

Further Study On Supra β -Irresolute Maps

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Abstract- In this paper, we investigate some other properties of supra β -irresolute maps, totally supra β -continuous functions, slightly supra β -continuous functions and strongly supra β -continuous functions. Also, the relationship between these functions are discussed.

Keywords- slightly supra β -continuous function, strongly supra β -continuous function, supra β -irresolute map, supra β -Ti, totally supra β -continuous function.

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I. INTRODUCTION

In 1983, A.S.Mashhour [3] introduced the supra topological spaces. In 2013, Saeid Jafari [4] introduced and studied a class of sets and a class of maps between topological spaces called supra β -open sets and supra β -continuous maps, respectively. In 2012, O.R.Sayed [5] researched supra β -connectedness by means of a supra β -separated sets. In [7], the authors introduced the notion of supra β -irresolute maps and supra β -Ti. In [6], the authors introduced the concepts of totally supra β -continuous functions, slightly supra β -continuous functions and strongly supra β -continuous functions.

Section 2 deals with the preliminary concepts. In Section 3, we investigate some other properties of supra β -irresolute maps. In Section 4, we discuss about slightly supra β -continuous functions, strongly supra β -continuous functions and totally supra β -continuous functions and the relationship between these functions.

II. PRELIMINARIES

A subcollection $\mu \subseteq P(X)$ where $P(X)$ is the power set of X , is called a supra topology [3] on X if $X, \emptyset \in \mu$ and μ is closed under arbitrary union. The ordered pair (X, μ) is called a supra topological space. The elements of μ are said to be supra open in (X, μ) and the complement of a supra open set is called a supra closed set. The supra

closure of a set A , denoted by $Cl^\mu(A)$, is the intersection of supra closed sets containing A . The supra interior of a set A , denoted by $Int^\mu(A)$, is the union of supra open sets contained in A . The supra topology μ on X is associated with the topology τ if $\tau \subseteq \mu$.

Throughout this paper, (X, τ) , (Y, σ) and (Z, ν) (or simply X , Y and Z) denote topological spaces on which no separation axioms are assumed unless explicitly stated where μ , ρ and η are the associated supra topologies with τ , σ and ν . For a subset A of X , the complement of A is denoted by $X - A$.

Definition 2.1 Let (X, μ) be a supra topological space. Let A be a subset of X . Then A is said to be a

- (1) **supra β -open set** [4] if $A \subseteq Cl^\mu(Int^\mu(Cl^\mu(A)))$.
- (2) **supra β -closed set** [4] if $X - A$ is supra β -open set.
- (3) **supra β -clopen set** [6] if A is both supra β -open and supra β -closed.

Definition 2.2 [4] The **supra β -closure** of a set A , denoted by $Cl_\beta^\mu(A)$, is the intersection of supra β -closed sets containing A . The **supra β -interior** of a set A , denoted by $Int_\beta^\mu(A)$, is the union of supra β -open sets contained in A .

Definition 2.3 [5] Let A and B be subsets of a supra topological space (X, μ) . Then A and B are said to be **supra β -separated** if $A \cap Cl_\beta^\mu(B) = Cl_\beta^\mu(A) \cap B = \emptyset$.

Definition 2.4 [5] A subset A of X is a **supra β -connected set** if it cannot be represented as a union of two nonempty supra β -separated sets. If X is supra β -connected, then X is called a **supra β -connected space**.

Definition 2.5 A map $f: X \rightarrow Y$ is said to be

- (1) **supra β -continuous map** [4] if the inverse image of each open set in Y is supra β -open in X .
- (2) **supra β -open map** [4] if the image of each open set in X is supra β -open in Y .
- (3) **supra β -irresolute map** [7] if the inverse image of each supra β -open set in Y is supra β -open in X .
- (4) **continuous map** [2] if the inverse image of each open set in Y is open in X .
- (5) **totally supra β -continuous function** [6] if the inverse image of each open set in Y is supra β -clopen in X .
- (6) **slightly supra β -continuous function at a point $x \in X$** [6] if for each clopen subset V in Y containing $f(x)$, there exists a supra β -open subset U in X containing x such that $f(U) \subseteq V$. The function f is said to be a **slightly supra β -continuous function** if f is a slightly supra β -continuous function at each point of X .
- (7) **strongly supra β -continuous function** [6] if the inverse image of every subset of Y is a supra β -clopen subset of X .

Definition 2.6 [2] A topological space (X, τ) is said to be a **T_0 -space** if for each pair of distinct points x and y in X , there exist an open set U in X which contains one of them but not the other.

Definition 2.7 [7] A supra topological space (X, μ) is said to be

- (1) **supra β - T_1** if for each pair of distinct points x and y in X , there exists supra β -open sets U and V in X such that $x \in U, y \notin U$ and $y \in V, x \notin V$.
- (2) **supra β - T_2** if for each pair of distinct points x and y in X , there exists disjoint supra β -open sets U and V in X such that $x \in U$ and $y \in V$.

Definition 2.8 [1] A space X is said to be

- (1) **clopen- T_1** if for each pair of distinct points x and y in X , there exist clopen sets U and V in X containing x and y respectively such that $y \notin U$ and $x \notin V$.
- (2) **clopen- T_2** if for each pair of distinct points x and y in X , there exist disjoint clopen sets U and V in X such that $x \in U$ and $y \in V$.

Theorem 2.9 [4] The following conditions are equivalent for a supra topological space (X, μ) .

- (1) (X, μ) is not supra β -connected.
- (2) There exist two non-empty disjoint supra β -open sets such that $X = A \cup B$.
- (3) There exist two non-empty disjoint supra β -closed sets such that $X = A \cup B$.

Theorem 2.10 Let $G \subseteq X$. Then G is supra β -connected if and only if there do not exist two supra β -closed sets A and B in X such that $G \not\subseteq A, G \not\subseteq B, G \subseteq A \cup B$ and $A \cap B \cap G = \emptyset$.

III. SUPRA β -IRRESOLUTE MAPS

In this section, some other properties of supra β -irresolute maps are investigated.

Theorem 3.1 If one of the following conditions holds, then the map $f: X \rightarrow Y$ is supra β -irresolute.

1. $f^{-1}(Int_{\beta}^{\rho}(A) \subseteq Int_{\beta}^{\mu}(f^{-1}(A)))$ for every subset A of Y .
2. $Cl_{\beta}^{\mu}(f^{-1}(A)) \subseteq f^{-1}(Cl_{\beta}^{\rho}(A))$ for every subset A of Y .
3. $f(Cl_{\beta}^{\mu}(B)) \subseteq Cl_{\beta}^{\rho}(f(B))$ for every subset B of X .

Proof: Let A be any supra β -open subset of Y . If Condition (1) $f^{-1}(Int_{\beta}^{\rho}(A) \subseteq Int_{\beta}^{\mu}(f^{-1}(A)))$ holds, then $f^{-1}(A) \subseteq Int_{\beta}^{\mu}(f^{-1}(A))$. We know that $Int_{\beta}^{\mu}(f^{-1}(A)) \subseteq f^{-1}(A)$. Thus $f^{-1}(A) = Int_{\beta}^{\mu}(f^{-1}(A))$. Then $f^{-1}(A)$ is supra β -open set in X . Hence f is supra β -irresolute. If condition (2) $Cl_{\beta}^{\mu}(f^{-1}(A)) \subseteq f^{-1}(Cl_{\beta}^{\rho}(A))$ holds, then we have $f^{-1}(Int_{\beta}^{\rho}(A) \subseteq Int_{\beta}^{\mu}(f^{-1}(A)))$. This is nothing but

condition (1). If condition (3) $f(Cl_{\beta}^{\mu}(B)) \subseteq Cl_{\beta}^{\rho}(f(B))$ holds for every subset B of Y, then $Cl_{\beta}^{\mu}(f^{-1}(A)) \subseteq f^{-1}(Cl_{\beta}^{\rho}(A))$ for every subset A of Y. This is nothing but condition (2). Hence f is supra β -irresolute.

The converse of Theorem 3.1 need not be true as shown by the following example.

Example 3.2 Let $X=\{a,b,c,d\}$. Clearly $\mu=\{\emptyset,X,\{a\},\{b\},\{a,b\}\}$ is a supra topology on X. Here the sets $\emptyset,X,\{a\},\{b\},\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}$ are supra β -open. Let $Y=\{a,b,c\}$. Clearly $\rho=\{\emptyset,Y,\{a\},\{a,b\},\{b,c\}\}$ is a supra topology on Y. Here all the subsets of X except $\{c\}$ are supra β -open. Define $f:X \rightarrow Y$ by $f(a)=c, f(b)=a, f(c)=f(d)=b$. Clearly the inverse image of every supra β -open in Y is supra β -open in X. Hence f is a supra β -irresolute map. Then

- (1) Let $A=\{b\}$. Then $f^{-1}(Int_{\beta}^{\rho}(A)) = f^{-1}(\{b\}) = \{c, d\}$ and $Int_{\beta}^{\mu}(f^{-1}(A)) = Int_{\beta}^{\mu}(\{c, d\}) = \emptyset$. Hence $f^{-1}(Int_{\beta}^{\rho}(A)) \not\subseteq Int_{\beta}^{\mu}(f^{-1}(A))$.
- (2) Let $A=\{a,c\}$. Then $Cl_{\beta}^{\mu}(f^{-1}(A)) = Cl_{\beta}^{\mu}(\{a, b\}) = X$ and $f^{-1}(Cl_{\beta}^{\rho}(A)) = f^{-1}(\{a, c\}) = \{a, b\}$. Hence $Cl_{\beta}^{\mu}(f^{-1}(A)) \not\subseteq f^{-1}(Cl_{\beta}^{\rho}(A))$.
- (3)
- (4) Let $A=\{a,b\}$. Then $f(Cl_{\beta}^{\mu}(A)) = f(X) = Y$ and $Cl_{\beta}^{\rho}(f(A)) = Cl_{\beta}^{\rho}(\{a, c\}) = \{a, c\}$. Hence $f(Cl_{\beta}^{\mu}(A)) \not\subseteq Cl_{\beta}^{\rho}(f(A))$.

Theorem 3.3 Let $f:X \rightarrow Y$ be a map. Then the following conditions are equivalent:

- (1) f is a supra β -irresolute map.
- (2) The inverse image of each supra β -closed set in Y is supra β -closed set in X.
- (3) $Cl_{\beta}^{\mu}(f^{-1}(B)) \subseteq f^{-1}(Cl_{\beta}^{\rho}(B))$ for every subset B of Y.
- (4) $f(Cl_{\beta}^{\mu}(A)) \subseteq Cl_{\beta}^{\rho}(f(A))$ for every subset A of X.

(5) $f^{-1}(Int_{\beta}^{\rho}(B)) \subseteq Int_{\beta}^{\mu}(f^{-1}(B))$ for every subset B of Y.

Proof:

(1) \Rightarrow (2) Let f be a supra β -irresolute map. Let A be a supra β -closed subset of Y. Then $Y-A$ is supra β -open in Y. Since f is a supra β -irresolute map, $f^{-1}(Y-A)$ is supra β -open in X. That is, $X-f^{-1}(A)$ is supra β -open in X. Hence $f^{-1}(A)$ is supra β -closed in X.

(2) \Rightarrow (3) Let B be any subset of Y. Since $Cl_{\beta}^{\rho}(B)$ is supra β -closed in Y and By (2), $f^{-1}(Cl_{\beta}^{\rho}(B))$ is supra β -closed in X. Therefore,

$$Cl_{\beta}^{\mu}(f^{-1}(B)) \subseteq Cl_{\beta}^{\mu}\left(f^{-1}\left(Cl_{\beta}^{\rho}(B)\right)\right) = f^{-1}(Cl_{\beta}^{\rho}(B))$$

Hence $Cl_{\beta}^{\mu}(f^{-1}(B)) \subseteq f^{-1}(Cl_{\beta}^{\rho}(B))$ for every subset B of Y.

(3) \Rightarrow (4) Assume that the condition $Cl_{\beta}^{\mu}(f^{-1}(B)) \subseteq f^{-1}(Cl_{\beta}^{\rho}(B))$ holds for every subset B of Y. Let A be any subset of X. Then $f(A) \in Y$. By (3), $f^{-1}(Cl_{\beta}^{\rho}(f(A))) \supseteq Cl_{\beta}^{\mu}(f^{-1}(f(A))) \supseteq Cl_{\beta}^{\mu}(A)$. Hence $f(Cl_{\beta}^{\mu}(A)) \subseteq Cl_{\beta}^{\rho}(f(A))$ for every subset A of X.

(4) \Rightarrow (5) Assume that the condition $f(Cl_{\beta}^{\mu}(A)) \subseteq Cl_{\beta}^{\rho}(f(A))$ holds for every subset A of X.

Let B be any subset of Y. Then $X-f^{-1}(B) \in Y$. By (4), $f(Cl_{\beta}^{\mu}(X-f^{-1}(B))) \subseteq Cl_{\beta}^{\rho}(f(X-f^{-1}(B)))$. Then $f(X-Int_{\beta}^{\mu}(f^{-1}(B))) \subseteq Cl_{\beta}^{\rho}(Y-B) = Y-Int_{\beta}^{\rho}(B)$.

Hence $f^{-1}(Int_{\beta}^{\rho}(B)) \subseteq Int_{\beta}^{\mu}(f^{-1}(B))$.

(5) \Rightarrow (1) Assume that the condition $f^{-1}(Int_{\beta}^{\rho}(B)) \subseteq Int_{\beta}^{\mu}(f^{-1}(B))$ holds for every subset B of Y.

Let B be any supra β -open subset of Y. By (5), $f^{-1}(B) \subseteq Int_{\beta}^{\mu}(f^{-1}(B))$. We know that

$$Int_{\beta}^{\mu}(f^{-1}(B)) \subseteq f^{-1}(B).$$

Thus $f^{-1}(B) = Int_{\beta}^{\mu}(f^{-1}(B))$. Therefore, $f^{-1}(B)$ is supra β -open in X. Hence f is a supra β -irresolute map.

Theorem 3.4 If $f: X \rightarrow Y$ is a supra β -irresolute surjective map and C and D are supra β -separated sets in Y , then $f^{-1}(C)$ and $f^{-1}(D)$ are supra β -separated in X .

Proof: Let $f: X \rightarrow Y$ be a supra β -irresolute map. Since C and D are supra β -separated in Y , $C \cap Cl_{\beta}^p(D) = Cl_{\beta}^p(C) \cap D = \emptyset$. By Theorem 3.3 (3), $Cl_{\beta}^{\mu}(f^{-1}(C)) \subseteq f^{-1}(Cl_{\beta}^p(C))$. Then $Cl_{\beta}^{\mu}(f^{-1}(C)) \cap f^{-1}(D) \subseteq f^{-1}(Cl_{\beta}^p(C) \cap D) = f^{-1}(\emptyset) = \emptyset$. Hence $Cl_{\beta}^{\mu}(f^{-1}(C)) \cap f^{-1}(D) = \emptyset$. Similarly we can prove that $f^{-1}(C) \cap Cl_{\beta}^{\mu}(f^{-1}(D)) = \emptyset$. Hence $f^{-1}(C)$ and $f^{-1}(D)$ are supra β -separated in X .

Theorem 3.5 If $f: X \rightarrow Y$ is supra β -irresolute bijective map and A is a supra β -connected subset of X , then $f(A)$ is a supra β -connected subset of Y .

Proof: Suppose $f(A)$ is not a supra β -connected subset of Y . Then $f(A) = C \cup D$ where C and D are non-empty supra β -separated sets in Y . By Theorem 3.4, $f^{-1}(C)$ and $f^{-1}(D)$ are supra β -separated sets in X . Since f is bijective, $f^{-1}(C)$ and $f^{-1}(D)$ are non-empty. Since f is bijective, $A = f^{-1}(C) \cup f^{-1}(D)$. Hence A is not a supra β -connected subset of X , which is a contradiction. Thus $f(A)$ is a supra β -connected subset of Y .

Theorem 3.6 If $f: X \rightarrow Y$ is a supra β -irresolute map and G is supra β -connected in X , then $f(G)$ is supra β -connected in Y .

Proof: Suppose $f(G)$ is not connected in Y , then there exist two supra β -closed sets A and B in Y such that $f(G) \not\subseteq A$, $f(G) \not\subseteq B$, $f(G) \subseteq A \cup B$ and $A \cap B \cap f(G) = \emptyset$. Then $G \not\subseteq f^{-1}(A)$, $G \not\subseteq f^{-1}(B)$, $G \subseteq f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$ and $f^{-1}(A) \cap f^{-1}(B) \cap G = \emptyset$. This implies that G is not supra β -connected in X , which is a contradiction. Hence $f(G)$ is supra β -connected in Y .

Corollary 3.7 If $f: X \rightarrow Y$ is a surjective supra β -irresolute map and X is supra β -connected, then Y is supra β -connected

Proof: By Theorem 3.6, $f(X)$ is supra β -connected. That is, Y is supra β -connected.

IV TOTALLY SUPRA β -CONTINUOUS FUNCTIONS, SLIGHTLY SUPRA β -CONTINUOUS FUNCTIONS AND STRONGLY SUPRA β -CONTINUOUS FUNCTIONS

In this Section, we investigate some other properties of totally supra β -continuous functions, slightly supra β -continuous functions and strongly supra β -continuous functions. Also, the relationship between these functions and supra β -irresolute maps are discussed.

Theorem 4.1 Let $f: X \rightarrow Y$ be a totally supra β -continuous injective function. If Y is a T_0 -space, then X is a supra β - T_2 space.

Proof: Let x and y be any pair of distinct points in X . Since f is injective, $f(x) \neq f(y)$ in Y . Since Y is a T_0 -space, there exists an open set V in Y containing $f(x)$ but not $f(y)$ or containing $f(y)$ but not $f(x)$. Thus for the first case, $x \in f^{-1}(V)$ and $y \notin f^{-1}(V)$. Since f is a totally supra β -continuous function, $f^{-1}(V)$ and $X - f^{-1}(V)$ are disjoint supra β -clopen sets in X containing x and y respectively. The proof of the other case is similar. Hence X is a supra β - T_2 space.

Theorem 4.2 Every slightly supra β -continuous function into a discrete space is strongly supra β -continuous function.

Proof: Let $f: X \rightarrow Y$ be a slightly supra β -continuous function where Y is a discrete space. Let A be any subset of Y . Then A is a clopen set in X . Since f is a slightly supra β -continuous function, $f^{-1}(A)$ is supra β -clopen in X . Hence f is a strongly supra β -continuous function.

Theorem 4.3 Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be functions. Then $g \circ f: X \rightarrow Z$.

- (1) If f is supra β -irresolute and g is slightly supra β -continuous then $g \circ f$ is slightly supra β -continuous.
- (2) If f is slightly supra β -continuous and g is continuous, then $g \circ f$ is slightly supra β -continuous.

Proof: Let V be any clopen subset of Z .

- (1) Since g is slightly supra β -continuous, $g^{-1}(V)$ is supra β -open in Y . Since f is supra β -irresolute, $f^{-1}(g^{-1}(V))$ is supra β -open in X . That is, $(g \circ f)^{-1}(V)$ is supra β -open in X .

- (2) Since g is continuous, $g^{-1}(V)$ is clopen in Y . Since f is slightly supra β -continuous, $f^{-1}(g^{-1}(V))$ is supra β -open in X . That is, $(gof)^{-1}(V)$ is supra β -open in X .

Hence gof is slightly supra β -continuous.

Corollary 4.4 If $f:X \rightarrow Y$ is supra β -irresolute and $g:Y \rightarrow Z$ is supra β -continuous then $gof:X \rightarrow Z$ is slightly supra β -continuous.

Proof: Since every supra β -continuous function is slightly supra β -continuous function, g is slightly supra β -continuous function. By Theorem 4.3 (1), gof is slightly supra β -continuous.

Theorem 4.5 Let $f:X \rightarrow Y$ be a supra β -irresolute supra β -open surjective map and $g:Y \rightarrow Z$ be a function. Then g is slightly supra β -continuous if and only if gof is slightly supra β -continuous.

Proof:

\Rightarrow Let g be slightly supra β -continuous. By Theorem 4.3 (1), gof is slightly supra β -continuous.

\Leftarrow Let gof be slightly supra β -continuous. Let V be any clopen subset of Z . Then $(gof)^{-1}(V)$ is supra β -open in X . Since f is supra β -open, $f((gof)^{-1}(V))$ is supra β -open in Y . Since f is surjective, $f((gof)^{-1}(V))=f(f^{-1}(g^{-1}(V)))=g^{-1}(V)$ is supra β -open in Y . Hence g is slightly supra β -continuous.

Theorem 4.6 Let $f:X \rightarrow Y$ be a slightly supra β -continuous injective map and Y is clopen- T_i , then X is supra β - T_i for $i=1,2$.

Proof: Let x and y be any two distinct points in X . Then $f(x)$ and $f(y)$ are distinct points in Y .

- (1) For $i=1$. Since Y is clopen- T_1 , there exist clopen sets V and W in Y such that $f(x) \in V, f(y) \notin V$ and $f(y) \in W, f(x) \notin W$. Since f is slightly supra β -continuous, $f^{-1}(V)$ and $f^{-1}(W)$ are supra β -open in X such that $x \in f^{-1}(V), y \notin f^{-1}(V)$ and $y \in f^{-1}(W), x \notin f^{-1}(W)$. This shows that X is supra β - T_1 .
- (2) For $i=2$. Since Y is clopen- T_2 , there exist disjoint clopen sets V and W in Y such that $f(x) \in V$ and $f(y) \in W$. Since f is slightly supra β -continuous, $f^{-1}(V)$ and $f^{-1}(W)$ are disjoint supra β -open in X containing x and y respectively. Hence X is supra β - T_2 .

Proposition 4.7 Every supra β -irresolute map is slightly supra β -continuous.

Proof: Let $f:X \rightarrow Y$ be a supra β -irresolute map. Let A be any clopen subset of Y . Then A is open in Y . Since ρ is the associated supra topology with σ , A is supra open in Y . Then A is supra β -open in Y . Since f is supra β -irresolute, $f^{-1}(A)$ is supra β -open in X . Hence f is slightly supra β -continuous.

The converse of Proposition 4.7 need not be true as shown by the following example.

Example 4.8 Let $X=\{a,b,c\}$ and $\tau=\{\emptyset, X, \{a,b\}\}$ be a topology on X . Clearly $\mu=\{\emptyset, X, \{a\}, \{a,b\}, \{b,c\}\}$ is a supra topology on X . Here the only clopen sets are \emptyset and X and all the subsets of X except $\{c\}$ are supra β -open in X . Define $f:X \rightarrow X$ by $f(a)=c, f(b)=a, f(c)=b$. Clearly the inverse image of every clopen set is supra β -open. Hence f is a slightly supra β -continuous function. But the inverse image of a supra β -open set $\{b\}$ is $\{c\}$ which is not a supra β -open set. Hence f is not a supra β -irresolute map.

Proposition 4.9 Every strongly supra β -continuous function is supra β -irresolute.

Proof: Let $f:X \rightarrow Y$ be a strongly supra β -continuous function. Let A be any supra β -open subset of Y . Since f is a strongly supra β -continuous function, $f^{-1}(A)$ is supra β -clopen in X . That is, $f^{-1}(A)$ is supra β -open in X . Hence f is supra β -irresolute.

The converse of Proposition 4.9 need not be true as shown by the following example.

Example 4.10 Let $X=\{a,b,c\}$ and $\tau=\{\emptyset, X, \{a,b\}\}$ be a topology on X . Clearly $\mu=\{\emptyset, X, \{a\}, \{a,b\}, \{b,c\}\}$ is a supra topology on X . Here all the subsets of X except $\{c\}$ are supra β -open and all the subsets of X except $\{c\}$ and $\{a,b\}$ are supra β -clopen in X . Define $f:X \rightarrow X$ by $f(a)=b, f(b)=a, f(c)=c$. Example 3.6 in [7] shows that this map f is a supra β -irresolute map. But the inverse image of the set $\{a,b\}$ is $\{a,b\}$ which is not supra β -clopen. Hence f is not a strongly supra β -continuous function.

Remark 4.11 Totally supra β -continuous functions and supra β -irresolute maps are independent.

For example, Let $X=\{a,b,c\}$ and $\tau=\{\emptyset, X, \{a,b\}\}$ be a topology on X . Clearly $\mu=\{\emptyset, X, \{a\}, \{a,b\}, \{b,c\}\}$ is a supra

topology on X . Here the sets $\varphi, X, \{a, b\}$ are open, all the subsets of X except $\{c\}$ are supra β -open and all the subsets of X except $\{c\}$ and $\{a, b\}$ are supra β -clopen in X .

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- (1) Define $f: X \rightarrow X$ by $f(a)=c, f(b)=a, f(c)=b$. Clearly the inverse image of every open set is supra β -clopen. Hence f is a totally supra β -continuous function. Example 4.8 shows that this map f is not a supra β -irresolute map.
- (2) Define $f: X \rightarrow X$ by $f(a)=b, f(b)=a, f(c)=c$. Example 3.6 in [7] shows that this map f is a supra β -irresolute map. Clearly the inverse image of the open set $\{a, b\}$ is $\{a, b\}$ which is not supra β -clopen. Hence f is not a totally supra β -continuous function.

V. CONCLUSION

In this paper, we have investigated some other properties of supra β -irresolute maps, totally supra β -continuous functions, slightly supra β -continuous functions and strongly supra β -continuous functions. Also, the relationship between these functions are discussed.

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