Separation Axioms on Neutrosophic Generalized Regular Topological Spaces

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Abstract- *In this paper, we introduce the concept of separation axiom on Neutrosophic generalized regular topological spaces. Some characterizations of separation axioms T_1 and T_2in NGR topological spaces will be introduced*

Keywords- Neutrosophic generalized regular T_1 space and Neutrosophic generalized regular T_2 space

I. INTRODUCTION

C.L.Chang[6] introduced and developed fuzzy topological space by using L.A. Zadeh[16] fuzzy sets. Coker[7] introduced the notion of Intuitionistic fuzzy topological spaces by using Atanassov[2] intuitionistic fuzzy set . Neutrality the degree of indeterminacy, as an independent concept, was introduced by Smarandache[9] He also defined the Neutrosophic set on three component Neutrosophic topological spaces(t,f,i)=(Truth,Falsehood,Indeterminacy).The Neutrosophic crisp set concept was converted to Neutrosophic topological spaces by A.A.Salama[14]. R.Dhavaseelan[8] introduced Neutrosophic generalized closed sets. S.Bayhan[5] developed the concepts On fuzzy separation axioms in intuitionistic fuzzy topological spaces .

II. PRELIMINARIES

Definition 2.1 [3]

Let X be a non-empty fixed set. A Neutrosophic set A has the form

$$A = \{(x,\mu_A(x), \sigma_A(x), \gamma_A(x)) : x \in X\}$$

where $\mu_A(x), \sigma_A(x), \gamma_A(x)$ are topological spaces and
$\mu_A(x)$ is the degree of membership function, $\sigma_A(x)$ is the degree of indeterminacy and
$\gamma_A(x)$ is the degree of non-membership function respectively of each $x \in X$ to the set A.

Remark 2.2 [3]

A Neutrosophic set $A = \{(x,\mu_A(x), \sigma_A(x), \gamma_A(x)) : x \in X\}$ can be identified to an ordered triple$(\mu_A, \sigma_A, \gamma_A)$ in $]0^-, 1^+[$on X.

Example 2.3 [3]

Since our main purpose is to construct the tools for developing Neutrosophic set and Neutrosophic topology, we must introduce the Neutrosophic set

We must introduce the Neutrosophic set
$0_N$and 1_Nin X as follows:
$0_N=\{(x,0,0,1):x\in X\}$
$1_N= \{(x, 1, 0, 0): x \in X\}$

Definition 2.4[3]

Let X be a non-empty set and A and B are Neutrosophic sets of the form

$$A = \{(x,\mu_A(x), \sigma_A(x), \gamma_A(x)) : x \in X\} \text{and}$$
$$B = \{(x,\mu_B(x), \sigma_B(x), \gamma_B(x)) : x \in X\}$$

then we consider the definition of subset $(A \subseteq B)$ is defined as
$A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x), \sigma_A(x) \leq \sigma_B(x), \gamma_A(x) \geq \gamma_B(x)$, for all $x \in X$.

Theorem 2.5[3]

For any Neutrosophic set A the following condition holds

(i) $0_N \subseteq A$, $0_N \subseteq 0_N$,
(ii) $A \subseteq 1_N$, $1_N \subseteq 1_N$.

Definition 2.6[3,4]

Let X be a non-empty set and

$A = \{x, \mu_A(x), \sigma_A(x), \gamma_A(x)\}$ and
$B = \{x, \mu_B(x), \sigma_B(x), \gamma_B(x)\}$ are Neutrosophic sets then $A \cap B$ is defined as

$A \cap B = \{x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \vee \gamma_B(x)\}$ then $A \cup B$ is defined as

$A \cup B = \{x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \wedge \gamma_B(x)\}$.

Definition 2.7[1,3]

A Neutrosophic topology is a non-empty set X is an family τ_N of Neutrosophic subsets in X satisfying the axioms :

(i)0_N , $1_N \in \tau_N$
(ii) $G_1 \cap G_2 \in \tau_N$ for any $G_1, G_2 \in \tau_N$
(iii) $\cup G_i \in \tau_N$ for every $\{ G_j : j \in J\} \subseteq \tau_N$

the pair (X, τ_N) is called Neutrosophic topological space.
The element in Neutrosophic topological space (X,τ_N) are called Neutrosophic open sets.

A Neutrosophic set F is closed if and only if $(F)^C$ is Neutrosophic open.

Definition 2.8[3]

Let (X, τ_N) Neutrosophic topological spaces and

$A = \{(x,\mu_A(x), \sigma_A(x), \gamma_A(x)) : x \in X\}$ be Neutrosophic set in X.

Then the
Neutrosophic closure and *Neutrosophic interior* are defined as
$Ncl(A) = \cap$ {K:K is Neutrosophic closed set in X and $A \subseteq K$},
$Nint(A) = \cup$ {G:G is Neutrosophic open set in X and $G \subseteq A$}.

Definition 2.9[3]

A is Neutrosophic open set if and only if A=*N int(A)*,
A is Neutrosophic closed set if and only if A=*Ncl(A)*.

Definition 2.10[1,3]

A subset A of Neutrosophic space (X,τ_N) is called Neutrosophic regular open (in short *NR open*) if $A = Nint(Ncl(A))$ .The Complement of *NR open* set is called *NRclosed*.

Definition 2.11[3]

A subset A of Neutrosophic space (X,τ_N) is called Neutrosophic generalized closed (in short *NG closed*) if $Ncl(A) \subseteq U$, whenever $A \subseteq U$ and U is Neutrosophic open. The Complement of a *NG closed* set is called *NG open* set.

Definition 2.12[3]

Let A be a subset of Neutrosophic space (X, τ_N) is called Neutrosophic generalized regular closed *(NGR closed)* if *Neutrosophic Regular cl*$(A) \subseteq U$ (in short $NRcl(A) \subseteq U$), whenever $A \subseteq U$ and U is Neutrosophic open.

The Complement of a *NGR closed* set is called *NGR open* set.

Definition 2.13[4]

Let (X, T) and (Y, S) be any two Neutrosophic topological space.
(i) A map $f : (X, T) \to (Y, S)$ is said to be *Neutrosophic generalized regular continuous* (in short *NGR continuous*)if the inverse image of every *Neutrosophic closed* set in (Y, S) is *NGR closed* set in (X, T).

Definition 2.14[1]

Let X be a non-empty set and $x \in X$ be a fixed element in X.
If r, t, s are fixed real numbers of $]0^-,1^+[$,suchthat$r+t+s \leq 3$. Then the $\langle x, r, t, s \rangle$ is called Neutrosophic points in X. where,
x_r denote the degree of membership of $x_{r,t,s}$
x_t denote the degree of in determinancy of $x_{r,t,s}$ and
x_s denote the degree of non-membership of $x_{r,t,s}$ and $x \in X$ the support of $x_{r,t,s}$.

The point $x_{r,t,s}$ is said to be contained in Neutrosophic set A if and only if $r < \mu_A(x)$, $t < \sigma_A(x)$ and $s > \gamma_A(x)$.

III. SEPARATION AXIOMS ON NEUTROSOPHIC GENERALIZED REGULAR TOPOLOGICAL SPACES

Definition 3.1

Let (X, τ) be an Neutrosophic topological space .

1. (X,τ) is called an$NGRT_1(i)$ space if and only if for each pair of distinct

Neutrosophic points $x_{r,t,s}$ and $y_{a,c,b}$ in X , there exist *NGR open* sets U, V in τ
Such that $x_{r,t,s} \in U$, $y_{a,c,b} \notin U$ and $y_{a,c,b} \in V$, $x_{r,t,s} \notin V$.

2. (X,τ)iscalledan$NGRT_1(ii)$space if and only if for all $x,y \in X$, $x \neq y$, there exist *NGR open* sets U, V in τ such that $U(x) = 1_N$, $U(y) = 0_N$ and $V(y) = 1_N$, $V(x) = 0_N$.

3. (X, τ) is called an *NGR* $T_1(iii)$ space if and only if for each pair of distinct Neutrosophic points $x_{r,t,s}$ and $y_{a,c,b}$ in X, there exist *NGRopen* sets U, V in τ such that $x_{r,t,s} \subseteq U \subseteq \overline{y_{a,c,b}}$ and $y_{a,c,b} \subseteq V \subseteq \overline{x_{r,t,s}}$.

4. (X, τ) is called an *NGR* $T_1(iv)$ space if and only if for each pair of distinct Neutrosophic points $x_{r,t,s}$ and $y_{a,c,b}$ in X, there exist *NGRopen* sets U, V in τ such that $x_{r,t,s} \subseteq U$, $U \cap y_{a,c,b} = 0_N$. (i.e., $U(y) = 0_N$) and $y_{a,c,b} \subseteq V$, $V \cap x_{r,t,s} = 0_N$. (i.e., $V(x) = 0_N$).

Theorem3.2

Let (X, τ) and (Y, σ) be any two Neutrosophic topological space and $f : X \to Y$ be an *NGR continuous* mapping. Then (X, τ) is an *NGR* $T_1(i)$ space if (Y, σ) is an Neutrosophic T_1space.

Proof :

Let $x_{r,t,s}$ and $y_{a,c,b}$ be any two distinct Neutrosophic points in (Y, σ)
Then there exist *Neutrosophic open* sets U, V in σ such that $x_{r,t,s} \subseteq U$, $y_{a,c,b} \not\subseteq U$ and $y_{a,c,b} \subseteq V$, $x_{r,t,s} \not\subseteq V$.
Since f is *NGR continuous* mapping ,
By definition 2.13(i) ,$f^{-1}(U)$ and $f^{-1}(V)$ are *NGR open* sets in X.
Such that $x_{r,t,s} \in f^{-1}(U)$, $y_{a,c,b} \notin f^{-1}(U)$ and $y_{a,c,b} \in f^{-1}(V)$, $x_{r,t,s} \notin f^{-1}(V)$
Hence by definition 3.1(1) , (X, τ) is an *NGR* $T_1(i)$ space.

Theorem3.3

Let (X, τ) be an Neutrosophic topological space and if X is an *NGR* $T_1(ii)$ space then it is an *NGR* $T_1(i)$ space.

Proof :

Let $x \neq y$ and Let $x_{r,t,s}$ and $y_{a,c,b}$ be any two distinct Neutrosophic points in X.

Then by definition 3.1(2) , there exist *NGR open* sets U, V in $\tau \ni U(x) = 1_N$, $U(y) = 0_N$ and $V(y) = 1_N$, $V(x) = 0_N$.

By definition 2.14 , We have
$\langle r, t, s \rangle \leq 3 = U(x)$ implies $x_{r,t,s} \subseteq U$ and
$\langle a, c, b \rangle \leq 3 = V(y)$ implies $y_{a,c,b} \subseteq V$
$U(y) = 0_N \Rightarrow \mu_U(y) = 0_N$, $\sigma_U(y) = 0_N$, $\gamma_U(y) = 1_N \Rightarrow y_{a,c,b} \not\subseteq U$ and
$V(x) = 0_N \Rightarrow \mu_V(x) = 0_N$, $\sigma_V(x) = 0_N$, $\gamma_V(x) = 1_N \Rightarrow x_{r,t,s} \not\subseteq V$

Therefore $x_{r,t,s} \subseteq U$, $y_{a,c,b} \not\subseteq U$ and $y_{a,c,b} \subseteq V$, $x_{r,t,s} \not\subseteq V$.
ThereforeBydefinition3.1(1),(X,τ)isan*NGR*$T_1(i)$space.

Theorem3.4

Let (X, τ) be an Neutrosophic topological space and Then X is an*NGR* $T_1(iv)$ space if and only if it is an *NGR* $T_1(ii)$ space.

Proof:

Let x and y be any two points in X with $x \neq y$ and $x_{r,t,s}$ and $y_{a,c,b}$ be any two distinct Neutrosophic points in X,

Then by definition 3.1(4) , there exist *NGR open* sets U, V in τ such that $x_{r,t,s} \subseteq U$, $U \cap y_{a,c,b} = 0_N$. (i.e., $U(y) = 0_N$) and $y_{a,c,b} \subseteq V$, $V \cap x_{r,t,s} = 0_N$. (i.e., $V(x) = 0_N$) .

Since $x_{r,t,s} \subseteq U$ and $y_{a,c,b} \subseteq V$
We have $U(x) = 1_N$ and $V(y) = 1_N$
On the other hand $U \cap y_{a,c,b} = 0_N$, $V \cap x_{r,t,s} = 0_N$
$\Rightarrow U(y) = 0_N$, $V(x) = 0_N$

Therefore By definition 3.1(2),X is an *NGR* $T_1(ii)$ space.
Conversely , Let $x \neq y$ and $x_{r,t,s}$ and $y_{a,c,b}$ be any two distinct Neutrosophic points in X.

By definition 3.1(2), there exist *NGR open* sets U, V in τ such that
$U(x) = 1_N$, $U(y) = 0_N$ and $V(y) = 1_N$, $V(x) = 0_N$.

Then by definition 2.14 , we have
$\langle r, t, s \rangle \leq 3 = U(x)$ implies $x_{r,t,s} \subseteq U$ and
$\langle a, c, b \rangle \leq 3 = V(y)$ implies $y_{a,c,b} \subseteq V$

Now$U \cap y_{a,c,b} = 0_N$(Since$U(y) = 0_N$)and Also $V \cap x_{r,t,s} = 0_N$(Since $V(x) = 0_N$).
By definition 3.1(4) , X is an *NGR* $T_1(iv)$ space .

Definiton 3.5

Let (X, τ) be an Neutrosophic topological space .

(i)(X,τ)iscalledan$NGRT_2(i)$space if and only if for all $x,y \in X$, $x \neq y$,there exist *NGR open* sets U, V in τsuch that $U(x) = 1_N$and $V(y) = 1_N$and $U \cap V = 0_N$.

(ii) (X,τ) is called an $NGRT_2(ii)$space if and only if for each pair of distinct Neutrosophic points $x_{r,t,s}$and $y_{a,c,b}$in X , there exist *NGR open* sets U, V in τ such that $x_{r,t,s} \subseteq U$, $y_{a,c,b} \subseteq V$ and $U \cap V = 0_N$.

(iii) (X,τ) is called an $NGRT_2(iii)$ space if and only if for each pair of distinct Neutrosophic points $x_{r,t,s}$ and $y_{a,c,b}$ in X , there exist *NGR open* sets U, V in τ such that $x_{r,t,s} \subseteq U \subseteq \overline{y_{a,c,b}}$ and $y_{a,c,b} \subseteq V \subseteq \overline{x_{r,t,s}}$and $U \subseteq \overline{V}$.

(iv) (X, τ) is called an $NGR\ T_2(iv)$ space if and only if for all $x, y \in X$, $x \neq y$,there exist *NGR open* sets U, V in τ such that $U(x) = 1_N = V(y)$ and $U(y) = 0_N = V(x)$ and $U \subseteq \overline{V}$.

Theorem3.6

Let (X, τ) and (Y,σ) be any two Neutrosophic topological space and $f : X \to Y$ be an *NGR continuous* mapping. Then (X, τ) is an $NGR\ T_2(i)$ space if (Y, σ) is an Neutrosophic T_2space.

Proof :

Let $x, y \in Y$, $x \neq y$, Then there exist *Neutrosophic open* sets U, V in σ such that $U(x) = 1_N$, $V(y) = 1_N$and $U \cap V = 0_N$.

Since f is *NGR continuous* mapping.

By definition 2.13(i) , $f^{-1}(x)$, $f^{-1}(y) \in X$, Such that $f^{-1}(x) \neq f^{-1}(y)$.

Then there exist *NGR open* sets $f^{-1}(U), f^{-1}(V) \in \tau$ in X ,

Such that $f^{-1}(U(x)) = f^{-1}(1_N) = 1_N$and $f^{-1}(V(y)) = f^{-1}(1_N) = 1_N$

Therefore $f^{-1}(U) \cap f^{-1}(V) = 0_N$

Hence By definition 3.5(i) , (X, τ) is called an $NGR\ T_2(i)$ space .

Theorem3.7

Let (X, τ) be an Neutrosophic topological space and Then X is an$NGR\ T_2(i)$ space if and only if it is an $NGR\ T_2(ii)$ space.

Proof :

Let $x_{r,t,s}$and $y_{a,c,b}$be any two distinct Neutrosophic points in X and $x \neq y$,

By definition 3.5(i) , there exist *NGR open* sets U, V in $\tau \ni U(x) = 1_N, V(y) = 1_N$and $U \cap V = 0_N$

Then We have, $\langle r, t, s\rangle \leq 3 = U(x) \Rightarrow x_{r,t,s} \subseteq U$ $\langle a, c, b\rangle \leq 3 = V(y) \Rightarrow y_{a,c,b} \subseteq V$and $U \cap V = 0_N$.

Hence by definition 3.5(ii) ,X is an $NGR\ T_2(ii)$ space.

Theorem3.8

Let (X, τ) be Neutrosophic topological space . Then every $NGR\ T_2(i)$ is an$NGR\ T_2(iv)$ space.

Proof:

Let $x, y \in X$ and $x \neq y$,

By definition 3.5(i) there exist *NGR open* sets U, V in τ such that $U(x) = 1_N = V(y)$. Similarly $U(y) = 0_N = V(x)$

Therefore , $U \subseteq \overline{V}$.

Hence By definition3.5(iv),Every $NGR\ T_2(i)$ is an $NGR\ T_2(iv)$ space.

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