roots).

Mathematical works:

Algebra:

Most of the information published by Brahmagupta is found in ancient texts such as "BRAHMASPUTASIDDHANTA" 25 chapters long. The meaning behind Brahmasputasiddhanta is "the system of the god of creation and astronomy". one well known explanation of algebra, he explained in this was multiplication using place-value system.

Brahmagupta gave the solution of the general linear equation in chapter eighteen of Brahmaputasiddhanta,

"The difference between rupas, when inverted and divided by the difference of the unknowns, is the unknown in the equation.The rupas are [subtracted on the side] below that from which the square and the unknown are to be subtracted".

Which is a solution for the equation $bx + c = dx + e$ equivalent to $x = e - c/b - d$, where rupas refers to the constants c and e. He further gave two equivalent solutions to the general quadratic equation

18.44.Diminish by the middle [number] the squareroot of the rupas multiplied by four times the square and increased by the square of the middle [number]; divide the remainder by twice the square.[The result is] the middle [number].

18.45.Whatever is the squareroot of the rupas multiplied by the square[and] increased by the square of half the unknown, diminish that by half the unknown[and] divide[the remainder] by its square.[The result is]the unknown.

Which are respectively solutions for the equation $ax^2 + bx = c$ equivalent to ,

$$x = (\sqrt{4ac} + b^2 - b)/2a$$ and

$$x = (\sqrt{ac} + \frac{b^2}{4} - \frac{b}{2})/a$$

In addition to his work on solutions to general linear equations and quadratic equations, Brahmagupta went yet further by considering systems of simultaneous equations (set of equations containing multiple variables) and like the algebra of Diophantus, the algebra of Brahmagupta was syncopated.

Arithmetic:

The four fundamental operations (addition, subtraction, multiplication and division) were known to many cultures before Brahmagupta. This current system is based on the Hindu Arabic number system and first appeared in Brahmasputasiddhanta.

In the beginning of chapter twelve of his Brahmasputasiddhanta, entitled calculation, Brahmagupta details operations on fractions. He explains how to find the cube and cube root of an integer and later gives rules facilitating the computation of squares and square roots. He then gives rules for dealing with five types of combinations of fractions:

$$\frac{a}{c} + \frac{b}{c}; \quad \frac{a}{c} \times \frac{b}{d}; \quad \frac{a}{1} + \frac{b}{d};$$

$$\frac{a}{c} + \frac{b}{d} \times \frac{a}{c} = \frac{a(d+b)}{cd}$$

and

$$\frac{a}{c} - \frac{b}{d} \times \frac{a}{c} = \frac{a(d-b)}{cd}$$

Series:

Brahmagupta then goes on to give the sum of the squares and cubes of the first *n* integers.

"12.20. The sum of the squares is that [sum] multiplied by twice the [number of] step[s] increased by one [and] divided by three. The sum of the cubes is the square of that [sum] Piles of these with identical balls [can also be computed]."

Here Brahmagupta found the result in terms of the *sum* of the first *n* integers, rather than in terms of *n* as the modern practice.

He gives the sum of the squares of the first *n* natural numbers as

$$\frac{n(n+1)(2n+1)}{6}$$

and the sum of the cubes of the first n natural numbers as

$$(\frac{n(n+1)}{2})^2.$$

Zero:

Brahmagupta's most famous and well known contribution to the mathematics world is his definition and use of "zero with negative numbers".

He stated "When zero is added to a number from a number, the number remains unchanged and a number multiplied by zero becomes zero".

Rules for addition and subtraction:

In chapter eighteen of his Brahmasputasiddhanta,

18.30. [The sum] of two positives is positives, of two negatives negative; of a positive and a negative [the sum] is their difference; if they are equal it is zero. The sum of a negative and zero is negative, [that] of a positive and zero positive, [and that] of two zeros zero.

18.32. A negative minus zero is negative, a positive [minus zero] positive; zero [minus zero] is zero. When a positive is to be subtracted from a negative or a negative from a positive, then it is to be added.

Multiplication:

18.33. The product of a negative and a positive is negative, of two negatives positive, and of positives positive; the product of zero and a negative, of zero and a positive, or of two zeros is zero.

Division:

But his description of division by zero differs from our modern understanding:

18.34. A positive divided by a positive or a negative divided by a negative is positive; a zero divided by a zero is zero; a positive divided by a negative is negative; a negative divided by a positive is [also] negative.

18.35. A negative or a positive divided by zero has that [zero] as its divisor, or zero divided by a negative or a positive [has that negative or positive as its divisor]. The square of a negative or of a positive is positive; [the square] of zero is zero. That of which [the square] is the square is [its] square-root.

Here Brahmagupta states that $0/0 = 0$ and as for the question of $a/0$ where $a \neq 0$ he did not commit himself. His rules for arithmetic on negative numbers and zero are quite close to the modern understanding, except that in modern mathematics division by zero is left undefined.

Diophantine analysis:

Brahmagupta provides a formula useful for generating Pythagorean triples and he went on to give a recurrence relation for generating solutions to certain instances of Diophantine equations of the second degree such as $Nx^2 + 1 = y^2$ (called pell's equation) by using Euclidean algorithm. The solution of the general pell's equation would have to wait for Bhaskara II in 1150 CE.

Geometry:

Cyclic quadrilaterals:

Brahmagupta's most famous result in geometry is his formula for cyclic quadrilaterals as well as theorems on the diagonals of cyclic quadrilateral.

If a,b,c,d are the sides of the quadrilateral, its area is

$$\sqrt{(s-a)(s-b)(s-c)(s-d)}$$

Where s is semiperimeter

$$S = a + b + c + d/2$$

He also derived formula for lengths m ,n of two diagonals for such quadrilateral

$$m^2 = \frac{(ab+cd)(ac+bd)}{ad+bc}$$

$$n^2 = \frac{(ac+bd)(ad+bc)}{ab+cd}$$

Triangles:

Brahmagupta dedicated a substantial portion of his work to geometry. A theorem gives the lengths of the two segments a triangle's base is divided into by its altitude.

12.22. The base decreased and increased by the difference between the squares of the sides divided by the base; when divided by two they are the true segments. The perpendicular [altitude] is the square-root from the square of a side diminished by the square of its segment.

Thus the lengths of the two segments are

$$1/2\left(b \pm \frac{c^2 - a^2}{b}\right).$$

He also gives theorem on rational triangles.

Pi (π):

In verse 40, he gives values of π,

12.40. The diameter and the square of the radius [each] multiplied by 3 are [respectively] the practical circumference and the area [of a circle]. The accurate [values] are the square-roots from the squares of those two multiplied by ten.

So Brahmagupta uses 3 as a "practical" value of π, and $\sqrt{10} \approx 3.1622\ldots$ as an "accurate" value of π. The error in this "accurate" value is less than 1%.

Measurements and constructions:

In some of the verses before verse 40, Brahmagupta gives constructions of various figures with arbitrary sides. He essentially manipulated right triangles to produce isosceles triangles, scalene triangles, rectangles, isosceles trapezoids, isosceles trapezoids with three equal sides, and a scalene cyclic quadrilateral.

After giving the value of pi, he deals with the geometry of plane figures and solids, such as finding volumes and surface areas (or empty spaces dug out of solids). He finds the volume of rectangular prisms, pyramids, and the frustum of a square pyramid. He further finds the average depth of a series of pits. For the volume of a frustum of a pyramid, he gives the "pragmatic" value as the depth times the square of the mean of the edges of the top and bottom faces, and he gives the "superficial" volume as the depth times their mean area.

Trigonometry:

In trigonometry he present a sine table in chapter 2 of his Brahmasputasiddhanta entitled planetary true longitudes, and he uses names of objects to represent the digits of place-value numerals as was common with numerical data in Sanskrit treatises.

Other Works:

His other works were the Cadamekela, the Khandakhadyaka, and the Durkeamynarda which consist of works written in only verses and no proofs.

He was honored by the title given to him by a fellow scientist “Ganita chakra chudamani” which is translated as “The gem of the circle of mathematicians”.