Near Skolem Mean Labeling For Some Path Related Graphs

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Abstract- *A graph G = (V,E) with p vertices and q edges is said to be a Near Skolem Mean graph if there exists a injective map f from the vertex set of G to{1,2,……p-1,p+1} such that the induced map* f^* *from the edge set of G to {2,3,……p} defined by ,*

$$f^*(e = uv) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \\ \frac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd} \end{cases} \text{, then}$$

Then the resulting edges get distinct labels from the set {2,3…….p}. A graph with Near skolem mean labeling is called a Near skolem mean graph . In this paper, we proved that the graphs path (P_n*), Comb (*P_n^+*),Broken Comb Br(n),H-graph (*H_n*),TwigT(n) are Near Skolem mean graphs.*

Keywords- Near Skolem mean graph, Near Skolem mean labeling.

I. INTRODUCTION

A graph G is a finite nonempty set of objects called vertices together with a set of unordered pairs of distinct vertices of G called edges. Each pair e={u,v} of vertices in E is called edges or a line of G. In this paper, we proved that the graphs Path (P_n), Comb(P_n^+),Broken Comb Br(n),H-graph (H_n),TwigT(n) are Near Skolem mean graphs.

II. PRELIMINARIES

A graph G = (V,E) with p vertices and q edges is said to be a Near Skolem Mean graph if there exists a injective map f from the vertex set of G to{1,2,……p-1,p+1} such that the induced map f^* from the edge set of G to {2,3,……p} defined by ,

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Then the resulting edges get distinct labels from the set {2,3…….p}. A graph with Near skolem mean labeling is called a Near skolem mean graph . In this paper, we proved that the graphs path (P_n), Comb (P_n^+),Broken Comb Br(n),H-graph (H_n),TwigT(n) are Near Skolem mean graphs.

DEFINITION 2.1:

Path is a graph whose vertices can be listed in the order ($u_1, u_2, \ldots . u_n$) such that the edges are {$u_i u_{i+1}$} where i=1,2,3…..n-1.

DEFINITION 2.2:

P_n^+ is a graph obtained from path of length n by attaching a pendant vertex from each vertex of the path.

DEFINITION 2.3:

H-graph H_n is a graph obtained from two copies of path P_n with vertices $(u_1, u_2, \ldots . u_n)$ and $(v_1, v, \ldots . v_n)$ by joining the vertices $\frac{u_{n+1}}{2}$ and $\frac{v_{n+1}}{2}$ if n is odd and $\frac{u_n}{2}$ and $\frac{v_n}{2}$ +1 if n is even.

DEFINITION 2.4:

Broken comb is a graph obtained from the comb by removing the end pendant vertices.

DEFINITION 2.5:

The graph obtained from a path by attaching exactly two pendent edges to each internal vertices of the path is called Twig and it is denoted by T(n).

III. MAIN RESULT

Theorem 3.1: Path P_n is a near skolem mean graph.

Proof: Let G be a graph.

Let $u_1, u_2, \ldots, u_n$ be the vertices of the path P_n with the length n-1.

Let $u_1u_2, u_2u_3 \ldots u_{n-1}u_n$ be the edges of the path P_n.

Define f : V(G) $\rightarrow$ {1,2,…p-1,p+1}

The vertex labels are,

$f(u_i) = i \quad ; 1 \le i \le n-1$

$f(u_n) = n+1$

The induced edge labels are,

$f^*(u_iu_{i+1}) = i+1 \quad ; 1 \le i \le n-1$

Clearly, the graph P_n is a near skolem mean graph**Example :**

The near skolem mean labelling of the graph P_7 & P_6 are shown in figures: 1 & 2

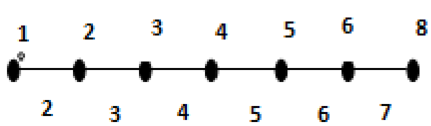

Fig: 1

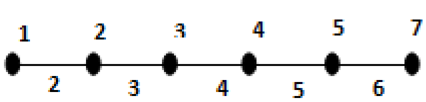

Fig: 2

Theorem 3.2: Comb $P_n \odot k_1$ is a near skolem mean graph .

Proof : Let G be a graph.

Let V(G) = { $u_1, u_2 \ldots u_n, v_1, v_2, \ldots v_n$ }

Let E(G) = $\{u_iu_{i+1} / 1 \le i \le n-1\} \cup \{u_iv_i / 1 \le i \le n\}$

Define f : V(G) $\rightarrow$ {1,2,…p-1,p+1}The vertex labels are,

$f(u_i) = 2i \quad ; 1 \le i \le n-1$

$f(u_n) = 2n-1$

$f(v_i) = 2i-1 \quad ; 1 \le i \le n-1$

$f(v_n) = 2n+1$

The induced edge labels are,

$f^*(u_iu_{i+1}) = 2i+1 \quad ; 1 \le i \le n-1$

$f^*(u_iv_i) = 2i \quad ; 1 \le i \le n$

Clearly, the graph $P_n \odot k_1$ is a near skolem mean graph.

Example :

The Near skolem mean labeling of the graph $P_5 \odot k_1$ and $P_6 \odot k_1$ are shown in figures: 3&4.

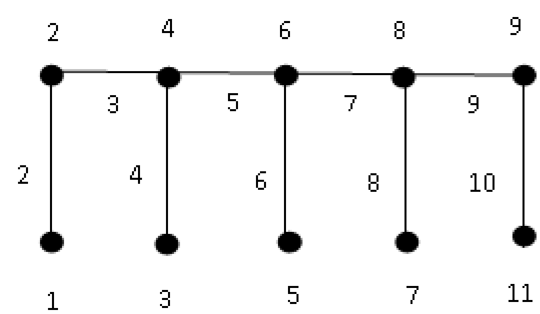

Fig: 3

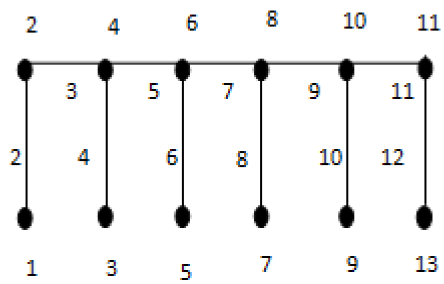

Fig: 4

Theorem 3.3: Broken Comb Br(n) is a near skolem mean graph.

Proof : Let G be a graph.

Let V(G) = { $u_1, u_2, \ldots u_n, v_1, v_2, \ldots v_{n-2}$ }

Let E(G) = {$u_i u_{i+1}$ / $1 \le i \le n-1$} $\cup$ {$u_{i+1} v_i$ / $1 \le i \le n-2$ }

Define f : V(G) $\rightarrow$ {1,2,……p-1,p+1}

The vertex labels are,

$f(u_i) = 2i-1$; $1 \le i \le n$

$f(v_i) = 2i$; $1 \le i \le n-2$

The induced edge labels are,

$f^*(u_i u_{i+1}) = 2i$; $1 \le i \le n-1$

$f^*(u_{i+1} v_i) = 2i+1$; $1 \le i \le n-2$

Clearly, the graph Br(n) is a near skolem mean graph.

Example : The Near skolem mean labeling of the graph Br(7) and Br(6) are shown in figures: 5 & 6.

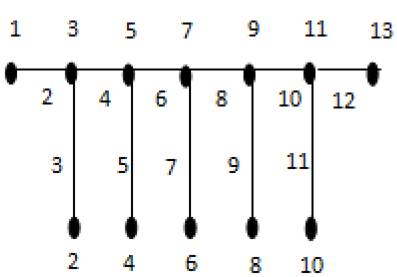

Fig: 5

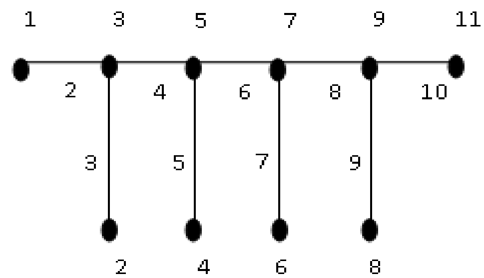

Fig: 6

Theorem 3.4: The H-graph H_n is a near skolem mean graph.

Proof: Let G be a graph.

When n is odd, n = 2k+1 and when n is even , n = 2k.

Let V(G) = { $u_1, u_2, \ldots\ldots u_n, v_1, v_2, \ldots\ldots v_n$ }

Let E(G) = {$u_i u_{i+1}$ / $1 \le i \le n-1$} $\cup$ {$v_i v_{i+1}$ / $1 \le i \le n-1$ } $\cup$ {$u_{k+1} v_{k+1}$} $\cup$ {$u_k v_{k+1}$}

Define f : V(G) $\rightarrow$ {1,2,……p-1,p+1}

The vertex labels are,

$f(v_i) = i$; $1 \le i \le n$

$f(u_i) = n+i$; $1 \le i \le n-1$

$f(u_n) = 2n+1$

The induced edge labels are,

$f^*(v_i v_{i+1}) = i+1$; $1 \le i \le n-1$

$f^*(u_{i-1} u_i) = n+i$; $2 \le i \le n$

When n is odd,

$f^*(u_{\frac{n+1}{2}} v_{\frac{n+1}{2}}) = n+1$

When n is even,

$f^*(v_{\frac{n}{2}+1} u_{\frac{n}{2}}) = n+1$

Clearly, the graph H_n is a near skolem mean graph.

Example:

The Near skolem mean labeling of the graph H_7 and H_6 are shown in figures: 7&8.

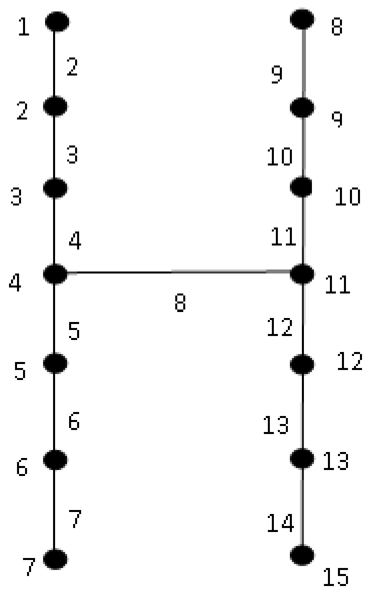

Fig: 7

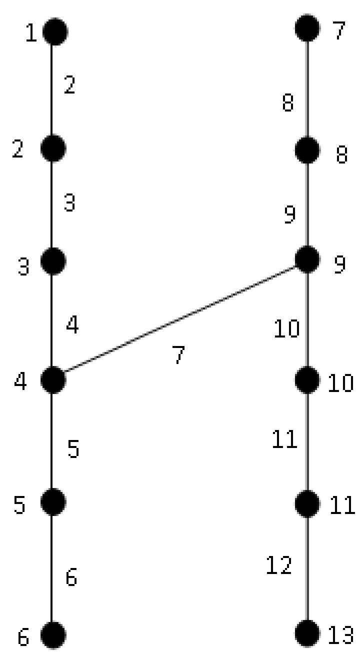

Fig: 8

Theorem 3.5: Twig graph T(n) is a near skolem mean graph.

Proof: Let G be a graph.

When n is odd, n = 2k+1 and when n is even , n = 2k.

Let
V(G) =
$\{u_1, u_2, \ldots u_n, v_1, v_2, \ldots v_{n-2}, w_1, w_2, \ldots w_{n-2}\}$

Let E(G) = $\{u_i u_{i+1} / 1 \le i \le n-1\} \cup \{v_i u_{i+1} / 1 \le i \le n-2\} \cup \{u_{i+1} w_i / 1 \le i \le n-2\}$

Define f : V(G) $\rightarrow$ {1,2,……p-1,p+1}

The vertex labels are,

$f(u_1) = 1$

$f(u_{i+1}) = 3i \quad ; 1 \le i \le n-1$

$f(v_i) = 3i+1 \quad ; 1 \le i \le n-2$

$f(w_i) = 3i-1 \quad ; 1 \le i \le n-2$

The induced edge labels are,

$f^*(u_1 u_2) = 2$

$f^*(u_{i+1} u_{i+2}) = 3i+2 \quad ; 1 \le i \le n-3$

$f^*(u_{n-1} u_n) = 3n-4$

$f^*(u_{i+1} v_i) = 3i+1 \quad ; 1 \le i \le n-2$

$f^*(u_2 w_1) = 3$

$f^*(u_{i+2} w_{i+1}) = 3i+3 \quad ; 1 \le i \le n-4$

$f^*(u_{n-1} w_{n-2}) = 3n-6$

Clearly, the graph Twig is a near skolem mean graph.

Example:

The Near skolem mean labeling of graph T(7) and T(6) are shown in figures: 9&10.

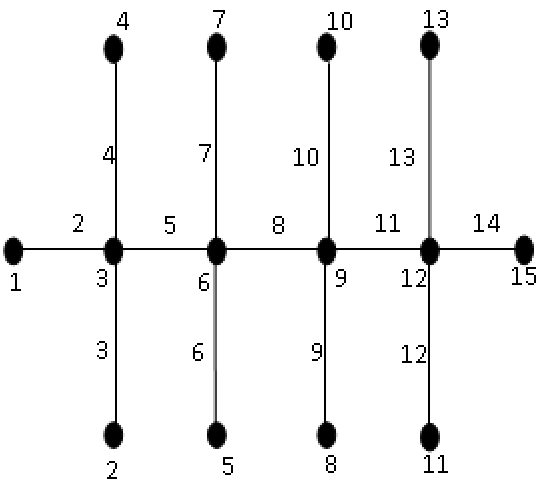

Fig: 9

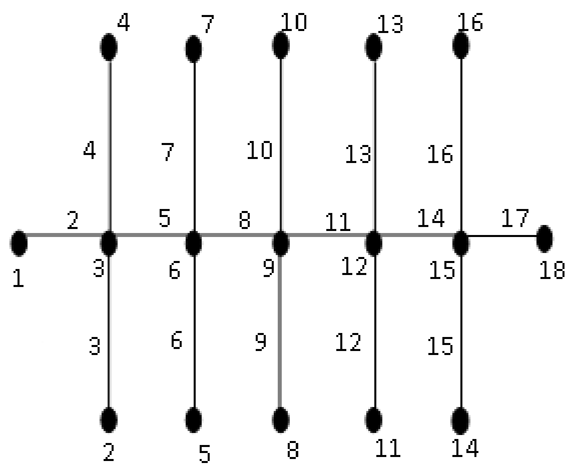

Fig: 10

REFERENCES

[1] V. Balaji, D.S.T. Ramesh and Subramanian, Near Skolem mean Labeling, Bulletin of Pure and Applied Sciences, Vol.26E No. 2(2007), 245-248.

[2] V. Balaji, D.S.T. Ramesh and Subramanian, Some Results on Near Skolem Mean Graphs, Bulletin of Pure and Applied Sciences, Vol. 27E No.1(2008),67-74.

[3] J.A. Gallian, A Dynamic Survey of Graph Labeling, The Electronic Journal of Combinatories 14(2007), #DS6.

[4] F.Harry, Graph theory, Addison-Wesley, Reading Mars, (1972).