Some Graphs on 3-Modulo Difference Cordial Labelling

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Abstract- *Let G = (V,E) be a simple graph with p vertices and q edges. G is said to have 3 – modulo difference cordial labeling if there is a injective map* $f: V(G) \rightarrow \{0,1,2,3,\dots\dots\dots,3p\}$ *such that for every edge* uv*,the induced labeling* f^* *isdefined asf*(uv) = 1 if* $|f(u) - f(v)| \equiv$ *0 (mod 3) and 0 elsewhere with the condition that* $|e_f(0) - e_f(1)| \leq 1$*,where* $e_f(0)$ *is the number of edges with label 0 and* $e_f(1)$ *is the number of edges with label 1.If G admits 3-modulo difference cordial labeling then G is a 3-modulo difference cordial graph. In this paper, we proved that the graphs Path(*P_n*), Comb(*P_n^+*)* , $Cycle(C_n)$, C_n^+ *are 3-modulo difference cordial graphs.*

Keywords- 3-modulo difference cordial graph,3-modulo difference cordial labeling

I. INTRODUCTION

A graph G is a finite nonempty set of objects called vertices together with a set of unordered pairs of distinct vertices of G called edges.Each pair e={u,v} of vertices in E is called edges or a line of G.In this paper,we proved that the graphs Path(P_n),Comb(P_n^+) , Cycle(C_n), C_n^+ are 3-modulo difference cordial graphs.For graph theoretic terminology we follow [2].

II. PRELIMINARIES

Let G = (V,E) be a simple graph with p vertices and q edges. G is said to have 3 – modulo difference cordial labeling if there is a injective map $f: V(G) \rightarrow \{0,1,2,3,\dots\dots\dots,3p\}$ such that for every edge uv, the induced labeling f^* is defined as f*(uv) = 1 if $|f(u) - f(v)| \equiv$ 0 (mod 3) and 0 elsewhere with the condition that $|e_f(0) - e_f(1)| \leq 1$,where $e_f(0)$ is the number of edges with label 0 and $e_f(1)$ is the number of edges with label 1.If G admits 3-modulo difference cordial labeling then G is a 3-modulo difference cordial graph. In this paper, we proved that the graphs Path(P_n),Comb(P_n^+) , Cycle(C_n), C_n^+ are 3-modulo difference cordial graphs.

DEFINITION 2.1:

Path is a graph whose vertices can be listed in the order ($u_1, u_2, u_3, \dots\dots u_n$) such that the edges are $\{u_i, u_{i+1}\}$where $i = 1,2,3,\dots n-1$.

DEFINITION 2.2:

P_n^+ is a graph obtained from path of length n by attaching a pendant vertex from each vertex of the path.

DEFINITION 2.3:

A closed path is called a cycle and a cycle of length n is denoted by C_n.

DEFINITION 2.4:

C_n^+ is a graph obtained from cycle of length n by attaching a pendant vertex from each vertex of the cycle

III. MAIN RESULT

Theorem 3.1:

Path (P_n) is a 3-modulo difference cordial graph.

Proof:

Let G be a graph

When n is odd , n = 2k + 1 and when n is even , n = 2k.

Let V(G) = { $u_1, u_2, u_3, \ldots \ldots u_n\}$

E(G) = { $u_i u_{i+1} / 1 \le i \le n-1\}$

Then $|V(G)| = n$ and
$|E(G)| = n-1$
Define $f: V(G) \rightarrow \{0,1,2,3, \ldots \ldots \ldots, 3n\}$

Case (i): n is even

Subcase (i): k is not a multiple of 3.

The vertex labels are,

$$f(u_i) = \begin{cases} 2i & 1 \le i \le k \\ 3i & k+1 \le i \le n \end{cases}$$

The induced edge labels are,

For $1 \le i \le k-1$
$f^*(u_i u_{i+1}) = 2$

For $k+1 \le i \le n-1$
$f^*(u_i u_{i+1}) = 3 \equiv 0 (mod\ 3)$
$f^*(u_k u_{k+1}) = k+3 \not\equiv 0 (mod\ 3)$

It is observed as

$e_f(0) = k$
$e_f(1) = k-1$

Subcase (ii): k is a multiple of 3.

The vertex labels are,

$$f(u_i) = \begin{cases} 2i & 1 \le i \le k \\ 3i & k+1 \le i \le n \end{cases}$$

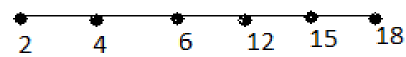
The induced edge labels are,

For $1 \le i \le k-1$
$f^*(u_i u_{i+1}) = 2$

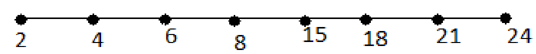
For $k+1 \le i \le n-1$
$f^*(u_i u_{i+1}) = 3 \equiv 0 (mod\ 3)$
$f^*(u_k u_{k+1}) = k+3 \equiv 0 (mod\ 3)$

It is observed as

$e_f(0) = k-1$
$e_f(1) = k$



P_6



P_8

Case (ii): n is odd

Subcase (i): k is not a multiple of 3

The vertex labels are,

$$f(u_i) = \begin{cases} 2i & 1 \le i \le k \\ 3i & k+1 \le i \le n \end{cases}$$

The induced edge labels are,

For $1 \le i \le k-1$
$f^*(u_i u_{i+1}) = 2$

For $k+1 \le i \le n-1$
$f^*(u_i u_{i+1}) = 3 \equiv 0 (mod\ 3)$
$f^*(u_k u_{k+1}) = k+3 \equiv 0 (mod\ 3)$

It is observed as

$e_f(0) = k$
$e_f(1) = k$

Subcase (ii): k is a multiple of 3

The vertex labels are,

$$f(u_i) = \begin{cases} 2i & 1 \le i \le k \\ 3ik + 1 \le i \le n-1 \end{cases}$$
$$f(u_n) = 3n - 1$$

The induced edge labels are,

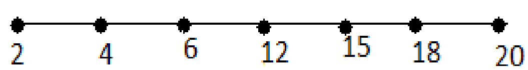
For $1 \le i \le k-1$
$$f^*(u_i u_{i+1}) = 2$$

For $k+1 \le i \le n-2$
$$f^*(u_i u_{i+1}) = 3 \equiv 0 (mod\ 3)$$
$$f^*(u_k u_{k+1}) = k + 3 \equiv 0 (mod\ 3)$$
$$f^*(u_{n-1} u_n) = 2$$

It is observed as

$$e_f(0) = k$$
$$e_f(1) = k$$

P_5



P_7

Clearly $|e_f(0) - e_f(1)| \le 1$

Then f is a 3 – modulo difference cordial labeling.

Hence P_n is a 3 – modulo difference cordial graph.

Theorem 3.2:

Comb ($P_n^+ \ or\ P_n \odot K_1$) is a 3-modulo differene cordial graph.

Proof:

Let G be a graph

When n is odd , n = 2k + 1 and when n is even , n = 2k.

Let V(G) = $\{u_1, u_2, u_3, \dots u_n, v_1, v_2, v_3 \dots v_n\}$

E(G) = $\{u_i u_{i+1} / 1 \le i \le n-1\} \cup \{u_i v_i / 1 \le i \le n\}$

Then $|V(G)| = 2n$ and $|E(G)| = 2n - 1$

Define $f: V(G) \to \{0,1,2,3, \dots \dots \dots, 6n\}$

The vertex labels are,

$$f(u_i) = 3i \quad , \quad 1 \le i \le n$$
$$f(v_i) = 3i - 1 \quad , \quad 1 \le i \le n$$

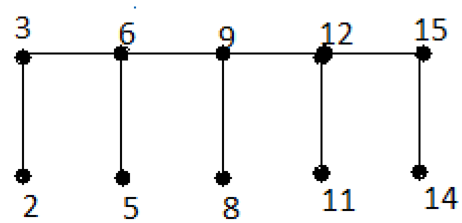
The induced edge labels are,

For $1 \le i \le n-1$,
$$f^*(u_i u_{i+1}) = 3 \equiv 0 (mod\ 3)$$

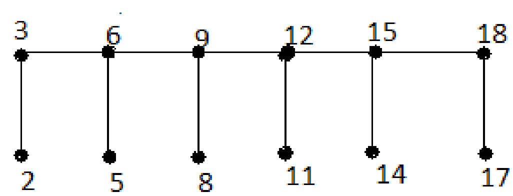
For $1 \le i \le n$,
$$f^*(u_i v_i) = 1$$

It is observed that

$$e_f(0) = n$$
$$e_f(1) = n - 1$$



$P_5 \odot K_1$



$P_6 \odot K_1$

Clearly $|e_f(0) - e_f(1)| \le 1$

Then f is a 3 – modulo differene cordial labeling

Hence P_n^+ is a 3-modulo difference cordial graph.

Theorem 3.3:

Cycle(C_n) is a 3-modulo difference cordial graph.

Proof:

Let G be a graph

When n is odd , n = 2k + 1 and when n is even , n = 2k.

Let V(G) = { $u_1, u_2, u_3, \ldots\ldots\ldots u_n\}$

E(G) = { $u_i u_{i+1} / 1 \le i \le n-1\} \cup \{u_n u_1\}$

Then $|V(G)| = n$ and $|E(G)| = n$

Define $f: V(G) \rightarrow \{0,1,2,3, \ldots\ldots\ldots, 3n\}$

Case (i): n is odd

Subcase (i): k is a multiple of 3.

The vertex labels are,

$$f(u_i) = \begin{cases} 2i & 1 \le i \le k \\ 3i & k+1 \le i \le n \end{cases}$$

The induced edge labels are,

For $1 \le i \le k-1$

$f^*(u_i u_{i+1}) = 2$

For $k+1 \le i \le n-1$

$f^*(u_i u_{i+1}) = 3 \equiv 0(mod\ 3)$

$f^*(u_k u_{k+1}) = k+3 \equiv 0(mod\ 3)$

$f^*(u_n u_1) = 3n-2 \equiv 1(mod\ 3)$

It is observed as

$e_f(0) = k$

$e_f(1) = k+1$

Subcase (ii): k is not a multiple of 3.

The vertex labels are,

$$f(u_i) = \begin{cases} 2i & 1 \le i \le k \\ 3i & k+1 \le i \le n \end{cases}$$

The induced edge labels are,

For $1 \le i \le k-1$

$f^*(u_i u_{i+1}) = 2$

For $k+1 \le i \le n-1$

$f^*(u_i u_{i+1}) = 3 \equiv 0(mod\ 3)$

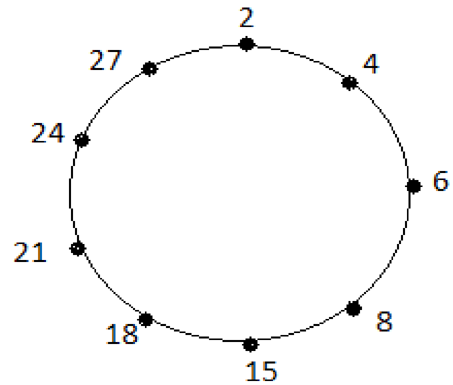
$f^*(u_k u_{k+1}) = k+3 \not\equiv 0(mod\ 3)$

$f^*(u_n u_1) = 3n-2 \equiv 1(mod\ 3)$

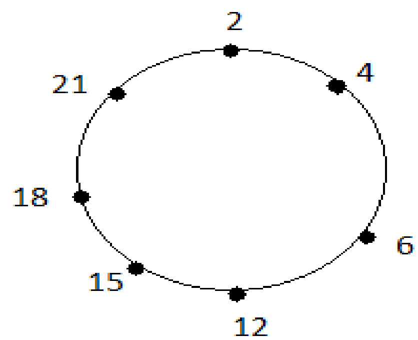
It is observed as

$e_f(0) = k+1$

$e_f(1) = k$



C_9



C_7

Case (ii): n is even

Subcase (i): k is not a multiple of 3

The vertex labels are,

$$f(u_i) = \begin{cases} 2i & 2 \le i \le k \\ 3i & k+1 \le i \le n-1 \end{cases}$$
$$f(u_1) = 1$$
$$f(u_n) = 3i + 1$$

The induced edge labels are,

$$f^*(u_1u_2) = 3 \equiv 0(mod\ 3)$$

For $2 \le i \le k-1$
$$f^*(u_iu_{i+1}) = 2$$

For $k+1 \le i \le n-2$
$$f^*(u_iu_{i+1}) = 3 \equiv 0(mod\ 3)$$
$$f^*(u_ku_{k+1}) = k+3 \not\equiv 0(mod\ 3)$$
$$f^*(u_{n-1}u_n) = 4 \equiv 1(mod\ 3)$$
$$f^*(u_nu_1) = 3n \equiv 0(mod\ 3)$$

It is observed as

$$e_f(0) = k$$
$$e_f(1) = k$$

Subcase (ii): k is a multiple of 3

The vertex labels are,

$$f(u_i) = \begin{cases} 2i & 1 \le i \le k \\ 3i & k+1 \le i \le n \end{cases}$$

The induced edge labels are,

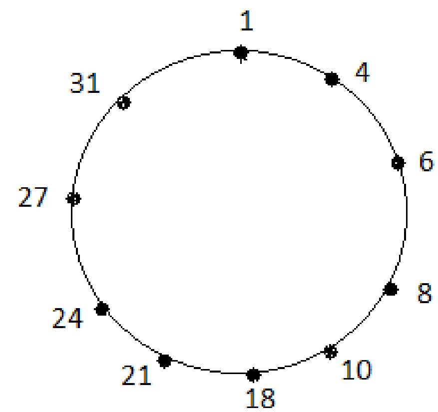
For $1 \le i \le k-1$
$$f^*(u_iu_{i+1}) = 2$$

For $k+1 \le i \le n-1$

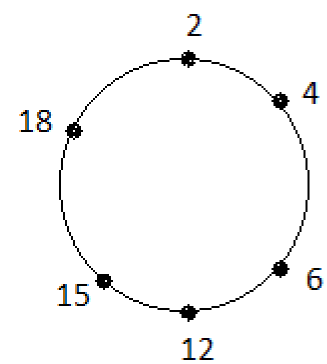
$$f^*(u_iu_{i+1}) = 3 \equiv 0(mod\ 3)$$
$$f^*(u_ku_{k+1}) = k+3 \equiv 0(mod\ 3)$$
$$f^*(u_nu_1) = 6k-2 \equiv 1(mod\ 3)$$

It is observed as

$$e_f(0) = k$$
$$e_f(1) = k$$



C_{10}



C_6

Clearly $|e_f(0) - e_f(1)| \le 1$

Then f is a 3 – modulo difference cordial labeling.

Hence C_n is a 3 – modulo difference cordial graph.

Theorem3.4:

C_n^+ is a 3-modulo differene cordial graph.

Proof:

Let G be a graph

When n is odd , n = 2k + 1 and when n is even , n = 2k.

Let $V(G) = \{ u_1, u_2, u_3, \ldots\ldots\ldots u_n, v_1, v_2, v_3, \ldots\ldots\ldots v_n \}$

$E(G) = \{ u_i u_{i+1} / 1 \leq i \leq n-1 \} \cup \{ u_i v_i / 1 \leq i \leq n \} \cup \{ u_n u_1 \}$

Then $|V(G)| = 2n$ and $|E(G)| = 2n$

Define $f: V(G) \rightarrow \{0,1,2,3, \ldots\ldots\ldots, 6n\}$

The vertex labels are,

$$f(u_i) = 3i \quad , \quad 1 \leq i \leq n$$
$$f(v_i) = 3i - 1 \quad , \quad 1 \leq i \leq n$$

The induced edge labels are,

For $1 \leq i \leq n-1$,

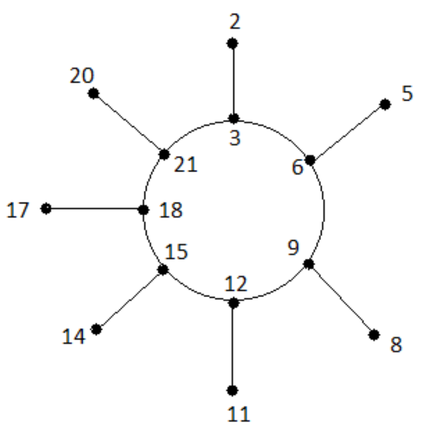
$$f^*(u_i u_{i+1}) = 3 \equiv 0 (mod\ 3)$$
$$f^*(u_n u_1) = n - 3 \equiv 0 (mod\ 3)$$

For $1 \leq i \leq n$,

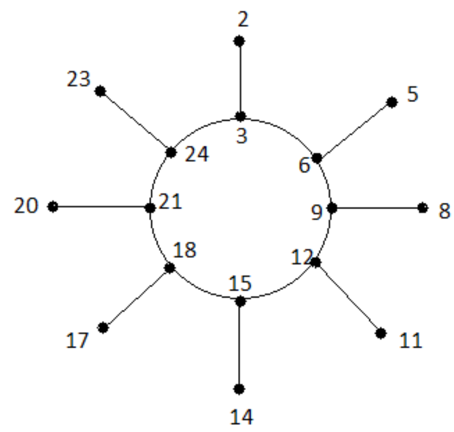
$$f^*(u_i v_i) = 1$$

It is observed that

$$e_f(0) = n$$
$$e_f(1) = n$$



C_7^+



C_8^+

Clearly $|e_f(0) - e_f(1)| \leq 1$

Then f is a 3 – modulo differene cordial labeling

Hence C_n^+ is a 3-modulo difference cordial graph.

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