Some Results on Difference Mean Cordial Labeling

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Abstract- *Let* $G=(V,E)$ *be a simple graph. A difference mean cordial labeling on* G *is a function in* $f:V(G)\to\{1,2,\dots.p\}$ *such that for each edge* uv *the induced map* f^* *defined by*

$$f^*(uv)=\begin{cases}1 & if\ f(u)-f(v)\equiv 0\ (mod\ 2)\\ 0 & else\end{cases}$$

and it satisfies the condition $|e_f(0)-e_f(1)|\le 1$ *where* $e_f(0)$ *and* $e_f(1)$ *represent the number of edges labeled with 0 and 1 respectively. A graph is called a* ***difference mean cordial graph*** *if it admits a* ***difference mean cordial labeling****. In this paper, we proved that the graphs path (*P_n*), Star (*$K_{1,n}$*) Comb (*P_n^+*), Broken Comb Br(n),Globe Gl(n) are Difference mean cordial graphs.*

Keywords- Difference mean cordial graph, Difference mean cordial labeling.

I. INTRODUCTION

A graph G is a finite nonempty set of objects called vertices together with a set of unordered pairs of distinct vertices of G called edges. Each pair e={u,v} of vertices in E is called edges or a line of G. In this paper, we proved that the graphs Path (P_n),Star ($K_{1,n}$), Comb (P_n^+), Broken Comb Br(n),Globe Gl(n) are Difference mean cordial graphs.

II. PRELIMINARIES

Let $G=(V,E)$ be a simple graph. A difference mean cordial labeling on G is a function in $f:V(G)\to\{1,2,\dots.p\}$ such that for each edge uv the induced map f^* defined by

$$f^*(uv)=\begin{cases}1 & if\ f(u)-f(v)\equiv 0\ (mod\ 2)\\ 0 & else\end{cases}$$

and it satisfies the condition $|e_f(0)-e_f(1)|\le 1$ where $e_f(0)$ and $e_f(1)$ represent the number of edges labeled with 0 and 1 respectively. A graph is called a **difference mean cordial graph** if it admits a **difference mean cordial labeling**.Then the resulting edges get distinct labels from the set {1,2,3…….p}. A graph with Difference mean cordial labeling is called a Difference mean Cordial graph .

DEFINITION 2.1:

Path is a graph whose vertices can be listed in the order ($u_1,u_2,\dots.u_n$) such that the edges are {u_iu_{i+1}} where i=1,2,3…..n-1.

DEFINITION 2.2:

P_n^+ is a graph obtained from path of length n by attaching a pendant vertex from each vertex of the path.

DEFINITION 2.3:

Broken comb is a graph obtained from the comb by removing the end pendant vertices.

DEFINITION 2.4:

$k_{1,m}$ is called a Star graph, $m\ge 1$

DEFINITION 2.5:

Globe is a graph obtained from two isolated vertices are joined by n paths of length 2. It is denoted by Gl_n.

III. MAIN RESULT

Theorem 3.1: P_n is a difference mean cordial graph

Proof: Let $V(P_n)=\{u_i:1\le i\le n\}$,

$E(P_n) = \{(u_i u_{i+1}) : 1 \le i \le n-1\}$

Define $f : V(G) \to \{1, 2, \dots \dots n\}$ by

Case (i) When n is even

Subcase(i) When $n \equiv 0 \ (mod4)$

Define $f(u_i) = i, 1 \le i \le \frac{n}{2}$

$f\left(u_{\frac{n}{2}+i}\right) = \frac{n}{2} + 2i, 1 \le i \le \frac{n}{4}$

$f(u_{n+1-i}) = \frac{n}{2} + 2i - 1$, $1 \le i \le \frac{n}{4}$

Subcase(ii) When $n \equiv 2 \ (mod4)$

Define

$f(u_i) = i, 1 \le i \le \frac{n}{2}$ $f\left(u_{\frac{n}{2}+i}\right) = \frac{n}{2} + 2i, 1 \le i \le \frac{n-2}{4}$

$f(u_{n+1-i}) = \frac{n}{2} + 2i - 1$

$1 \le i \le \frac{n+2}{4}$

Case(ii) When n is odd

Subcase(i) When $n \equiv 1 \ (mod4)$

Define $f(u_i) = i, \ 1 \le i \le \frac{n-1}{2}$

$f\left(u_{\frac{n-1}{2}+i}\right) = \frac{n-1}{2} + 2i,$

$f(u_{n+1-i}) = \frac{n-1}{2} + 2i - 1, \qquad 1 \le i \le \frac{n-1}{4} + 1$

Subcase(ii) When $n \equiv 3 \ (mod4)$

Define

$f(u_i) = 1, \ 1 \le i \le \frac{n-1}{2}$ $f\left(u_{\frac{n-1}{2}+i}\right) = \frac{n-1}{2} + 2i, \qquad 1 \le i \le \frac{n+1}{4}$

$f(u_{n+1-i}) = \frac{n-1}{2} + 2i - 1, \qquad 1 \le i \le \frac{n+1}{4}$

The induced edge labels are,

$$f^*(u_i u_{i+1}) = \begin{cases} 1 & if\ f(u_i) - f(u_{i+1}) \equiv 0 \ (mod\ 2) \\ 0 & else \end{cases}$$

Here, $e_f(0) = \frac{n}{2}$ and $e_f(1) = \frac{n}{2} - 1$, When $n \equiv 0(mod4)$

$e_f(0) = \frac{n}{2}$ and $e_f(1) = \frac{n}{2} - 1$, When $n \equiv 2(mod4)$

Also, $e_f(0) = \frac{n-1}{2} = e_f(1)$,

When $n \equiv 1(mod4), n \equiv 3(mod4)$

Therefore, it satisfies the condition $|e_f(0) - e_f(1)| \le 1$.

Hence, P_n is a difference mean cordial graph.

Example: The difference mean cordial labeling of P_6 with $n \equiv 2 \ (mod4)$ is shown in fig 1

$1 \le i \le \frac{n-1}{4}$

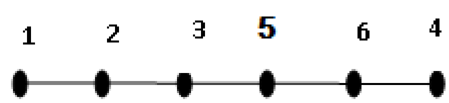

Fig 1

The difference mean cordial labeling of P_8 with $n \equiv 0 \ (mod4)$ is shown in fig 2

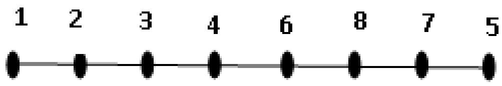

Fig 2

The difference mean cordial labeling of P_7 with $n \equiv 3\ (mod 4)$ is shown in fig 3

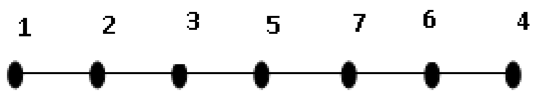

Fig 3

The difference mean cordial labeling of P_5 with $n \equiv 1 (mod 4)$ is shown in fig 4

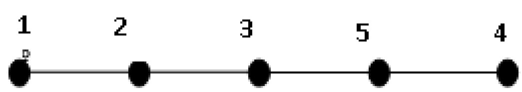

Fig 4

Theorem 3.2: Star $K_{1,n}$ is a difference mean cordial graph.

Proof : Let G be a graph.

Let V(G) = { $u_i : 1 \leq i \leq n$ }

Let E(G) = { uu_i / $1 \leq i \leq n$ }

Define f : V(G) $\rightarrow$ {1,2,….,n}

The vertex labels are,

$f(u) = 1$

$f(u_i) = i+1 \quad ; 1 \leq i \leq n$

The induced edge labels are,

$$f^*(uu_i) = \begin{cases} 1 & if\ f(u) - f(u_i) \equiv 0\ (mod\ 2) \\ 0 & else \end{cases}$$

Here, $e_f(0) = \frac{n}{2} = e_f(1)$,When n is even

$e_f(0) = \frac{n+1}{2}$ and $e_f(1) = \frac{n-1}{2}$,when n is odd

Therefore, it satisfies the condition $|e_f(0) - e_f(1)| \leq 1$

Clearly, the graph $Gl(n)$ is a difference mean cordial graph.

Example: The difference mean cordial labeling of the graph $K_{1,6}$ and $K_{1,7}$ are shown in figures: 5 &6.

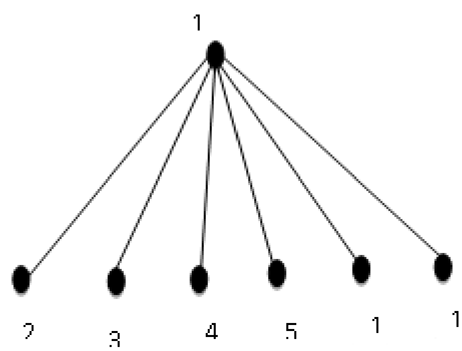

Fig 5

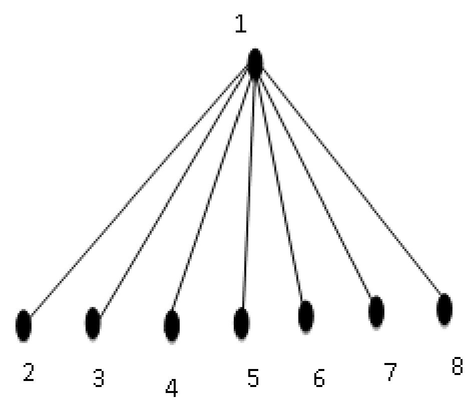

Fig 6

Theorem 3.3: Comb $P_n \odot k_1$ is a difference mean cordial graph .

Proof : Let G be a graph.

Let V(G) = { $u_1, u_2 ... u_n, v_1, v_2, ... v_n$ }

Let E(G) = { $u_i u_{i+1}$ / $1 \leq i \leq n-1$ } $\cup$ { $u_i v_i$ / $1 \leq i \leq n$ }

Define f :V(G) $\rightarrow$ {1,2,…,n}

The vertex labels are,

$f(u_i) = 2i-1;\ 1 \le i \le n$
$f(v_i) = 2i \quad ;\ 1 \le i \le n$

The induced edge labels are,

$$f^*(u_i u_{i+1}) = \begin{cases} 1 & if\ f(u_i) - f(u_{i+1}) \equiv 0\ (mod\ 2) \\ 0 & else \end{cases}$$
$$f^*(u_i v_i) = \begin{cases} 1 & if\ f(u_i) - f(v_i) \equiv 0\ (mod\ 2) \\ 0 & else \end{cases}$$

Here, $e_f(0) = n$, When n is even , odd
$e_f(1) = n - 1$, When n is even , odd

Therefore, it satisfies the condition $|e_f(0) - e_f(1)| \le 1$

Clearly, the graph $P_n \odot k_1$ is a difference mean cordial graph.

Example:

The difference mean cordial labeling of the graph $P_5 \odot k_1$ and $P_6 \odot k_1$ are shown in figures: 7&8.

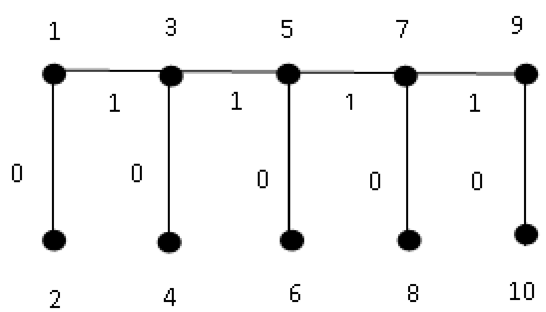

Fig: 7

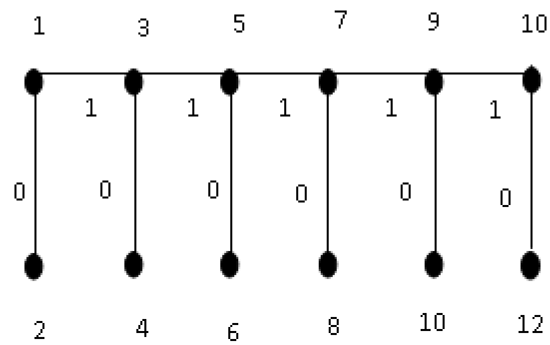

Fig: 8

Theorem 3.4: Broken Comb Br(n) is a difference mean cordial graph.

Proof : Let G be a graph.

Let V(G) = $\{u_1, u_2, \dots, u_n, v_1, \dots, v_{n-2}\}$
E(G) = $\{u_i u_{i+1} / 1 \le i \le n-1\} \cup \{u_{i+1} v_i / 1 \le i \le n-2\}$

Define f : V(G) $\rightarrow$ {1,2,….,n}

The vertex labels are,

$f(u_{i+1}) = 2i+1 \quad ;\ 1 \le i \le n-2$
$f(v_i) = 2i \quad ;\ 1 \le i \le n-2$
$f(u_1) = 1$
$f(u_n) = 2n-2$

The induced edge labels are,

$$f^*(u_i u_{i+1}) = \begin{cases} 1 & if\ f(u_i) - f(u_{i+1}) \equiv 0\ (mod\ 2) \\ 0 & else \end{cases}$$
$$f^*(u_{i+1} v_i) = \begin{cases} 1 & if\ f(u_{i+1}) - f(v_i) \equiv 0\ (mod\ 2) \\ 0 & else \end{cases}$$

Here, $e_f(0) = n - 1,$

When n is even, odd

$e_f(1) = n - 2$,

When n is even, odd

Therefore, it satisfies the condition $|e_f(0) - e_f(1)| \le 1$

Clearly, the graph Br(n) is a difference mean cordial graph.

Example: The difference mean cordial labeling of the graph Br(7) and Br(6) are shown in figures: 9 & 10.

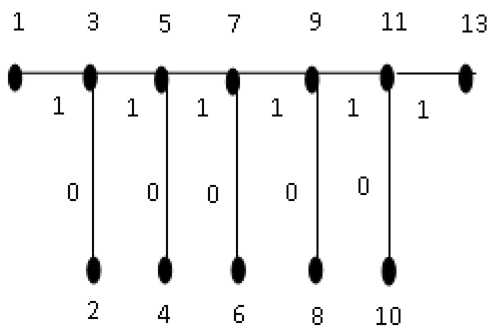

Fig: 9

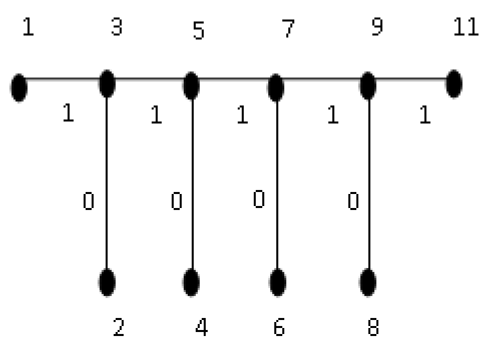

Fig: 10

Theorem 3.5: Globe $Gl(n)$ is a difference mean cordial graph.

Proof : Let G be a graph.

Let V(G) = { $u, v, u_i : 1 \le i \le n$ }

E(G) = {uu_i / $1 \le i \le n$} $\cup$ {vu_i / $1 \le i \le n$ }

Define f : V(G) $\rightarrow$ {1,2,….,n}

The vertex labels are,

$f(u) = 1$

$f(v) = 2$

$f(u_i) = i+2 \quad ; 1 \le i \le n$

The induced edge labels are,

$$f^*(uu_i) = \begin{cases} 1 & if\ f(u) - f(u_i) \equiv 0\ (mod\ 2) \\ 0 & else \end{cases}$$

$$f^*(vu_i) = \begin{cases} 1 & if\ f(v) - f(u_i) \equiv 0\ (mod\ 2) \\ 0 & else \end{cases}$$

Here, $e_f(0) = n = e_f(1)$, When n is even , odd

Therefore, it satisfies the condition $|e_f(0) - e_f(1)| \le 1$

Clearly, the graph $Gl(n)$ is a difference mean cordial graph.

Example: The difference mean cordial labeling of the graph $Gl(6)$ and $Gl(7)$ are shown in figures: 11 & 12.

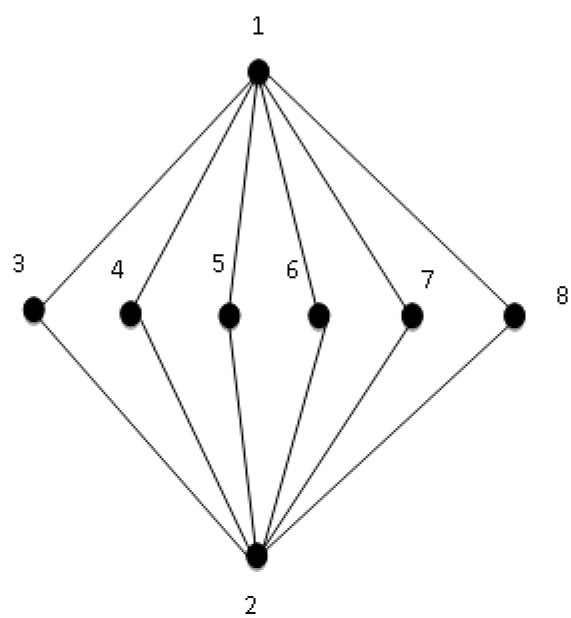

Fig: 11

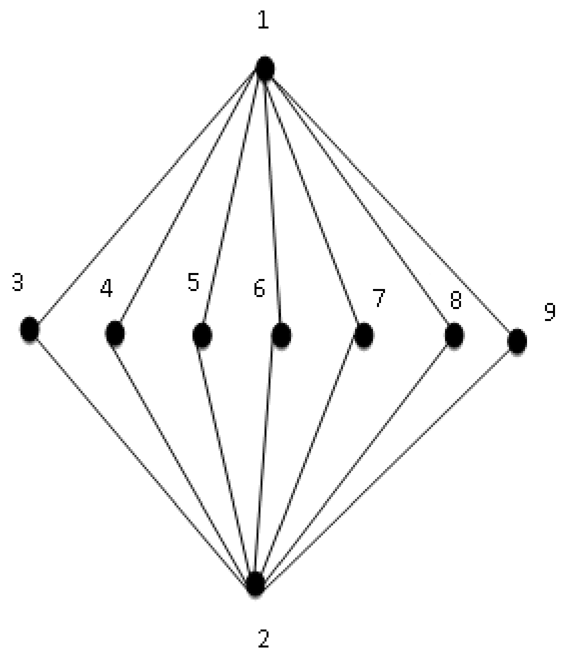

Fig: 12

REFERENCES

[1] I. Cahit, Cordial graphs: a weaker version of graceful and harmonious graphs, Ars combinatorial 23 (1987) , 201-207.

[2] J. A. Gallian, A Dynamic survey of graph labeling, the electronic journal of combinatorics 18 (2013) #Ds6.

[3] F. Harary, Graph theory, Addision Wesley, New Delhi (1969).