# Some Mappings on Metric Space

[1]R.Deepa, [2]G.Subathra, [3]T.Surendiran
Department of Mathematics
[1,2] Bharath institute of Higher education and research, Chennai-73, Tamilnadu, India
[3] University college of Engineering, Ariyalur Tamilnadu, India

***Abstract***- *In this paper, we study some mappings on metric space, which includes the linearity and compact closureness.*

***Keywords***- Complete, Compactness, Boundedness, Equi-continuity.

## I. INTRODUCTION

The Arzela – Ascoli theorem can be proven using the Heine- Borel theorem. Throughout this, compactness is characterized by proving a series of small theorems in metric spaces. This has the aim of proving that, in metric spaces, the notions of compactness and sequential compactness are equivalent.

From the Arzela – Ascoli theorem, the Peano Existence theorem has derived, which is used for the existence of the solutions of ordinary differential equations of the form.[2,8].

$$x'(t) = f(x, x(t)) \quad (1)$$

$$x(t_0) = x_0 \quad (2)$$

In order to understand many of the following theorems, proofs and definitions, a few basic notions, used commonly in analysis.

$$B_r(x) = \{y \in R^n : \|x - y\| < r\}, \quad (3)$$

is the open ball of radius r and center x on $R^n$ and the set

$$\overline{Br(x)} = \{y \in R^n : \overline{\|x - y\| \leq r}\}, \quad (4)$$

is the closed ball of radius r and center x on $R^n$ any open ball with center x, $x \in R^n$ is called a neighbourhood of X. The following definitions and theorems are needed for the study on mappings[1,4].

**DEFINITION 1.1**

A set $A \subset R^n$ is an open set if for every point $x \in A$ there exists an $\varepsilon > 0$ such that $B_\varepsilon(x)$ is contained in A.

**DEFINITION 1.2**

A set $A \subset R^n$isa closed set if its compliment, $A^c := \{x \in R^n : x \notin A\}$ is open.

**Definition 1.3**

A point $A \in R^n$ is a Closure point of a set $A \subset R^n$if and only if every neighborhood of x contains atleast one point of A.

The Closure of a set A is the set of all closure points of A and is denoted A.

**Definition 1.4**

A point x is a **limit point** of a set A if every neighbourhood of x contains at least one point of A that is not x.

**Theorem 1.1 [9].THE BOLZANO – WEIERSTRASS THEOREM).**

If a subset $A \in R^n$ is closed and bounded, it is sequentially compact.

In $R^n$, the notions of compactness and sequential compactness are equivalent, here this is a theorem associating compactness with closed bounded sets. Through the relation of closed, boundedness to a type of compactness[10], it is evident that the Bolzano- Weierstrass theorem will be a powerful tool to form a proof for the Arzela – Ascoli theorem.

A set C, is a cover of a space A if it is a collection of subsets $Ci \subset A$ whose union is the space A.

$$A \subseteq \cup_{i \in I} Ci$$

A Space is **Compact** if and only if each open cover, $C_i$, for generality, has a finite sub cover $K_j \subset C_i$ such that K is finite (where I and j index C and K respectively)

A family of functions, $F = \{f1, f2, \ldots\ldots, fn \mid n \in N\}$, on a set X is uniformly bounded if there exists an $m \in N$ suchthatfnis bounded by m, $\forall fn \in F$.

**THEOREM1. 2. [9].** Take a subset A $\in R^n$, then A is closed and bounded if and only if it is compact.

To prove this, they began by proving that every compact set is closed and bounded. Then, in order to prove that every closed and bounded set is compact, they reduced the problem to one on a closed bounded box. After assuming that the box is not compact they used a method of bisection and Cantors intersection theorem (also proved) to form a contradiction.

**Definition 1.5.[5]**

Let X be a set, a metric (is a function) d: X xX$\rightarrow$ R for which the following conditions are satisfied.

1. $d$(x, y) >0 for all x, y ,z $\in$ X
2. d(x, y) = d(y, x) for all x, y $\in$X
3. d(x, z) $\leq$ d(x, y) + d(y, z) for all x, y, z $\in$ X (triangle inequality)

**THEOREM 1.3.[5].** Every sequentially compact metric space is totally bounded.

**THEOREM 1.4 [9].** A family of functions F on a metric space, X, is compact if and only if F is bounded, closed and equi-continuous.

The Heine- Borel theorem, states that every F is closed and bounded if and only if it is compact. It remains to prove that if F is compact, it is equi-continuous and that if F is equi-continuous, it is compact. They begin with the former, as it requires only the definitions of compactness, equi-continuity and an appropriate ε - net.

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