Near Difference Cordial Labeling Of Some Graphs

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Abstract- *Let G be a (p,q) graph. Let f be a map from $V(G)$ to $\{1,2,\dots,p-1,\ p+1\}$. For each edge uv, assign the label $|f(u)-f(v)|$. f is called "near difference cordiallabeling", if f is 1-1 and $|e_f(0)-e_f(1)|\le 1$ where $e_f(1)$ and $e_f(0)$ denote the number of edges labeled with 1 and not labeled with 1 respectively. A graph with a near difference cordial labeling is called a "near difference cordial graph".*

Keywords- Near difference cordial graph, Near difference cordial labeling.

I. INTRODUCTION

A graph G is a finite nonempty set of objects called vertices together with a set of unordered pairs of distinct vertices of G called edges. Each pair e={u,v} of vertices in E is called edges or a line of G. In this paper, we proved that the graphs Path (P_n) , Fan (F_n) ,H-graph (H_n) , $C_n{}^+$, Ladder L_n are Near difference cordial graphs.

II. PRELIMINARIES

Let G be a (p,q) graph. Let f be a map from $V(G)$ to $\{1,2,\dots,p-1,\ p+1\}$. For each edge uv, assign the label $|f(u)-f(v)|$. f is called "near difference cordial labeling", if f is 1-1 and $|e_f(0)-e_f(1)|\le 1$ where $e_f(1)$ and $e_f(0)$ denote the number of edges labeled with 1 and not labeled with 1 respectively. A graph with a near difference cordial labeling is called a "near difference cordial graph". In this paper, we proved that the graphs path (P_n), Fan (F_n), H-graph (H_n), $C_n{}^+$, Ladder L_n are Near difference cordial graphs.

DEFINITION 2.1:

Path is a graph whose vertices can be listed in the order ($u_1, u_2, \dots . u_n$) such that the edges are $\{u_i u_{i+1}\}$ where i=1,2,3…..n-1.

DEFINITION 2.2:

The join of G_1 and G_2 is the graph $G = G_1 + G_2$ with vertex set $V = V_1 \cup V_2$ and edge set $E = E_1 \cup E_2 \cup \{uv: u \in V_1, v \in V_2\}$.The graph $P_n + K_1$ is called a Fan.

DEFINITION 2.3:

H-graph H_n is a graph obtained from two copies of path P_n with vertices $(u_1, u_2, \dots . u_n)$ and $(v_1, v, \dots . v_n)$ by joining the vertices $u_{\frac{n+1}{2}}$ and $v_{\frac{n+1}{2}}$ if n is odd and $u_{\frac{n}{2}}$ and $v_{\frac{n}{2}+1}$ if n is even.

DEFINITION 2.4:

$C_n{}^+$ is a graph obtained from cycle of length n by attaching a pendant vertex from each vertex of the cycle.

DEFINITION 2.4:

The ladder graph L_n is a planar undirected graph with 2n vertices and 3n-2 edges. The ladder graph can be obtained as the Cartesian product of two path graphs,one of which has only one edge: $L_{n,1} = P_n \times P_2$

III. MAIN RESULT

Theorem 3.1: P_n is a near difference cordial graph

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Proof: Let G be P_n.

Let $V(G) = \{u_i : 1 \le i \le n\}$ and $E(G) = \{u_i u_{i+1} : 1 \le i \le n-1\}$.

Define,

$f: V(G) \to \{1,2,\dots,p-1,p+1\}$

Case 1: When n is odd

Subcase 1: $n \equiv 1 \ (mod\ 4)$

Define $f(u_i) = i, \quad 1 \le i \le \frac{n+1}{2}$

$$f\left(u_{\frac{n+1}{2}+i}\right) = \frac{n+1}{2} + 2i, \ 1 \le i \le \frac{n-5}{4}$$

$$f(u_{n+1-i}) = \frac{n-1}{2} + 2i, \ 1 \le i \le \frac{n+3}{4}$$

Subcase 2: $n \equiv 3 \ (mod\ 4)$

Define $f(u_i) = i, \quad 1 \le i \le \frac{n+1}{2}$

$$f\left(u_{\frac{n+1}{2}+i}\right) = \frac{n+1}{2} + 2i, \ 1 \le i \le \frac{n+1}{4}$$

$$f(u_{n+1-i}) = \frac{n-1}{2} + 2i, \ 1 \le i \le \frac{n-3}{4}$$

The edge uv, assign the label $|f(u) - f(v)|$.

Here, $e_f(0) = \frac{n-1}{2}$, $e_f(1) = \frac{n-1}{2}$.

Therefore, the graph G satisfies the conditions $|e_f(0) - e_f(1)| \le 1$.

Hence P_n is a near difference cordial graph.

Case 2: When n is even

Subcase 1: $n \equiv 0 \ (mod\ 4)$

Define $f(u_i) = i, \quad 1 \le i \le \frac{n+2}{2}$

$$f\left(u_{\frac{n+2}{2}+i}\right) = \frac{n+2}{2} + 2i, \ 1 \le i \le \frac{n}{4}$$

$$f(u_{n+1-i}) = \frac{n}{2} + 2i, \quad 1 \le i \le \frac{n-4}{4}$$

Subcase 2: $n \equiv 2 \ (mod\ 4)$

Define $f(u_i) = i, \quad 1 \le i \le \frac{n+2}{2}$

$$f\left(u_{\frac{n+2}{2}+i}\right) = \frac{n+2}{2} + 2i, \ 1 \le i \le \frac{n-6}{4}$$

$$f(u_{n+1-i}) = \frac{n}{2} + 2i, \quad 1 \le i \le \frac{n+2}{4}$$

The edge uv, assign the label $|f(u) - f(v)|$.

Here, $e_f(0) = \frac{n-2}{2}$ and $e_f(1) = \frac{n}{2}$.

Therefore, the graph G satisfies the conditions $|e_f(0) - e_f(1)| \le 1$.

Hence P_n is a near difference cordial graph.

Example: When n is odd

The near difference cordial labeling of P_5 with $n \equiv 1 \ (mod\ 4)$ as shown in Figure 1

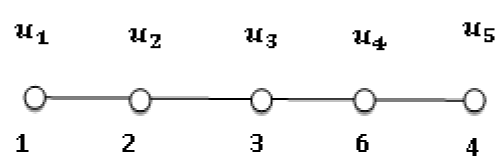

Fig.1

The near difference cordial labeling of P_7 with $n \equiv 3 \ (mod\ 4)$ as shown in Figure 2

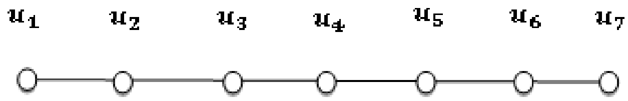

Fig.2

When n is even

The near difference cordial labeling of P_4 with $n \equiv 0\ (mod\ 4)$ as shown in Figure 3

Fig.3

The near difference cordial labeling of P_6 with $n \equiv 2\ (mod\ 4)$ as shown in Figure 4

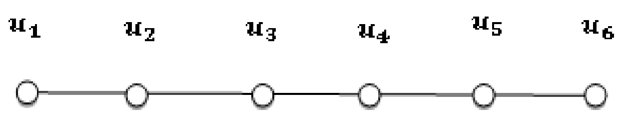

Fig.4

Theorem 3.2: The Fan $F_n$is near difference cordial graph for all n.

Proof: Let $F_n = P_n + k_1$

Where, P_n is the path $u_1, u_2, \dots, u_n$ and $V(k_1) = \{u\}$.

Define,

$$f: V(G) \to \{1,2,\dots,p-1,p+1\}$$

Case 1: When n is odd

The vertex labels are,

$$f(u) = 1$$
$$f(u_i) = i+1\ ,1 \le i \le n-1$$
$$f(u_n) = n+2$$

The induced edge labels are,

$$f^*(uu_i) = i\ ,1 \le i \le n-1$$
$$f^*(u_i u_{i+1}) = 1\ ,1 \le i \le n-2$$
$$f^*(u_{n-1}u_n) = 2$$
$$f^*(uu_n) = n+1$$

Here, $e_f(0) = n$ and $e_f(1) = n-1.$

Therefore, the graph F_n satisfies the conditions $|e_f(0) - e_f(1)| \le 1.$

Hence F_n is near difference cordial graph.

Case 2: When n is even

The vertex labels are,

$$f(u) = 1$$
$$f(u_i) = i+1\ , \qquad 1 \le i \le n-1$$
$$f(u_n) = n+2$$

The induced edge labels are,

$$f^*(uu_i) = i\ ,1 \le i \le n-1$$
$$f^*(u_i u_{i+1}) = 1\ ,1 \le i \le \frac{n+2}{2}$$
$$f^*(u_{n-1}u_n) = 2$$
$$f^*(uu_n) = n+1$$

Here, $e_f(0) = n$ and $e_f(1) = n-1$

Therefore, the graph F_n satisfies the conditions $|e_f(0) - e_f(1)| \le 1.$

Hence, F_n is near difference cordial graph.

Example: The near difference cordial labeling of F_7 and F_6 are shown in Figure 5 &6

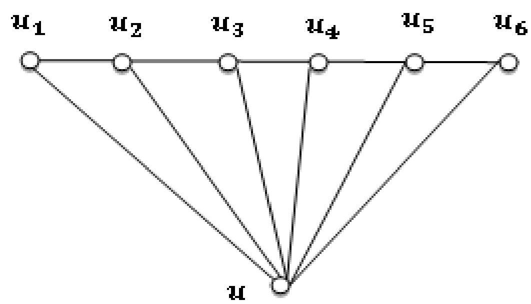

Fig.5

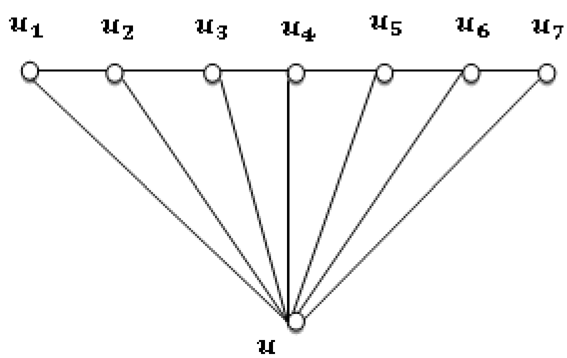

Fig.6

Theorem 3.3: The H-graph G is a near difference cordial graph.

Proof: Let $v_1, v_2, \dots, v_n$ and $u_1, u_2, \dots, u_n$ be the vertices of the graph G.

We define a labeling,

$$f: V(G) \to \{1,2,\dots,p-1,p+1\}$$

Case 1: When n is odd

The vertex labels are,

$$f(u_i) = i, 1 \le i \le n$$
$$f(u_i) = n + 2i, 1 \le i \le \frac{n+1}{2}$$
$$f\left(u_{\frac{n+1}{2}+i}\right) = 2n - 2i, 1 \le i \le \frac{n-1}{2}$$

The induced edge labels are,

$$f^*(v_i v_{i+1}) = 1, 1 \le i \le n-1$$
$$f^*(u_i u_{i+1}) = 2, 1 \le i \le \frac{n-1}{2},$$
$$\frac{n+3}{2} \le i \le n-1 f^*\left(u_{\frac{n+1}{2}} u_{\frac{n+3}{2}}\right) = 3,$$
$$f^*\left(v_{\frac{n+1}{2}} u_{\frac{n+1}{2}}\right) = 2n - i, \quad i = \frac{n-1}{2}$$

Here, $e_f(0) = n$ and $e_f(1) = n-1.$

Therefore, the graph G satisfies the conditions $|e_f(0) - e_f(1)| \le 1.$

Hence H-graph G is a near difference cordial graph.

Case 2: When n is even

The vertex labels are,

$$f(v_i) = i, 1 \le i \le n$$
$$f(u_i) = n + 2i - 1, 1 \le i \le \frac{n+2}{2}$$
$$f\left(u_{\frac{n+2}{2}+i}\right) = 2n - 2i, 1 \le i \le \frac{n-2}{2}$$

The induced edge labels are,

$$f^*(v_i v_{i+1}) = 1, \ 1 \le i \le n-1$$
$$f^*(u_i u_{i+1}) = 2, \ 1 \le i \le \frac{n}{2},$$
$$\frac{n+4}{2} \le i \le n-1$$
$$f^*\left(u_{\frac{n+2}{2}} u_{\frac{n+4}{2}}\right) = 3$$
$$f^*\left(v_{\frac{n}{2}+1} u_{\frac{n}{2}}\right) = n - 1 + i, \qquad i = \frac{n-2}{2}$$

Here, $e_f(0) = n$ and $e_f(1) = n - 1$

Therefore, the graph G satisfies the conditions $|e_f(0) - e_f(1)| \leq 1$.

Hence, H-graph G is near difference cordial graph.

Example: The near difference cordial labeling of H_5 & H_6 are shown in Figure 7 &8

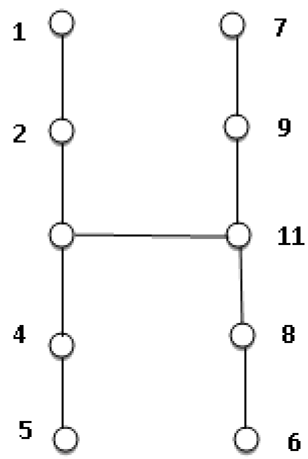

Fig.7

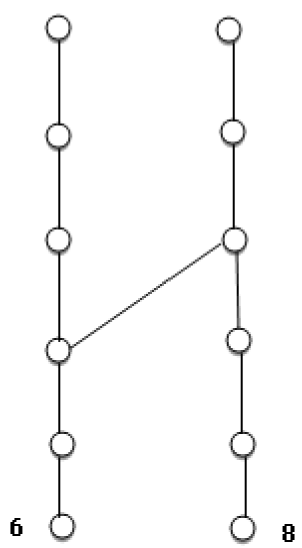

Fig.8

Theorem 3.4: C_n^+ is a near difference cordial graph for all n.

Proof: Let G be a graph.

Let

$$V(G) = \{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n\}$$
$$E(G) \rightarrow \{u_i u_{i+1} \mid 1 \leq i \leq n - 1\} \cup \{u_i v_{n-i+1} \mid 1 \leq i \leq n\} \cup \{u_n u_1\}$$

.

Define,

$$f: V(G) \rightarrow \{1, 2, \dots, p-1, p+1\}$$

When n is odd, even

The vertex labels are,

$$f(u_i) = i, \quad 1 \leq i \leq n$$
$$f(v_i) = n + i, \quad 1 \leq i \leq n - 1$$
$$f(v_n) = 2n + 1$$

The induced edge labels are,

$$f^*(u_i u_{i+1}) = 1, 1 \leq i \leq n - 1 f^*(v_{i+1} u_{n-i}) = 2i + 1 \leq i \leq n - 2 f^*(u_n u_1) = n - 1$$
$$f^*(v_1 u_n) = 1$$
$$f^*(v_n u_1) = 2n$$

Here, $e_f(0) = n$ and $e_f(1) = n$.

Therefore, the graph C_n^+ satisfies the conditions $|e_f(0) - e_f(1)| \leq 1$.

Hence C_n^+ is near difference cordial graph.

Example: The near difference cordial labeling of C_7^+ & C_6^+ are shown in Figure 9 & 10.

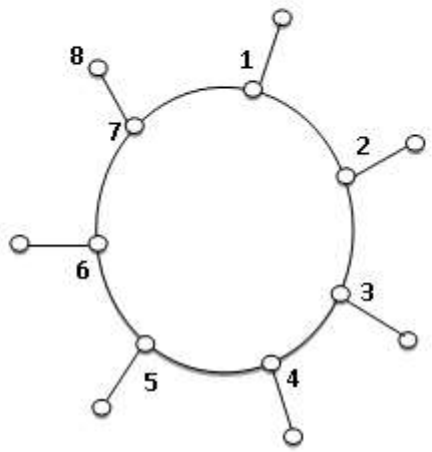

Fig.9

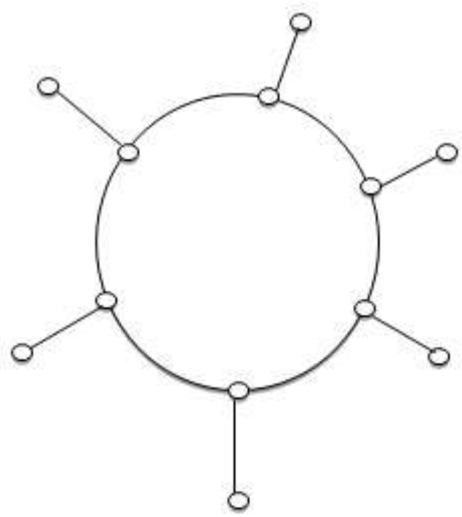

Fig.10

Theorem 3.5: Ladder L_n is a near difference cordial graph.

Proof: Let G be a graph.

Let

$$V(G) = \{u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n\}$$
$$E(G) \to \{u_i u_{i+1} \mid 1 \le i \le n-1\} \cup \{v_i v_{i+1} \mid 1 \le i \le n-1\} \cup \{u_i v_i \mid 2 \le i \le n-1\}$$

Define,

$$f: V(G) \to \{1, 2, \ldots, p-1, p+1\}$$

Case 1: When n is odd

Define $f(u_i) = i, \ 1 \le i \le n$

$f(v_i) = n + i, \quad 1 \le i \le \frac{n-1}{2} f(v_{n-3}) = 2n-2$

$$f(v_{n-i+1}) = \frac{3n-3}{2} + 2i, 1 \le i \le \frac{n-1}{2}$$

The edge uv, assign the label $|f(u) - f(v)|$.

Here, $e_f(0) = \frac{3n-3}{2}$ and $e_f(1) = \frac{3n-5}{2}$.

Therefore the graph L_n satisfies the conditions $|e_f(0) - e_f(1)| \le 1$.

Hence L_n is near difference cordial graph.

Case 2: When n is even

Define $f(u_i) = i, \ 1 \le i \le n$

$f(v_i) = n + i, 1 \le i \le \frac{n}{2}$

$f(v_{n-3}) = 2n - 2$

$f(v_{n-i+1}) = \frac{3n}{2} + 2i - 1,$

$1 \le i \le \frac{n-2}{2}$

The edge uv, assign the label $|f(u) - f(v)|$.

Here, $e_f(0) = \frac{3n-4}{2}$ and $e_f(1) = \frac{3n-4}{2}$.

Therefore the graph L_n satisfies the conditions $|e_f(0) - e_f(1)| \le 1$.

Hence L_n is near difference cordial graph.

Example: The near difference cordial labeling of L_7 & L_8 are shown in Figure 11 &12.

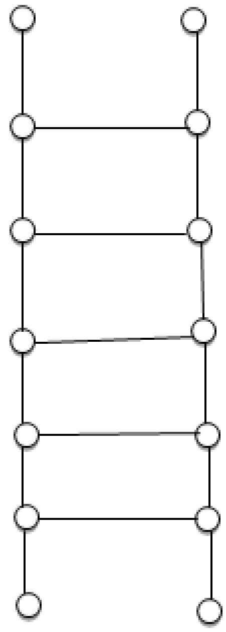

Fig.11

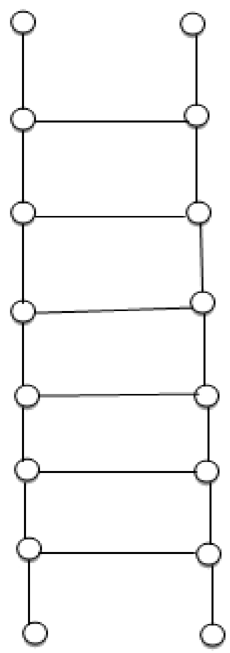

Fig.12

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