Characterizations of Secondary Idempotent Matrices

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Abstract- *The concept of secondary idempotent matrices is introduced.Characterization of secondary idempotent matrices are obtained and derived some theorems.*

Keywords- Transpose of a matrix, conjugate transpose of a matrix, Secondary transpose of a matrix,idempotent matrix, secondary idempotent matrix

I. INTRODUCTION

Anna Lee [1] has initiated the study of secondary symmetric matrices. Also she has shown that for a complex matrix A, the usual transpose A^T and secondary transpose A^S are related as $A^S=VA^TV$ [2] where 'V' is the permutation matrix with units in its secondary diagonal. Alaa A.Hammodat, Ali A.Bilal, Akram S.Mohammed has given some basic properties of idempotent matrices in [3]. Statistical inference econometric analysis and matrix algebra was given by Festschrift in Honour of Gotz Trenkler in [4]. Some properties of polynomial idempotent matrices are discussed and characterization of polynomial idempotent matrices are obtained by Ramesh.G,Sudha P.N [5].

In this paper the s-idempotent matrices is defined and its characterizations are discussed. Some theorems relating to s-idempotent matrices are derived.

II. PRELIMINARIES AND NOTATIONS

Let C_{nxn} be the space of nxn complex matrices of order n. For $A\in C_{nxn}$. Let $A^T, \overline{A}, A^*, A^S, A^\theta$ denote transpose, conjugate, conjugate transpose, secondary transpose, conjugate secondary transpose of a matrix A respectively. Let 'V' be the associated permutation matrix whose elements on the secondary diagonal are 1, other elements are zero. Also 'V' satisfies the following properties .Also V satisfies the following properties $V^T = \overline{V} = V^* = V^{-1} = V, V^2 = I$ and $VV^* = V^*V = I$

A matrix $A\in C_{nxn}$ is called Hermitian if A = A*

III. SECONDARY IDEMPOTENT MATRICES

Definition 3.1:

A is a s-idempotent matrix if AVA =A

PROPERTIES OF S-IDEMPOTENT MATRICES

Theorem 3.1.1:

Let $A \in C_{n\times n}$. If A is a s-idempotent matrix then A^T is also s-idempotent matrix.

Proof:

We know that if A is a s-idempotent matrix then AVA =A. Taking transpose on both sides

$(AVA)^T=A^T$

$\Rightarrow A^TV^TA^T=A^T$

$\Rightarrow A^TVA^T=A^T$ [$\because V^T=V$]

i.e., $A^TVA^T=A^T$

Therefore A^T is a s-idempotent matrix.

Theorem 3.1.2:

Let $A \in C_{n\times n}$. If A is a s-idempotent theorem then $\overline{A}$ is also s-idempotent matrix.

Proof:

We know that if A is a s-idempotent matrix then AVA=A. Taking conjugate on both sides

$(\overline{AVA}) = \overline{A}$

$\Rightarrow \overline{A}\overline{V}\overline{A} = \overline{A}$

$\Rightarrow \overline{A}V\overline{A} = \overline{A}$ [$\because \overline{V}$ =V]

Therefore $\overline{A}$ is a s-idempotent matrix.

Theorem 3.1.3:

Let $A \in C_{n\times n}$. If A is a s-idempotent matrix then A^* is s-idempotent matrix.

Proof:
we know that if A is a s-idempotent matrix then AVA=A
Taking conjugate transpose on both sides

$(AVA)^*=A^*$
$\Rightarrow A^*V^*A^*=A^*$
$\Rightarrow A^*VA^*=A^*$ $[V^*=V]$

$\therefore$ A* is a s-idempotent matrix.

Theorem 3.1.4:

Let $A \in C_{n\times n}$. If A is a s-idempotent matrix then A^{-1} is also s-idempotent.

Proof:

We know that if A is a s-idempotent matrix then AVA=A.
Taking inverse on both sides

$(AVA)^{-1}=A^{-1}$
$\Rightarrow A^{-1}V^{-1}A^{-1}=A^{-1}$
$\Rightarrow A^{-1}VA^{-1}=A^{-1}$ $[\because V^{-1}=V]$

Therefore A^{-1} is a s-idempotent matrix.

Theorem 3.1.5:

Let $A \in C_{n\times n}$. If A is a s-idempotent matrix, then A^S is s-idempotent matrix.

Proof:

We know that if A is a s-idempotent matrix, then AVA=A.

Claim: $A^SVA^S=A^S$

We know that AVA=A
Taking secondary transpose on both sides
$(AVA)^S=A^S$
$A^SV^SA^S=A^S$
$\therefore A^SVA^S=A^S$ $[\because V^S=V]$

Therefore A^S is s-idempotent matrix.

Theorem 3.1.6:

Let $A \in C_{n\times n}$. If A and B are s-idempotent matrices then A+B is s-idempotent matrix if

$AVB=-BVA$

Proof:
We know that if A and B are s-idempotent matrices, then AVA=A and BVB=B

Claim: (A+B)V(A+B)=A+B
$(AV+BV)(A+B)=AVA+AVB+BVA+BVB$
$=A+AVB+BVA+B$
$=A+B$ $[\because AVB=-BVA]$

Therefore (A+B)V(A+B)=A+B.Hence A+B is a s-idempotent matrix.

Theorem 3.1.7:

If A and B are s-idempotent matrices and AB = BA, A is idempotent then AB is s-idempotent matrix.

Proof:

We know that if A is s-idempotent matrix , then AVA=A. We know that if B is s-idempotent matrix, then BVB=B.

Claim: ABVAB=AB

$ABVAB = ABVBA$ $[\because AB = BA]$
$= ABA$ $[\because BVB = B]$
$= AAB$ $[\because BA=AB]$
$= A^2B$ $[\because A^2=A]$
$= AB$
$\therefore ABVAB = AB$

$\therefore$ AB is s-idempotent matrix.

Theorem 3.1.8:

If A and B are s-idempotent matrix and BA=AB, B is idempotent then BA is s-idempotent matrix.

Proof:

We know that if A is s-idempotent matrix, then AVA = A.
and if B is s-idempotent matrix, then BVB = B.

Claim: BAVBA = BA

$BAVBA = BAVAB$ $[\because BA = AB]$

$= BAB$ [∵ AVA=A]
$= BBA$ [∵ AB = BA]
$= B^2A$ [∵ $B^2=B$]
$= BA$
∴ BAVBA = BA
∴ BA is s-idempotent matrix.

IV. CONCLUSION

In this paper the concept of s-idempotent matrices was defined and theorem relating to characterizations of s-idempotent matrices were derived.

REFERENCES

[1] Anna Lee., "Secondary symmetric, secondary skew symmetric Secondary orthogonal matrices", period. Math. Hungary 7, pp.63-70 (1976).
[2] Anna Lee., "On s-symmetric, s-skew symmetric and s-orthogonal matrices", period. Math. Hungary 7, pp.71-76 1976).
[3] Alaa A.Hammadat, Ali A.Bilal, Akram S.Mohammed, "Some basic properties of Idempotent matrices",J.Edu.& Sci., vol.21, No.4(2008).
[4] Festschrift in Honour of Gotz Trenkler, "Statistical Inference Econometric Analysis and Matrix Algebra", physica-Veriag Heidelberg, Springer, (2009).
[5] Ramesh .G.,Sudha P.N., "On the Determinant of a product of Two polynomial matrices" IOSR Journal of Mathematics IOSR –JM vol.10,Issue 6,Ver.II,pp10-13,Nov-Dec(2014).