Grey Relational Analysis: Optimization Tool For Gear Material Selection

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Abstract- *In the initial stage of manufacturing, selection of proper material for particular application is often referred as multi objective decision making process and the material should be selected on the basis of optimized parameters so as to maximize functional requirement and minimize undesirable effect. The purpose of this paper is to throw light on the use of the Grey Relational Analysis (GRA) for selection of gear material as a case study. Multi objective optimization is done using GRA method for surface hardness, bending fatigue limit, surface fatigue limit, ultimate tensile strength and cost. At the end of the paper, analysis shows that carburized steel is best material for power transmitting gears.*

Keywords- Grey Relational Analysis (GRA), Multi Objective Decision Making, Optimization, Material Selection.

I. INTRODUCTION

For any industry the material of the product has major contribution to its performance and quality of the industry and at the same time it is the major source of failure of product and companies reputation. Hence proper material selection must be there so as to meet the functional requirement and for success of the product in market. Proper selection of material improves not only the quality of product but also quality of industry in eye of customer as the customer requirement gets fulfilled. Wrong selection of material leads to failure of functional requirement of product and customer satisfaction. Thus it affects productivity, profitability and reputation of organization.[6] Material selection for any industrial application is complex and time consuming process and it requires prefect knowledge to understand which material will meet the required specification. The designer should consider requirement, availability and cost. Now the question arises why to select proper material. During the last decades many new materials and material types have been developed. At present of the order of millions of engineering materials exist due to advance research and rapid development in material science. Therefore there are number of material there to serve our requirement. Designer needs to select the best of them with the explicit objective of maximizing the functional requirement and minimizing the cost. This paper provides 'Grey Relational Analysis' as a optimize tool and investigate the effectiveness of the Grey system theory to determine the ranking of materials for power transmitting gears.

II. OPTIMIZATION

It is the act of obtaining the best result under the given circumstances. It is the process of selection of the material with the objective of maximizing most significant functional requirement or minimizing most significant undesirable effect. A large number of techniques have been implemented in the field of the material parameters optimization [1]. Optimization may be a single objective like minimizing the weight, minimizing the cost or maximizing the energy carrying capacity etc. while multi objective optimization consider effect of all at same time. While choosing the proper material, there is not always a single definite criterion and the designers need to take into account a large number of material selection parameters such as strength, hardness, fatigue limit, cost, etc. hence it becomes multi objective optimization. In single objective optimization problems, it is straightforward to find an optimal solution but multi-objective ones provide users with sets of solutions, out of which one should be chosen so as to apply in physical systems [1-2]. Optimum material is one which minimizes cost while meeting the product performance objectives. The paper reviles a well known technique Grey Relational Analysis. Deng proposed use of principle of grey relational analysis to solve engineering problems. He measures the degree of approximation among sequences using grey relational grade. The grey relational grade can provide knowledge of the factors affecting response variables [1, 3].

Grey relational analysis is used for systems that are incompletely described with relatively few data available, and for which standard statistical assumptions are not satisfied. The term "Grey" stands for poor, incomplete and uncertain where "Relational" stands for uncertain relation among all factors. In grey relational analysis, black represents having no information and white represents having all information. A grey system has a level of information between black and white [1, 8]. In other words in a grey system, some information is known and some information is unknown. The advantage of the Grey system theory is that it is designed to study uncertainty and is designed to work with a system where the available information is insufficient to fully characterize the system. Through the grey relational analysis, a grey relational grade is obtained to evaluate the multiple performance characteristics. As a result, optimization of the complicated multiple performance characteristics can be converted into optimization of a single grey relational grade [1, 8].

III. METHODOLOGY OF GREY RELATIONAL ANALYSIS

The procedure of GRA follows the steps described below.

A. Normalization of S/N ratios

The first step in Taguchi based grey relational analysis is normalization of the S/N ratio which is performed to prepare raw data for the analysis where the original sequence is transferred to a comparable sequence. Normalization of the S/N ratio in the range between zero and unity is also called as the grey relational generation. In this investigation "larger the-better" criterion is used for normalization of all the responses as except cost where smaller is better condition is used which are mentioned in 1.1 and 1.2 respectively. (Normalization data is mentioned in Table-3)

SMALLER THE-BETTER

(1.1) $$x_i^*(k) = \frac{max\ x_i^0(k) - x_i^0(k)}{max\ x_i^0(k) - min\ x_i^0(k)}$$

LARGER THE- BETTER

(1.2) $$x_i^*(k) = \frac{x_i^*(k) - min\ x_i^0(k)}{max\ x_i^0(k) - min\ x_i^0(k)}$$

B. Determination of deviation sequence

The deviation sequence $\Delta_{0i}(k)$ is the absolute difference between the reference sequence $x_0^*(k)$ and the comparability sequence $x_i^*(k)$ after normalization. It is determined using 1.3 as:

(1.3) $$\Delta_{0i}(k) = \|x_0^*(k) - x_i^*(k)\|$$

C. Determination of Grey Relational Coefficient

GRC for all the sequences expresses the relationship between the ideal (best) and actual normalized S/N ratio. If the two sequences agree at all points, then their grey relational coefficient is 1. The grey relational coefficient $\xi_i(k)$ can be expressed by Eq. 4.

(1.4) $$\xi_i(k) = \frac{\Delta_{min} + \zeta . \Delta_{max}}{\Delta_{0i}(k) + \zeta . \Delta_{max}}$$

Where, Δmin is the smallest value of $\Delta_{min} = \min_{\forall j \in i} \min_{\forall k} \|x_0^*(k) - x_i^*(k)\|$ and Δmax is the largest value of $\Delta_{max} = \max_{\forall j \in i} \min_{\forall k} \|x_0^*(k) - x_i^*(k)\|$, $x_0^*(k)$ is the ideal normalized S/N ratio, $x_i^*(k)$ is the normalized comparability sequence, and ζ is the distinguishing coefficient. The value of ζ can be adjusted with the systematic actual need and defined in the range between 0 and 1; here it is taken as 0.5.

(Grey Relational Coefficient mentioned in Table-4)

D. Determination of Grey Relational Grade

The overall evaluation of the multiple performance characteristics is based on the Grey Relational Grade (GRG). The grey relational grade is an average sum of the grey relational coefficients which is defined as follows:

$$\gamma_i = \frac{1}{n}\sum_{k=1}^{n} \xi_i(k)$$

(Grey Relational Grade mentioned in Table-5)

E. Determination of Optimal parameters

The grey relational grade calculated for each sequence is taken as a response for the further analysis. The larger-the better quality characteristic was used for analysing the GRG, since a larger value indicates the better performance of the process. Results are discussed in Table-5.

IV. ILLUSTRATIVE EXAMPLE

A power transmission gear material selection problem from Milam et al. [4], Chatterjee & Chakraborty [5] is considered as a case study. Gears are generally torque transmitting element. Gears mesh together and make things turn. Gears are generally used for transmitting power, to increase or decrease speed of rotation and to move rotational motion to a different axis. Gears are also used to transfer motion from one moving part to another. For selection of the suitable gear material for any type of application, mainly three types of criteria, i.e. atomic bond strength, arrangement and packing of the atoms in solid material, and tooth failure are emphasized. Microstructure-insensitive properties (like density, elastic modulus and thermal properties) and microstructure-sensitive properties (e.g. strength, ductility, fracture toughness and hardness) are also predominant in gear material selection problems [5]. Keeping in view the above requirements, considered seven alternative gear materials and five selection parameters. [4, 6].

TABLE-1 PARAMETERS AND UNITS

Parameter	Unit
Surface hardness (S.H)	BHN
Bending Fatigue Limit (B.F.L)	N/mm2
Surface fatigue limit (S.F.L)	N/mm2
Ultimate tensile strength (U.T.S)	N/mm2
Cost (C)	USC/lb

Following table consists of the already available information about the various mechanical parameters and cost.

TABLE-2.QUANTITATIVE DATA FOR GEAR MATERIAL

MATERIAL	S.H	B.F.L	S.F.L	U.T.S	C
Cast iron	200	100	330	380	0.171
Ductile iron	220	360	460	880	0.342
Cast alloy steel	270	435	630	590	0.119
Through hardened alloy steel	270	540	670	1190	1.283
Surface hardened alloy steel	585	680	1160	1580	3.128
Carburised steel	700	920	1500	2300	2.315
Nitrided steel	750	760	1250	1250	4.732

TABLE- 3 NORMALIZED DATA.

MATERIAL	S.H	B.F.L	S.F.L	U.T.S	C
Cast iron	0	0	0	0	0.98
Ductile iron	0.036	0.31	0.11	0.26	0.95
Cast alloy steel	0.12	0.40	0.25	0.10	1
Through hardened alloy steel	0.12	0.53	0.29	0.42	0.74
Surface hardened alloy steel	0.7	0.70	0.70	0.62	0.34
Carburised steel	0.90	1	1	1	0.52
Nitrided steel	1	0.80	0.78	0.45	0

TABLE-4 GREY RELATIONAL COEFFICIENT

MATERIAL	S.H	B.F.L	S.F.L	U.T.S	C
Cast iron	0.33	0.33	0.33	0.33	0.96
Ductile iron	0.34	0.42	0.35	0.40	0.90
Cast alloy steel	0.36	0.45	0.40	0.35	1
Through hardened alloy steel	0.36	0.51	0.41	0.46	0.65
Surface hardened alloy steel	0.62	0.62	0.62	0.56	0.43
Carburised steel	0.83	1	1	1	0.51
Nitrided steel	1	0.71	0.69	0.47	0.33

V. RESULT AND CONCLUSION

GRA technique is applied in order to get optimized material selection of gear by using all the necessary steps and formulas of Grey Relational Analysis. The resulting table is given below. (Bold number indicates optimal parameter level)

TABLE-5 RANK AND GREY RELATIONAL GRADE (G.R.G)

Rank	MATERIAL	S.H	B.F.L	S.F.L	U.T.S	C	G.R.G
1	**Carburised steel**	**700**	**920**	**1500**	**2300**	**2.315**	**0.86**
2	Nitrided steel	750	760	1250	1250	4.732	0.64
3	Surface hardened alloy steel	585	680	1160	1580	3.128	0.57
4	Cast alloy steel	270	435	630	590	0.119	0.51
5	Ductile iron	220	360	460	880	0.342	0.48
6	Through hardened alloy steel	270	540	670	1190	1.283	0.47
7	Cast iron	200	100	330	380	0.171	0.45

Grey Relational Analysis is very effective technique for optimization of material selection which involves multiple criteria. At the end of paper we come to conclusion that carburised steel is most preferable material for gears according to Grey Relational Analysis. Carburised steel has grey relational grade of 0.86 making it optimum material for gears. However practically we also need to consider other parameters like availability, manufacturability, durability, etc which tend to use other materials too. It is clear that we have focused on GRA hence results of this paper would not be compatible with practical application. For this to happens all the parameters mentioned above needs to be considered as well.

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