

# Study of Free Vibration Analysis of Rectangular Laminated Plate With Circular Cut-Out

Ashish Daksh<sup>1</sup>, Satyendra Dubey<sup>2</sup>, Y.K. Bajpai<sup>3</sup>

<sup>1</sup>Student-M.Tech.(Structural Engineering)Department of Civil Engineering

<sup>2</sup>Asst.Professor-Department of Civil Engineering

<sup>3</sup>Professor- Head Department Civil Engineering

<sup>1,2,3</sup>Gyan Ganga Institute of of Technology and Science, Jabalpur

**Abstract-** Composite materials are made from the combination of two or more materials with different chemical and physical properties, which when combined give a new material with enhanced material properties. Our study is concerned with the analysis of free vibration of rectangular plate with circular cut-out. We study the effects of aspect/size ratio, thickness ratio, and angle of the laminate, number of laminates, material properties and different boundary conditions.

**Keywords-** FEM, Composite materials, FSDT

## I. INTRODUCTION

A composite material is defined as a material which is composed of two or more materials at a microscopic scale and chemically distinct face. So, a composite material is heterogeneous at a microscopic scale but statistically homogeneous at macroscopic scale. The materials which form the composite are also called as constituents or constituent materials. The constituent materials of a composite have significantly different properties. Further, it should be noted that the properties of the composite formed may not be obtained from these constituents. However, a combination of two or more materials with significant properties will not suffice to be called as a composite material. Composite material are mainly used in sstructural, construction, medical, house-hold, industrial, transportation, electrical; electronics, etc. Metals are the most commonly used materials in these applications.

## II. LITERATURE REVIEW

S. Liu [1] studied a vibration analysis of composite laminated plates. D. Ngo-Cong et al [2] studied the free vibration analysis of laminated plates based on FSDT using one- dimensional IRBFN method.. Mohamed-Ouejdi Belarbi et al [3] uses a layer wise finite element formulation on the free vibration analysis of laminated composite and sandwich plates. T. Kant and K. Swaminathan [4] present the analytical solutions for free vibration of laminated composite and sandwich plates based on a higher order refined theory.

Avadesh Kumar Sharma and N. D. Mittal [5] studied the free vibration analysis of laminated composite plates with elastically restrained edges using FEM. Harsh Kumar Bhardwaj et al [6] studied the free vibration analysis of laminated composite plates with skew cutouts based on FSDT. Pushpendra Kumar Sharma and Jyoti Vimal [7] studied the vibration analysis of laminated composite plates using FEM.

## III. MATERIAL PROPERTIES

Shell 181 elements is used for the analytical analysis of rectangular plate with circular hole. Following material properties are considered in the analysis:

$$E_{11}/E_{22}= 25, \nu_{12}= 0.25, G_{12}=G_{13}= 0.5E_{22}, G_{23}=0.2E_{22}$$

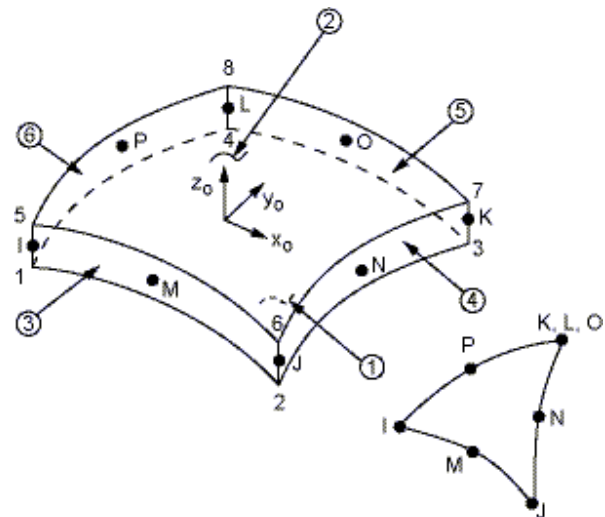


Fig. Geometry of SHELL 181

Non-dimensional frequency is given by

$$\bar{\omega} = \frac{\omega b^2}{h} \sqrt{\frac{\rho}{E_{22}}}$$

**IV. RESULT AND DISCUSSION**

Table1. Convergence study of non-dimensional frequencies for a cross-ply laminate, (a/b=1, h/b=0.01) for CCFF boundary condition for different mesh size (M×N)

M=N	Mode							
	1	2	3	4	5	6	7	8
7	5.4113	10.9971	25.7563	30.8719	34.1487	43.9286	49.1970	63.4234
Sharma et al [7]	5.4691	11.1076	25.9892	31.3080	34.6154	44.4667	49.5819	64.0844
9	5.4111	10.9930	25.6902	30.8605	34.1208	43.7862	48.7458	62.7503
Sharma et al [7]	5.4686	11.1034	25.9228	31.2964	34.5858	44.4667	49.5819	64.0844
11	5.4111	10.9909	25.6696	30.8574	34.1136	43.7541	48.5982	62.5748
Sharma et al [7]	5.4686	11.1018	25.9017	31.2933	34.5790	44.2849	48.9807	63.2036
13	5.4110	10.9909	25.6613	30.8553	34.1115	43.7428	48.5393	62.5108
Sharma et al [7]	5.4686	11.1018	25.8932	31.2917	34.5763	44.2736	48.9211	63.1403
15	5.4110	10.9909	25.6572	30.8553	34.1105	43.7387	48.5114	62.4829
Sharma et al [7]	5.4686	11.1012	25.8895	31.2912	34.5753	44.2689	48.8931	63.1140
17	5.4110	10.9909	25.6551	30.8553	34.1105	43.7356	48.4970	62.4695
Sharma et al [7]	5.4686	11.1012	25.8874	31.2906	34.5748	44.2668	48.8789	63.0981
19	5.4110	10.9909	25.6541	30.8543	34.1105	43.7345	48.4887	62.4612
Sharma et al [7]	5.4686	11.1012	25.8864	31.2906	34.5748	44.2657	48.8710	63.0876

**a) Rectangular Plates with Circular Hole**

Table2. Variation of first ten natural frequencies with size ratio (a/b=1.5, 2, 2.5, 3) for a cross-ply laminate having circular cut-out at the centre, (r/b=0.1, h/b=0.01) for fully clamped boundary condition

Mode	a/b			
	1.5	2	2.5	3
1	19.256	19.105	16.139	17.137
2	29.016	25.843	24.797	22.169
3	40.159	35.066	28.297	25.564
4	48.942	41.505	34.601	29.864
5	53.238	43.089	37.929	37.907
6	63.587	53.005	46.705	39.886
7	68.969	61.104	48.705	45.051
8	77.956	66.539	58.422	49.983
9	91.046	74.034	59.326	60.195
10	95.437	81.366	63.651	61.851

From table 2 it is clear that natural frequencies of a rectangular plate with circular cut-out at the centre decreases with size ratio of plate except for first mode.

Table3. Variation of first ten natural frequency parameters

$\bar{\omega} = \frac{\omega b^2}{h} \sqrt{\frac{\rho}{E_{22}}}$  with angle of ply ( $\Theta = 30^\circ, 45^\circ, 60^\circ, 90^\circ$ ) for an angle-ply laminate having size ratio a/b= 1.5 with circular cut-out at the centre, (r/b=0.1, h/b=0.01) for fully clamped boundary condition

Mode	$\Theta$			
	30°	45°	60°	90°
1	62.452	18.140	18.565	19.256
2	95.365	28.480	28.848	29.016
3	126.128	39.033	39.777	40.159
4	151.628	46.008	47.109	48.942
5	165.483	52.201	52.844	53.238
6	197.04	62.706	63.589	63.587
7	208.854	67.305	68.383	68.969
8	234.210	73.550	75.052	77.956
9	263.881	87.845	89.420	91.046
10	272.688	93.603	94.537	95.437

As seen from table 3 natural frequencies for first 10 modes of a rectangular plate with circular cut-out at the centre has a non-uniform variation.

Table4. Variation of first ten natural frequencies with different boundary conditions (CCCC, SSSS, CSCS, CFCF) for a cross-ply laminate having size ratio a/b= 1.5 with circular cut-out at the centre, (r/b=0.1, h/b=0.01)

Mode	Boundary Condition			
	CCCC	SSSS	CSCS	CFCF
1	19.256	7.307	10.451	7.229
2	29.016	16.109	23.665	8.232
3	40.159	23.185	25.218	15.323
4	48.942	29.043	38.685	17.626
5	53.238	33.397	44.775	22.091
6	63.587	41.853	52.075	29.386
7	68.969	47.232	59.035	34.537
8	77.956	53.169	60.574	35.686
9	91.046	64.382	69.181	41.846
10	95.437	67.247	80.414	46.727

As seen from table 4 the values of natural frequency is maximum for fully clamped condition and minimum for all sides simply supported conditions.

Table5. Variation of first ten natural frequencies with thickness ratio (h/b=0.01, 0.05, 0.1, 0.5) for a cross-ply laminate having size ratio a/b= 1.5 with circular cut-out at the centre, (r/b=0.1) for fully clamped boundary condition

Mode	h/b			
	0.01	0.05	0.1	0.5
1	19.256	2.215	125.101	1.733
2	29.016	3.322	16.900	2.464
3	40.159	4.629	21.641	3.251
4	48.942	5.655	25.130	3.891
5	53.238	6.202	26.871	4.142
6	63.587	7.546	30.429	4.785
7	68.969	8.067	32.562	5.145
8	77.956	9.012	35.935	5.713
9	91.046	10.748	39.548	6.371
10	95.437	11.421	40.340	6.469

Table 5 shows the variation of first 10 natural frequencies of rectangular plate with circular cut-out with thickness. Natural frequency first decreases with thickness and then increases and again decreases.

Table6. Variation of first ten natural frequencies with material properties ( $E_{11}/E_{22}= 10, 15, 20, 25$ ) for a cross-ply laminate having size ratio  $a/b= 1.5$  with circular cut-out at the centre, ( $r/b=0.1, h/b=0.01$ ) for fully clamped boundary condition

Mode	$E_{11}/E_{22}$			
	10	15	20	25
1	14.090	16.128	12.644	19.256
2	21.831	24.744	27.060	29.016
3	30.924	34.648	37.629	40.159
4	36.023	41.172	45.353	48.942
5	39.220	44.712	49.271	53.238
6	48.438	54.710	59.974	63.587
7	51.968	58.631	64.159	68.969
8	58.266	65.923	72.343	77.956
9	68.434	77.401	84.735	91.046
10	70.655	80.312	88.404	95.437

As shown in table 6 natural frequencies of rectangular plate with circular cut-out for first 10 modes of vibration increases with material property.

Table7. Variation of first ten natural frequencies with no. of layers ( $n= 3, 5, 7, 9$ ) for a cross-ply laminate having size ratio  $a/b= 1.5$  with circular cut-out at the centre, ( $r/b=0.1, h/b=0.01$ ) for fully clamped boundary condition

Mode	n			
	3	5	7	9
1	19.256	24.662	25.352	25.305
2	29.016	36.035	36.730	36.493
3	40.159	52.412	54.430	54.560
4	48.942	60.859	61.950	61.608
5	53.238	67.144	69.203	69.161
6	63.587	80.479	83.350	83.680
7	68.969	84.820	86.811	86.581
8	77.956	101.471	106.007	106.884
9	91.046	112.717	115.505	115.288
10	95.437	117.229	121.224	121.534

As seen from table 7 the natural frequencies for a rectangular plate with circular cut-out for first 10 modes increases with increase in number of laminates of the plate.

### V. CONCLUSIONS

Following conclusions derived from the analytical analysis of rectangular plate with circular cut-out:

- Natural frequencies for first 10 modes of vibration for a rectangular plate with circular cut-out increases with angle of ply.
- Natural frequencies for first 10 modes of vibration for a rectangular plate with circular cut-out increases with material properties of plate.

- Natural frequencies for first 10 modes of vibration for a rectangular plate with circular cut-out increases with number of laminate in the plate.
- Natural frequencies for first 10 modes of vibration for a rectangular plate with circular cut-out decreases with aspect/ size ratio.
- Natural frequencies for first 10 modes of vibration for a rectangular plate with circular cut-out increases with thickness ratio of the plate.
- Natural frequencies for first 10 modes of vibration for a rectangular plate with circular cut-out are maximum for fully clamped condition and minimum for all sides simply supported condition.

### REFERENCES

- [1] S. Liu, "A vibration analysis of composite laminated plates" Finite Elements in Analysis and Design, 9(4), 295-307, 1991.
- [2] D. Ngo-Cong, N. Mai-Duy, W. Karunasena and T. Tran-Cong, "Free vibration analysis of laminated plates based on FSDT using one- dimensional IRBFN method", Computers and Structures, 89 (1-2), 1-13, 2011.
- [3] Xinkang Li, Jifa Zhang and Yao Zheng, "Static and free vibration analysis of laminated composite plates using isogeometric approach based on the third order shear deformation theory", Advances in Mechanical Engineering, 2015.
- [4] Mohamed-Ouejdi Belarbi, Abdelouahab Tati, Houdayfa Ounis and Abdelhak Khechai, "On the free vibration analysis of laminated composite and sandwich plates: A layer wise finite element formulation", Latin American Journal of Solids Structure, 14 (12), 2017.
- [5] T. Kant and K. Swaminathan, "Analytical solutions for free vibration of laminated composite and sandwich plates based on a higher order refined theory", Composite Structures, 53, 73-85, 2001.
- [6] J. Suresh Kumar, T. Dharma Raju and K. Vijaya Kumar Reddy, "Vibration analysis of laminated plates using higher- order shear deformation theory with zig-zag function", Indian journal of Science and Technology, 4 (8), 2011.
- [7] Avadesh Kumar Sharma and N. D. Mittal, "Free vibration analysis of laminated composite plates with elastically restrained edges using FEM", Central European Journal of Engineering, 3(2), 306-315, 2013.
- [8] Harsh Kumar Bhardwaj, Jyoti Vimal and Avadesh Kumar Sharma, "Study of free vibration analysis of laminated composite plates with skew cutouts based on FSDT", Journal of Civil Engineering and Environmental Technology, 1(3), 71-75, 2014.

- [9] Pushpendra Kumar Sharma and Jyoti Vimal, “Study of the vibration analysis of laminated composite plates using FEM”, *International Journal of Advanced Mechanical Engineering*, 4(6), 675-680, 2014.
- [10] Arpita Mandal, Chaitali Ray and Salil Halder, “Free vibration analysis of laminated composite skew plates with cut-out”, *Archive of Applied Mechanics*, 9, 2017.