A Robust Active Contour Segmentation Based On Fractional-Order Differentiation And Fuzzy Energy

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Abstract- *This paper presents a novel fast model for activecontours to detect objects in an image, based on techniques of curve evolution. The proposed model can detect objects whose boundaries are not necessarily defined by gradient, based on the minimization of a fuzzy energy, which can be seen as a particular case of a minimal partition problem. This fuzzy energy is used as the model motivation power evolving the active contour, which will stop on the desired object boundary. However, the stopping term does not depend on the gradient of the image, as most of the classical active contours, but instead is related to the image color and spatial segments. The fuzziness of the energy provides a balanced technique with a strong ability to reject "weak" local minima. Moreover, this approach converges to the desired object boundary very fast, since it does not solve the Euler-Lagrange equations of the underlying problem, but, instead, calculates the fuzzy energy alterations directly. The theoretical properties and various experiments presented demonstrate that the proposed fuzzy energy-based active contour is better and more robust than classical snake methods based on the gradient or other kind of energies.*

Keywords**-** Active contour, deformable curve, segmentation, curve evolution, object detection, fast convergence, fuzzy logic, energy-based.

I. INTRODUCTION

Image segmentation is one of the first and most important tasks in image analysis and computer vision. Although various methods has been proposed in the literature, the design of robust and efficient segmentation algorithms is still a very challenging research topic, due to the variety and complexity of images [1]. Since the introduction of snakes [2], active contours have been applied to a variety of problems, such as image segmentation, feature extraction, image registration, etc. The original snake model and its variations, geodesic active contour models [3]–[8], are prone to getting "trapped" by extraneous edges. Cohen *et al.* [9], [10] has proposed the minimal path technique, which captures the global minimum of a contour energy between two fixed user-defined end points. Other implementations have also been proposed for capturing more global minimizers by restricting the search space. Dual snakes, dual-band active contour and similar methods restrict their search spaces exploiting normals lengths on the initial contour. Active contours have been also combined with the optimization tool of graph-cuts. All these classical snakes and active contour models are known as "edge-based" models, since they rely on edge functional to stop the curve evolution detecting only objects with edges defined by gradient. Thus, the performance of the purely edge-based models is often inadequate. There has been much research into the design of complex region-based energy functionals that are less likely to yield undesirable local minima when compared to simpler edge-based energy functionals. In general, region-based models utilize image information not only near the evolving contour, but image statistics inside and outside the contour. Chan and Vese proposed an active contour based on a region-based energy functional inspired by Mumford-Shah functional. This energy can be seen as a particular case of the minimal partition problem, and in the level set formulation the active contour is evolved. This model, as well as most of the region-based energy functionals, can handle objects with boundaries not necessarily defined by gradient, but assume highly constrainedmodels for pixel intensities within each region having highcomputational cost.as well as other researchers in the literature, propose a number variations of the Chan and Vese method in order to overcome its limitations, especially the high computational cost. Some of them utilize the simplicity of the k-means algorithm,while others instead of solving the PDE equations of the underlying energy functionals; they directly calculate the energy alterations. However, their main drawback, is that they are more sensitive to noise and cannot handle objects with ill-defined boundaries. This paper deals with the above mentioned problems. It presents a novel fuzzy energy-based active contour, which can handle objects whose boundaries are not necessarily defined by gradient, objects with very smooth or even with discontinuous boundaries. The fuzzy logic is a tool that has been intensively used in data clustering, but not in active contour methods. Generally, fuzzy methods provide more accurate and robust data clustering, thus, we combine it with active contour methodology, introducing here a model as a fuzzy energy-based minimization. The fuzziness of the energy provides a balanced technique with a strong ability to reject "weak" local minima. Furthermore, we formulate the model in terms of pseudo-level set functions, but instead of computing the associated Euler-Lagrange equations, we apply a direct method to solve the corresponding PDE without numerical stability constraints. This methodology improves very much the computational speed in relation to other methods exploiting Euler-Lagrange equations. Thus, the fuzziness of the energy and the fast "solution" of the PDE, provides us an attractive and easily implemented stable active contour with a desirable resistance to noise. The remainder of the paper is organized as follows. The description of the model, its fuzzy motivation energy and its properties are presented in Section II. Experimental results are presented in Section III and conclusions are drawn in Section IV.

II. DESCRIPTION OF THE MODEL

First, the basic idea of the model will be introduced. Let us define the evolving curve C in the image domain. Let us assume that the image I is formed by two regions of approximately piecewise-constant intensities, of distinct values Ii and Io. Assume that the object to be detected is represented by the region with the value Ii, and its boundary by C0. So, we have I ≈ Ii inside the object (inside C0) and I ≈ Io outside the object (outside C0). Now, let us consider the following functionals:

$$\begin{aligned} F_1(C) + F_2(C) = & \\ & \int_\Omega [u(x,y)]^m \, |I(x,y) - c_1|^2 dxdy \\ & + \int_\Omega [1-u(x,y)]^m \, |I(x,y) - c_2|^2 dxdy, \end{aligned}$$

Where c1 and c2 are constants depending on C, expressing the average prototypes of the image regions inside and outside respectively of C. The membership function u(x, y) 2 [0, 1] is the degree of membership of I(x, y) to the inside of C, and m is a weighting exponent on each fuzzy membership. In this simple case, it is obvious that the boundary of the object C0, is the minimizer of the "fitting" term:

$$\inf_C \{F_1(C) + F_2(C)\} \approx 0 \approx F_1(C_0) + F_2(C_0).$$

The five cases are illustrated in Figure 1. If the curve C is outside the object, then F1(C) > 0 and F2(C) ≈ 0 or F1(C) ≈ 0 and F2(C) > 0 depending on object position (inside or outside the curve). If the curve C is inside the object, then F1(C) ≈ 0 and F2(C) > 0. If C is both inside and outside the object, then F1(C) > 0 and F2(C) > 0. Finally, the fitting term is minimized when C = C0, i.e., when the curve C is on the boundary of the object. That is, the fitting term is minimized when the curve C is converged to the object boundary C0.

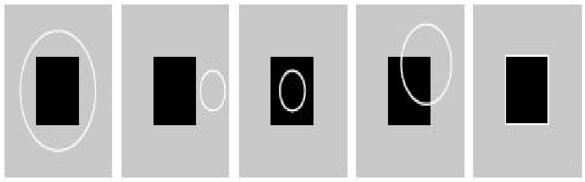

Fig. 1. *All possible cases in the position of the curve* C *in relation to the object under consideration.*

The proposed active contour is based on the minimization of the above fitting term, taking into account the length term of the model C as a regularization term. Therefore, the energy functional F(C, c1, c2, u) is introduced as:

$$\begin{aligned} F(C, c_1, c_2, u) = & \ \mu \cdot Length(C) \\ & + \lambda_1 \int_\Omega [u(x,y)]^m \, |I(x,y) - c_1|^2 dxdy \\ & + \lambda_2 \int_\Omega [1-u(x,y)]^m \, |I(x,y) - c_2|^2 dxdy, \end{aligned}$$

A. Pseudo Level-Set Formulation

Let us define a pseudo level set formulation, similar to the level set method [5], based on the membership values u, where C ⊂I is represented by the pseudo zero level set of Lipschitz similar function u : I → IR, such that:

$$\begin{cases} C & = \{(x,y) \in I : u(x,y) = 0.5\}, \\ inside(C) & = \{(x,y) \in I : u(x,y) > 0.5\}, \\ outside(C) & = \{(x,y) \in I : u(x,y) < 0.5\}, \end{cases} \qquad (3)$$

For more details, we refer the reader to [5]. Keeping u fixed and minimizing the energy F(C, c1, c2, u) (1) with respect to c1 and c2, it is easy to express these constants functions of u by:For more details, we refer the reader to [5]. Keeping u fixed and minimizing the energy F(C, c1, c2, u)

(1) with respect to c1 and c2, it is easy to express these constants functions of u by:

$$c_1 = \frac{\int_\Omega [u(x,y)]^m I(x,y)dxdy}{\int_\Omega [u(x,y)]^m dxdy}, \qquad (4)$$

and

$$c_2 = \frac{\int_\Omega [1-u(x,y)]^m I(x,y)dxdy}{\int_\Omega [1-u(x,y)]^m dxdy}. \qquad (5)$$

B. Numerical Approximation

In equation (1), the two fitting terms are easy to be computed directly. We can also, approximate the length term by:

$$\sum_{i,j} \sqrt{(Q_{i+1,j} - Q_{i,j})^2 + (Q_{i,j+1} - Q_{i,j})^2}, \qquad (7)$$

where Qi,j = H(u(i, j) − 0.5), u(i, j) is the value ofu at the (i, j) pixel and H(・) is the Heaviside function.The summand can only take the values 0, 1, or √2, dependingon whether the 3 distinct pair of points from theset {u(i, j), u(i + 1, j), u(i, j + 1)} belong to the same ordifferent regions. Thus, the length term can be easily computedknowing only the H(u − 0.5), and there is no need to knowu. This computed value can be interpreted as the discretizedlength of the pseudo zero level set. Note, that to apply ouralgorithm, we do not need to differentiate F in (1), whichwould have necessitated _(u) in Euler-Lagrange equation of(1).The usual approach to solve a minimization problem as in(1) is to derive its Euler-Lagrange equation and then to useexplicit time marching or implicit iteration. In the proposedmethod, the time step is not restricted as in the explicit timemarching. The algorithm for the fuzzy energy model is:

1) Give an initial partition of the image, set u > 0.5 for one part and u < 0.5 for the other.
2) Compute c1 using (4) and c2 using (5).
3) Assume that the value of the current pixel is Io and uo its corresponding degree of membership.

$$\Delta F = \lambda_1 s_1 \frac{[u_n^m - u_o^m](I_o - c_1)^2}{s_1 + u_n^m - u_o^m} + \lambda_2 s_2 \frac{[(1-u_n)^m - (1-u_o)^m](I_o - c_2)^2}{s_2 + (1-u_n)^m - (1-u_o)^m},$$

C. Analysis of the Algorithm

The model always converges in a finite number of sweeps (usually less than 10). If we do not consider the length term (μ = 0), it even converges in less than 5 sweeps. This leads us to analyze a simplified case of the algorithm that leaves out the regularization term of the initial model (1). Also, the parameters _1 and _2 are both thought equal to 1 (_1 = _2 = 1). Let us consider a two phase image where the object is represented by A (multi-connected or not) and the background by B, and the corresponding value for A and B is a and b respectively. Consider also an initial position (partition) u > 0.5 and u < 0.5 of the model in the image, denoted by U1 and U2 respectively. Let c1 and c2 to be the constants of the model position, 0 ≤ ui ≤ 1 the degree of membership of image point Ii, (i = 1, 2, . . .), and F the corresponding model energy. Furthermore, let us assume a point P with value Io 2 {a, b} and degree of membership uo. If we calculate the new degree of membership un (6) for point P, let ˜ c1, ˜ c2 be the new model constants for U1 and U2, respectively, and ˜ Fbe the new model energy. Then we can easily calculate ((A.3), (A.4) and (A.8)):

$$\begin{aligned} \tilde{c_1} &= c_1 + \frac{u_n^m - u_o^m}{s_1 + u_n^m - u_o^m}(I_o - c_1), \\ \tilde{c_2} &= c_2 + \frac{(1-u_n)^m - (1-u_o)^m}{s_2 + (1-u_n)^m - (1-u_o)^m}(I_o - c_2), \\ \tilde{F} &= F + s_1 \frac{u_n^m - u_o^m}{s_1 + u_n^m - u_o^m}(I_o - c_1)^2 \\ &\quad + s_2 \frac{(1-u_n)^m - (1-u_o)^m}{s_2 + (1-u_n)^m - (1-u_o)^m}(I_o - c_2)^2, \end{aligned}$$

III. EXPERIMENTAL RESULTS

In this Section, we show the performance of the proposed method by presenting numerical results using the fuzzy energy model on various synthetic and real images, with different types of contours and shapes. We show the active contour evolving in the original image, and the associated piecewise constantapproximation of (given by constants c1 and c2). In our numerical experiments, we generally choose the parameters to be _1 = _2 = 1. Only the length parameter μ, which has a scaling role, is not the same in all experiments. If we have to detect all or as many objects as possible and of any size, then μ should be small. If we have to detect only large objects and not to detect small objects (like points, dueto noise), then μ has to be larger.

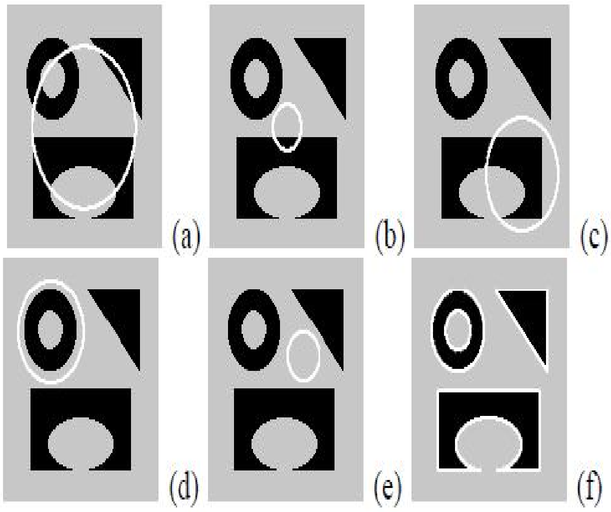

Fig. 3. *Segmentation of a two-phase image.* ***(a)***, ***(b)***, ***(c)***, ***(d)*** *and* ***(e)*** *are five different initial conditions, which have the same result after one sweep.* ***(f)****the result segmented image. The interior contour is automatically detected. In this experiment, the length parameter was omitted (*$\mu = 0$*).*

First, the segmentation results on a two-phase image (Figure 3) are presented. The length term is omitted ($\mu = 0$) since there is no noise. Five different initial conditions were used and all of them converged to the correct solution in one sweep. Even, the interior contour was automatically detected without considering a second initial model, something that shows the robustness of the algorithm. Thus, we can generalize that when the length term is omitted ($\mu = 0$), then the algorithm is completely independent to the model's initial condition. When the length term is taken into account, then the algorithm is dependent to the initial condition. This is occurred due to the fact that the lengthterm when increases, the model tends to behave as a rigid one, which in practice means that the model can be spatially moved without any deformation. So, one could claim that the length term localizes the proposed model. But, this is also the property that enforces the model to have a remarkable resistance to image noise, as it is shown below.

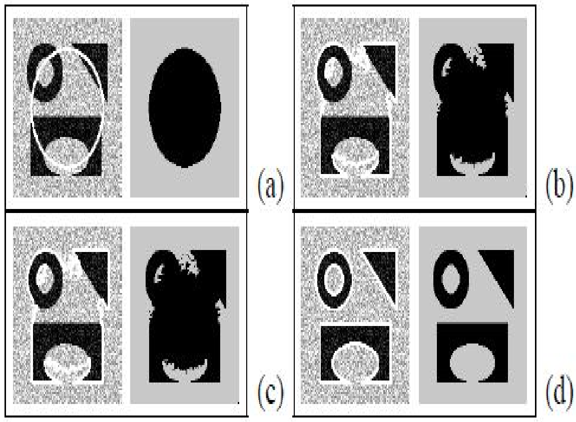

Fig. 4. *Detection of different objects from a noisy image, with variousshapes and with interior contour (uniform noise). The model converges fromthe initial* ***(a)*** *to the final position* ***(d)*** *with two intermediate steps* ***(b)*** *and* ***(c)****. The length parameter was set equal to* $\mu = 0.150$.

Figure 4 shows how the model works on a noisy syntheticimage, with various shapes and an interior contour. All thecontours are automatically detected, without considering a second initial curve. Jacobi iteration, as well as, the natureof the model allows the automatical change of the topology.

TABLE I
SEGMENTATION MEASURE RESULTS.

Image	Proposed Method	Chan-Vese Method	Gibou-Fedkiw Method
Clear	0.002	0.005	0.003
Noisy (gaussian)	0.017	0.020	0.054
Noisy (uniform)	0.016	0.024	0.036
Real images	0.023	0.080	0.084

Finally, another advantage of the proposed algorithm is its easy extension to segmentation of images with multiple regions. In order to achieve that, one should add to the initialenergy equation (1) as much centers ci as he wants, and then to analyze the new equation as it is presented in this paper.

IV. CONCLUSION

In this paper, a novel fast and robust model for active contours to detect objects in an image was introduced. The model can detect objects whose boundaries are not necessarily defined by gradient, due to the fact that it is based on an energy minimization algorithm and not on an edge-function as the most classical active contour models. This energy is based on fuzzy logic, which can be seen as a particular case of a minimal partition problem, and is used as the model motivation power evolving the active contour until to catch the desired object boundary. Furthermore, the stopping term of the model evolution does not depend on the gradient of the image, as most of the classical active contours, but instead is related to the image color and spatial segments. The fuzziness of the energy provides a balanced technique with a strong ability to reject "weak" local minima. Also, it is not needed to smooth the initial images, even if they are very noisy, since the model very well detect and preserve the locations of the boundaries. The interior contours of the objects can be automatically detected, starting only with the initial curve (model). The initial position of the model can be anywhere in the image, and it does not necessarily surrounds the objects to be detected. Finally, the small computation time of the evolution of the model renders the proposed method as a very promising tool even for real time applications. This lies in the fact that, the introduced method does not solve the Euler-Lagrange equation of the underlying problem.

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