Near Difference Mean Cordial Labeling Of Some Path Related Graphs

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Abstract- *Let $G=(V,E)$ be a simple graph. A Near difference mean cordial labeling on G is a function in $f: V(G) \to \{1,2,\dots,p-1,p+1\}$ such that for each edge uv the induced map f^* defined by*

$$f^*(uv) = \begin{cases} 1 & if|f(u)-f(v)| \equiv 0 (mod\ 2) \\ 0 & elsewhere \end{cases}$$

*and it satisfies the condition $|e_f(0)-e_f(1)| \le 1$, where $e_f(0)$ and $e_f(1)$ represent the number of edges labeled with 0 and 1 respectively. A graph that admits a Near difference mean cordial labeling is called a **Near difference mean cordial graph.***

In this paper, we proved that the graphs Path (P_n),Comb ($P_n \odot K_1$), Broken Comb Br(n), Star ($K_{1,n}$), Globe Gl(n) are Near difference mean cordial graphs.

Keywords- Near difference mean cordial graph, Near difference mean cordial labeling.

I. INTRODUCTION

A graph G is a finite nonempty set of objects called vertices together with a set of unordered pairs of distinct vertices of G called edges. Each pair e={u,v} of vertices in E is called edges or a line of G.In this paper, we proved that the graphs Path (P_n),Comb ($P_n \odot K_1$), Broken Comb Br(n), Star ($K_{1,n}$), Globe Gl(n) are Near difference mean cordial graphs.

II. PRELIMINARIES

Let $G=(V,E)$ be a simple graph. A Near difference mean cordial labeling on G is a function in $f: V(G) \to \{1,2,\dots,p-1,p+1\}$ such that for each edge uv the induced map f^* defined by

$$f^*(uv) = \begin{cases} 1 & if|f(u)-f(v)| \equiv 0 (mod\ 2) \\ 0 & elsewhere \end{cases}$$

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DEFINITION 2.1:

Path is a graph whose vertices can be listed in the order ($u_1, u_2, \dots . u_n$) such that the edges are {$u_i u_{i+1}$} where $i = 1,2,\dots,n$.

DEFINITION 2.2:

A graph obtained by joining a single pendant edge to eachvertex of path is called a comb and it is denoted by $P_n \odot K_1$.

DEFINITION 2.3:

Broken comb is a graph obtained from the comb by removing the end pendant vertices.

DEFINITION 2.4:

$k_{1,m}$ is called a Star graph, $m \ge 1$

DEFINITION 2.5:

Globe is a graph obtained from two isolated vertices are joined by n paths of length 2. It is denoted by Gl_n.

III. MAIN RESULT

Theorem 3.1: PathP_n is a near difference mean cordial graph

Proof: Let $V(P_n) = \{u_i : 1 \le i \le n\}$, $E(P_n) = \{u_i u_{i+1} : 1 \le i \le n-1\}$.

Case (i): When n is even.

We define a labeling

$f: V(G) \to \{1,2,\dots,p-1,p+1\} as follows:$

$$f(u_{2i-1}) = i,\ \ 1 \le i \le \frac{n}{2}$$
$$f(u_{2i}) = \frac{n+2}{2} + (i-1), \qquad 1 \le i \le \frac{n-2}{2}$$
$$f(u_n) = n+1$$

The induced edge labeling are,

$$f^*(u_i u_{i+1}) = \begin{cases} 1 & if\,|f(u_i) - f(u_{i+1})| \equiv 0 (mod\ 2) \\ 0 & elsewhere \end{cases}$$

Here, $e_f(0) = \frac{n}{2}$ and $e_f(1) = \frac{n-2}{2}$, when $n \equiv 0 (mod\ 4)$

$e_f(0) = \frac{n-2}{2}$ and $e_f(1) = \frac{n}{2}$, when $n \equiv 2 (mod\ 4)$

Case (ii): When n is odd.

We define a labeling

$f: V(G) \to \{1,2,\dots,p-1,p+1\} as follows:$

$$f(u_{2i-1}) = i,\ \ 1 \le i \le \frac{n-1}{2}$$
$$f(u_{2i}) = \frac{n+3}{2} + (i-1), \qquad 1 \le i \le \frac{n-3}{2}$$
$$f(u_{n-1}) = n+1$$
$$f(u_n) = \frac{n+1}{2}$$

The induced edge labeling are,

$$f^*(u_i u_{i+1}) = \begin{cases} 1 & if\,|f(u_i) - f(u_{i+1})| \equiv 0 (mod\ 2) \\ 0 & elsewhere \end{cases}$$

Here, $e_f(0) = \frac{n-1}{2} = e_f(1)$.

Therefore, it satisfies the condition $|e_f(0) - e_f(1)| \le 1$.

Hence, f is a near difference mean cordial labeling.

Thus, path P_n is a near difference mean cordial graph.

Example: The Near difference mean cordial labeling of P_8 with $n \equiv 0 (mod\ 4)$ is shown in fig 1

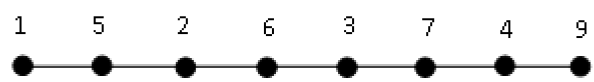

Figure 1

The Near difference mean cordial labeling of P_{10} with $n \equiv 2 (mod\ 4)$ is shown in fig 2

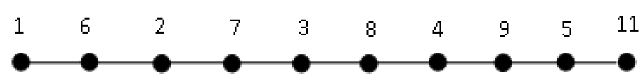

Figure 2

The Near difference mean cordial labeling of P_7 is shown in fig 3

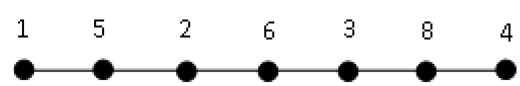

Figure 3

Theorem 3.2: Comb $P_n \odot K_1$ is a near difference mean cordial graph.

Proof: Let $G = P_n \odot K_1$ be a graph.

Let

$V(G) = \{u_i, v_i : 1 \le i \le n\}$,
$E(G) = \{u_i u_{i+1} : 1 \le i \le n-1\} \cup \{u_i v_i : 1 \le i \le n\}$.

We define a labeling $f: V(G) \to \{1,2,\dots,p-1,p+1\}$ as follows:

$$f(u_i) = 2i-1, \quad 1 \le i \le n$$
$$f(v_i) = 2i, \quad 1 \le i \le n-1$$
$$f(v_n) = 2n+1$$

The induced edge labeling are,

$$f^*(u_i u_{i+1}) = \begin{cases} 1 & if\ |f(u_i) - f(u_{i+1})| \equiv 0 (mod\ 2) \\ 0 & elsewhere \end{cases}$$

$$f^*(u_i v_i) = \begin{cases} 1 & if\ |f(u_i) - f(v_i)| \equiv 0 (mod\ 2) \\ 0 & elsewhere \end{cases}$$

Here, $e_f(0) = n-1$ and $e_f(1) = n$.

Therefore, it satisfies the condition $|e_f(0) - e_f(1)| \le 1$.

Hence, f is a near difference mean cordial labeling.

Thus, comb $P_n \odot K_1$ is a near difference mean cordial graph.

Example: The Near difference mean cordial labeling of $P_7 \odot K_1$ is shown in fig 4

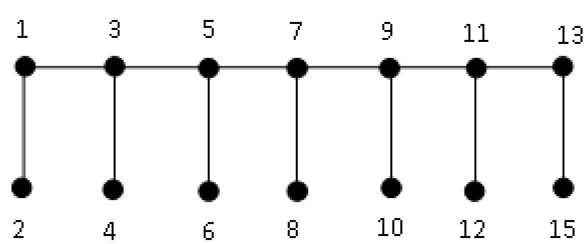

Figure 4

Theorem 3.3: Broken $Br(n)$ Comb is a near difference mean cordial graph.

Proof: Let $G = Br(n)$ be a graph.

Let $V(G) = \{u_i : 1 \le i \le n, v_i : 1 \le i \le n-1\}$ and $E(G) = \{u_i u_{i+1} : 1 \le i \le n-1\} \cup \{u_{i+1} v_i : 1 \le i \le n-1\}$.

We define a labeling $f: V(G) \to \{1,2,\dots,p-1,p+1\} as follows:$

$$f(u_i) = 2i-1, \quad 1 \le i \le n$$
$$f(v_i) = 2i, \quad 1 \le i \le n-2$$

The induced edge labeling are,

$$f^*(u_i u_{i+1}) = \begin{cases} 1 & if\ |f(u_i) - f(u_{i+1})| \equiv 0 (mod\ 2) \\ 0 & elsewhere \end{cases}$$

$$f^*(u_{i+1} v_i) = \begin{cases} 1 & if\ |f(u_{i+1}) - f(v_i)| \equiv 0 (mod\ 2) \\ 0 & elsewhere \end{cases}$$

Here, $e_f(0) = n-2$ and $e_f(1) = n-1$.

Therefore, it satisfies the condition $|e_f(0) - e_f(1)| \le 1$.

Hence, f is a near difference mean cordial labeling.

Thus, broken comb $Br(n)$ is a near difference mean cordial graph.

Example: The Near difference mean cordial labeling of $Br(8)$ is shown in fig

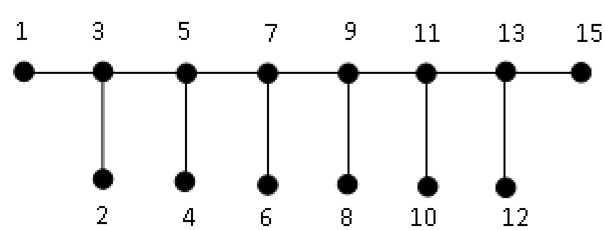

Figure 5

Theorem 3.4: Star $K_{1,n}$ is a near difference mean cordial graph.

Proof: Let $G = K_{1,n}$.

Let $V(G) = \{u, u_i : 1 \le i \le n\}$ and $E(G) = \{uu_i : 1 \le i \le n-1\}$.

We define a labeling

$f: V(G) \to \{1,2,\ldots,p-1,p+1\}$ *as follows*:

$$f(u_i) = i, \quad 1 \le i \le n$$
$$f(u) = n+2$$

The induced edge labeling are,

$$f^*(uu_i) = \begin{cases} 1 & if\ |f(u) - f(u_i)| \equiv 0 (mod\ 2) \\ 0 & elsewhere \end{cases}$$

Here, $e_f(0) = \frac{n-1}{2}$ and $e_f(1) = \frac{n+1}{2}$, when n is odd.
$e_f(0) = \frac{n}{2} = e_f(1)$, when n is even.

Therefore, it satisfies the condition $|e_f(0) - e_f(1)| \le 1$.
Hence, f is a near difference mean cordial labeling.

Thus, star $K_{1,n}$ is a near difference mean cordial graph.

Example: The Near difference mean cordial labeling of $K_{1,5}$ is shown in figure 6

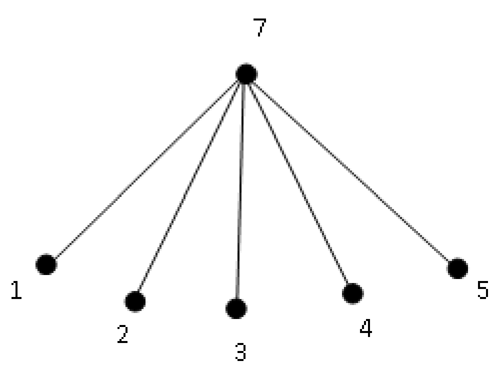

Figure 6

The Near difference mean cordial labeling of $K_{1,6}$ is shown in figure 7

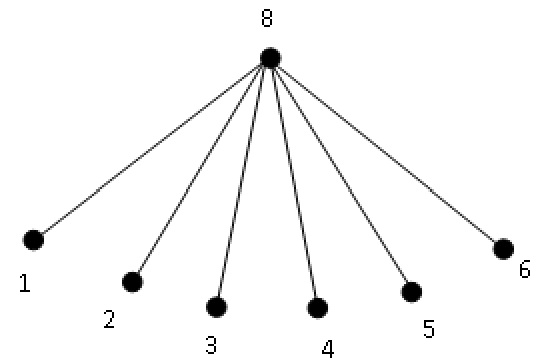

Figure 7

Theorem 3.5: Globe $Gl(n)$ is a near difference mean cordial graph.

Proof : Let $G = Gl(n)$.

Let $V(G) = \{u, v, u_i : 1 \le i \le n\}$ and $E(G) = \{uu_i, vu_i : 1 \le i \le n\}$.

We define a labeling $f: V(G) \to \{1,2,\ldots,p-1,p+1\}$ *as follows*:

$$f(u) = 1, f(v) = 2$$
$$f(u_i) = i+2, \quad 1 \le i \le n-1$$
$$f(u_n) = n+3$$

The induced edge labeling are,

$$f^*(uu_i) = \begin{cases} 1 & if\ |f(u) - f(u_i)| \equiv 0 (mod\ 2) \\ 0 & elsewhere \end{cases} \quad f^*(vu_i) = \begin{cases} 1 & if\ |f(v) - f(u_i)| \equiv 0 (mod\ 2) \\ 0 & elsewhere \end{cases}$$

Here, $e_f(0) = n = e_f(1)$.

Therefore, it satisfies the condition $|e_f(0) - e_f(1)| \le 1$.

Hence, f is a near difference mean cordial labeling.

Thus, star $K_{1,n}$ is a near difference mean cordial graph.

Example: The Near difference mean cordial labeling of $Gl(6)$ is shown in fig 8

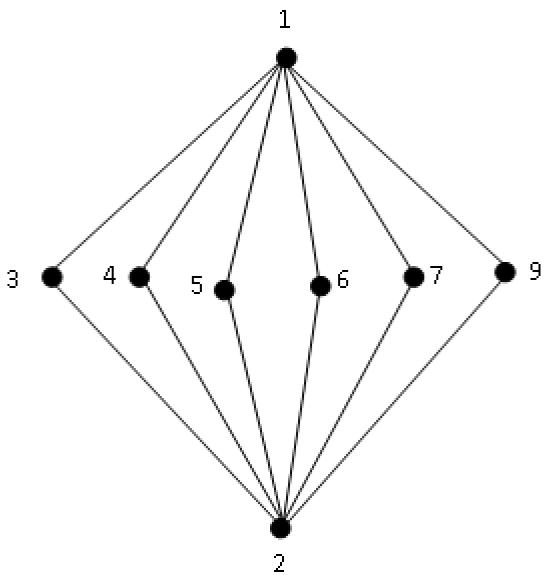

Figure 8

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