A Study on Triangular Cordial Graphs

S. Anusuya[1], Dr. A. Nagarajan[2]
[1]Dept of MATHEMATICS
[2]Head and Associate Professor, Dept of MATHEMATICS
[1] Aditanar College of Arts and Science, Tiruchendur – TamilNadu
[2]V.O.Chidambaram College, Thoothukudi, India–628 008.

Abstract- *Let G = (V,E) be a simple graph with p vertices and q edges. G is said to have Triangular cordial labeling if there is injective map f : V(G) → { 1,2,3,…., T_P } such that for every edge uv, the induced edge f^* is defined as follows*

$$f^*(uv) = \begin{cases} 1 & \text{if } f(u)+f(v) \text{ is a triangular number} \\ 0 & \text{if } f(u)+f(v) \text{ is not a triangular number} \end{cases}$$

With condition that $|e_f(0) - e_f(1)| \leq 1$ *where,* $e_f(0)$ is *the number of edges with label 0 and* $e_f(1)$ *is the number of edges with label 1. If G admits Triangular cordial labeling then G is called a Triangular cordial graph.*

In this chapter we identified the graphs Path - P_n*, Star -* $K_{1,n}$ *, Cycle -* C_n*, Subdivided star*
$< K_{1,n} : n >$ *are Triangular cordial graphs.*

Keywords- Triangular cordial graph, Triangular cordial labeling.

AMS Classification 05C78

I. INTRODUCTION

A graph G is a finite non-empty set of objects called vertices together with a set of unordered pairs of distinct vertices of G called edges. The vertex set and the edge set of G are denoted by V(G) and E(G) respectively.

The concept of cordial labeling introduced by **Cahit [1]** and proved certain results in cordial labeling in [2]. It motivated us to define triangular cordial labeling.

II. PRELIMINARIES

Definition : 2.1

A **Triangular number** counts objects arranged in an equilateral triangle.

The n^{th}**Triangular** number is the number of dots in the triangular arrangement With n dots on a side, and is **equal to the sum of the n natural numbers from 1 to n.**

$$T_n = \sum_{i=1}^{n} n = n(n+1)/2$$

Definition : 2.2

Let G = (V,E) be a simple graph with p vertices and q edges. G is said to have **Triangular cordial labeling** if there is injective map

f : V(G) → { 1,2,3,…., T_P } such that for every edge uv, the induced edge f^* is defined as follows

$$f^*(uv) = \begin{cases} 1 & \text{if } f(u)+f(v) \text{ is a triangular number} \\ 0 & \text{if } f(u)+f(v) \text{ is not a triangular number} \end{cases}$$

With condition that $|e_f(0) - e_f(1)| \leq 1$ where , $e_f(0)$ is the number of edges with label 0 and $e_f(1)$ is the number of edges with label 1. If G admits Triangular cordial labeling then G is called a Triangular cordial graph.

Definition : 2.3

Path is a graph whose vertices can be listed in the order $(u_1, u_2, u_3, \ldots\ldots u_n)$ such that the edges are $\{u_i, u_{i+1}\}$ where $i = 1,2,3,\ldots\ldots n-1$.

Definition : 2.4

A **star** $K_{1,n}$ is a tree with one internal vertex and n edges.

Definition : 2.5

A closed path is called a **cycle** and a cycle of length n is denoted by C_n.

Definition : 2.6

A **Subdivided graph** is obtained by replacing every edge of G by P_3. It is denoted by S(G).

Definition : 2.7

Subdivided star is a graph with one internal vertex and 2n edges . It is denoted by $< K_{1,n} : n >$.

III. MAIN RESULT

THEOREM 3.1

Path P_n is a Triangular cordial graph.

Proof:

Let G = P_n be a graph.
Let V(G) = {u_1, u_2,….., u_n}
Let E(G) = { (u_i, u_{i+1}) : 1 $\leq i \leq n$-1}
Then $|V(G)|$ = n and $|E(G)|$ = n - 1

Case (i)

Suppose n is even, n = 2k (say)

Let f : V(G) $\rightarrow$ {1,2,3,….., T_{2k}} be defined as follows:

f(u_1) = T_1

$$f(u_i) = \begin{cases} T_i - f(u_{i-1}) & , 2 \leq i \leq k+1 \\ T_i - f(u_{i-1}) + 1 & , k+2 \leq i \leq 2k \end{cases}$$

The induced edge labels are given below

For $1 \leq i \leq k$,

f(u_i) + f(u_{i+1}) = f(u_i) + T_{i+1} $-$ f(u_i) = T_{i+1}

$f^*(u_i u_{i+1})$ = 1.

f(u_{k+1}) + f(u_{k+2}) = f(u_{k+1}) + T_{k+2} $-$ f(u_{k+1}) + 1 = T_{k+2} + 1

$f^*(u_{k+1} u_{k+2})$ = 0.

For $k+2 \leq i \leq 2k-1$,

f(u_i) + f(u_{i+1}) = f(u_i) + T_{i+1} $-$ f(u_i) + 1 = T_{i+1} + 1

(not a triangular number)

$f^*(u_i u_{i+1})$ = o.

Case (ii)

Suppose n is odd, n = 2k+1 (say)

Let f : V(G) $\rightarrow$ {1,2,3,….., T_{2k+1}} be defined as follows:

f(u_1) = T_1

$$f(u_i) = \begin{cases} T_i - f(u_{i-1}) & , 2 \leq i \leq k+1 \\ T_i - f(u_{i-1}) + 1 & , k+2 \leq i \leq 2k+1 \end{cases}$$

The induced edge labels are given below

For $1 \leq i \leq k$,

f(u_i) + f(u_{i+1}) = f(u_i) + T_{i+1} $-$ f(u_i) = T_{i+1}

$f^*(u_i u_{i+1})$ = 1.

f(u_{k+1}) + f(u_{k+2}) = f(u_{k+1}) + T_{k+2} $-$ f(u_{k+1}) + 1 = T_{k+2}+1

$f^*(u_{k+1} u_{k+2})$ = 0.

For $k+2 \leq i \leq 2k$,

f(u_i) + f(u_{i+1}) = f(u_i) + T_{i+1} $-$ f(u_i) + 1 = T_{i+1}+1

(not a triangular number)

$f^*(u_i u_{i+1})$ = o.

It is observed that,

e_f(0) = k-1 and e_f(1) = k , if n is even.

e_f(0) = k and e_f(1) = k , if n is odd.

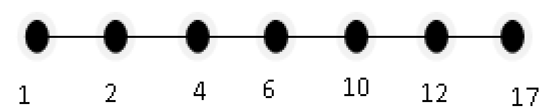
Clearly, $|e_f(0) - e_f(1)| \leq 1$.

Then f is a Triangular cordial labeling.

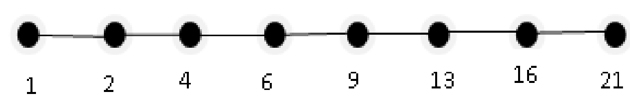
Hence , the Path P_n is a Triangular cordial graph.

Example 3.2

P_7



P_8



THEOREM 3.3

The star $K_{1,n}$ is a Triangular cordial graph.

Proof:

Let G = $K_{1,n}$ be a graph where ,
V(G) = {u_1, u_2,….., u_n, v} and
E(G) = { (v, u_i) : 1 $\le i \le n$}
Then $|V(G)|$ = n + 1 and $|E(G)|$ = n

Case(i)
Suppose when n is even, n = 2k (say)
Define f : V(G) $\to$ {1,2,3,….., T_{2k}} as follows:

f(v)= T_1

$$f(u_i) = \begin{cases} T_{i+1} - 1, & 1 \le i \le k \\ T_{i+1}, & k+1 \le i \le 2k \end{cases}$$

The induced edge labels are,
For $1 \le i \le k$,
$f(u_i) + f(v) = T_{i+1} - 1 + T_1 = T_{i+1} - 1 + 1 = T_{i+1}$

$f^*(u_i v)$ = 1
For $k + 1 \le i \le 2k$,
$f(u_i) + f(v) = T_{i+1} + T_1 = T_{i+1} + 1$
(not a triangular number)
$f^*(u_i v)$ = o.

Case(ii)

Suppose when n is odd, n = 2k + 1 (say)
Define f : V(G) $\to$ {1,2,3,….., T_{2k+1}} as follows:

f(v) = T_1

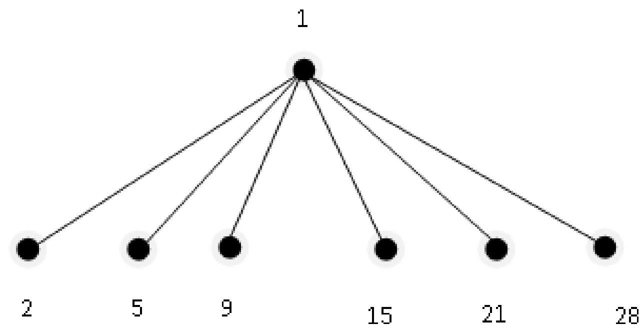
$$f(u_i) = \begin{cases} T_{i+1} - 1 & ,1 \le i \le k \\ T_{i+1}, & k+1 \le i \le 2k+1 \end{cases}$$

The induced edge labels are given below
For $1 \le i \le k$,
$f(u_i) + f(v) = T_{i+1} - 1 + T_1 = T_{i+1} - 1 + 1 = T_{i+1}$

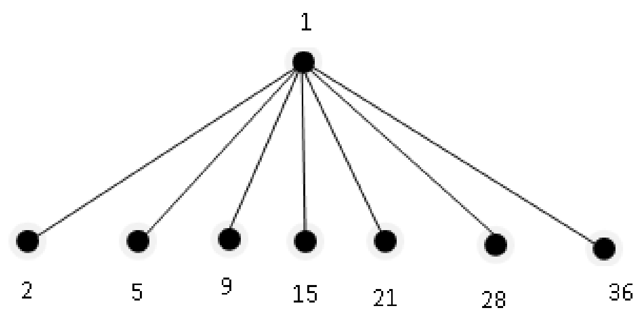
$f^*(u_i v)$ = 1.
For $k+1 \le i \le 2k+1$,
$f(u_i) + f(v) = T_{i+1} + T_1 = T_{i+1} + 1$
(not a triangular number)
$f^*(u_i v)$ = o.
It is observed that,
$e_f(0)$ = k and $e_f(1)$ = k , if n is even.
$e_f(0)$ = k + 1 and $e_f(1)$ = k , if n is odd.
Clearly, $|e_f(0) - e_f(1)| \le 1$.
Then f is a Triangular cordial labeling.
Hence , the star $K_{1,n}$ is a Triangular cordial graph.

Example 3.4

$K_{1,6}$



$K_{1,7}$



THEOREM 3.5

The Cycle C_n is a Triangular Cordial graph.

Proof:

Let G = C_n be a graph where,
Let V(G) = { $u_1, u_2, u_3, \ldots, u_n$ } and
E(C_n) = {(u_i, u_{i+1}) : $1 \le i \le n-1$} ∪ { (u_n, u_1) }
Then $|V(G)|$ = n and $|E(G)|$ = n

Case (i)

Suppose n is even, n = 2k (say)
Let f : V(G) $\to$ { 1,2,3,….., T_{2k} } be defined as follows:

f(u_1)= T_1

$$f(u_i) = \begin{cases} T_i - f(u_{i-1}) & , 2 \le i \le k+1 \\ T_i - f(u_{i-1}) + 1, & k+2 \le i \le 2k-1 \end{cases}$$

$f(u_n) = f(u_{2k}) = T_{2k} - f(u_{2k-1}) + r$ so that $0 < r \leq 2$ and $T_{2k} - f(u_{2k-1}) + r + 1$ and $T_{2k} + r$ are not triangular numbers ----------- (1)

Then the induced edge labels are given below

For $1 \leq i \leq k$,

$f(u_i) + f(u_{i+1}) = f(u_i) + T_{i+1} - f(u_i) = T_{i+1}$

$f^*(u_i u_{i+1}) = 1$

$f(u_{k+1}) + f(u_{k+2}) = f(u_{k+1}) + T_{k+2} - f(u_{k+1}) + 1 = T_{k+2} + 1$

(not a triangular number)

$f^*(u_{k+1} u_{k+2}) = 0$

For $k + 2 \leq i \leq 2k - 2$,

$f(u_i) + f(u_{i+1}) = f(u_i) + T_{i+1} - f(u_i) + 1 = T_{i+1} + 1$

(not a triangular number)

$f^*(u_i u_{i+1}) = 0$

$f(u_{2k-1}) + f(u_{2k}) = f(u_{2k-1}) + T_{2k} - f(u_{2k-1}) + r = T_{2k} + r$

(not a triangular number ,as per the choice of r in (1))

$f^*(u_{2k-1} u_{2k}) = 0$

$f(u_{2k}) + f(u_1) = T_{2k} - f(u_{2k-1}) + r + T_1 = T_{2k} - f(u_{2k-1}) + r + 1$

(not a triangular number ,as per the choice of r in (1))

$f^*(u_{2k} u_1) = 0$

Case (ii)

Suppose n is odd, n = 2k + 1 (say)

Let f : V(G) $\rightarrow$ { 1,2,3,….., T_{2k+1} } be defined as follows:

$f(u_1) = T_1$

$$f(u_i) = \begin{cases} T_i - f(u_{i-1}) & , 2 \leq i \leq k+1 \\ T_i - f(u_{i-1}) + 1 & , k+2 \leq i \leq 2k \end{cases}$$

$f(u_n) = f(u_{2k+1}) = T_{2k+1} - f(u_{2k}) + r$ so that $0 < r \leq 2$ and $T_{2k+1} - f(u_{2k}) + r + 1$ and $T_{2k+1} + r$ are not triangular numbers --------- (2)

Then the induced edge labels are given below

For $1 \leq i \leq k$,

$f(u_i) + f(u_{i+1}) = f(u_i) + T_{i+1} - f(u_i) = T_{i+1}$

$f^*(u_i u_{i+1}) = 1$

$f(u_{k+1}) + f(u_{k+2}) = f(u_{k+1}) + T_{k+2} - f(u_{k+1}) + 1 = T_{k+2} + 1$

(not a triangular number)

$f^*(u_{k+1} u_{k+2}) = 0$

For $k + 2 \leq i \leq 2k - 1$,

$f(u_i) + f(u_{i+1}) = f(u_i) + T_{i+1} - f(u_i) + 1 = T_{i+1} + 1$

(not a triangular number)

$f^*(u_i u_{i+1}) = 0$

$f(u_{2k}) + f(u_{2k+1}) = f(u_{2k}) + T_{2k+1} - f(u_{2k}) + r = T_{2k} + r$

(not a triangular number ,as per the choice of r in (2))

$f^*(u_{2k} u_{2k+1}) = 0$

$f(u_{2k+1}) + f(u_1) = T_{2k+1} - f(u_{2k}) + r + T_1 = T_{2k+1} - f(u_{2k}) + r + 1$

(not a triangular number ,as per the choice of r in (2))

$f^*(u_{2k+1} u_1) = 0$

It is observed that

$e_f(0) = k$ and $e_f(1) = k$, if n is even.

$e_f(0) = k + 1$ and $e_f(1) = k$, if n is odd.

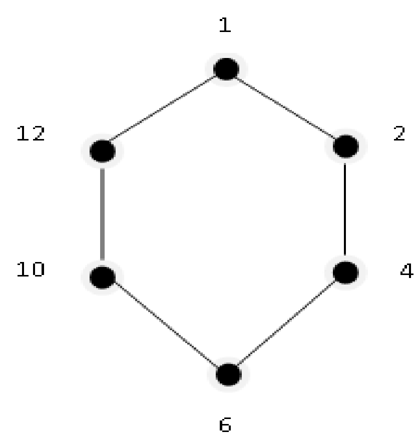
Clearly, $|e_f(0) - e_f(1)| \leq 1$.

Then f is a Triangular cordial labeling.

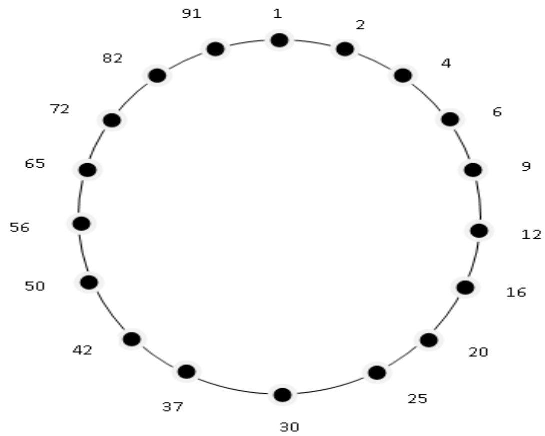
Hence , the Cycle C_n is a Triangular cordial graph.

Example 3.6

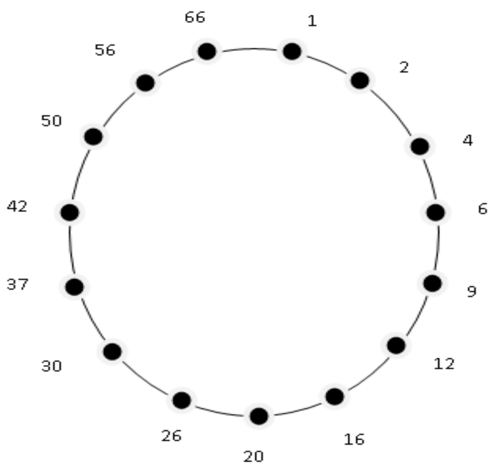
$i)$ C_6 (r=1)



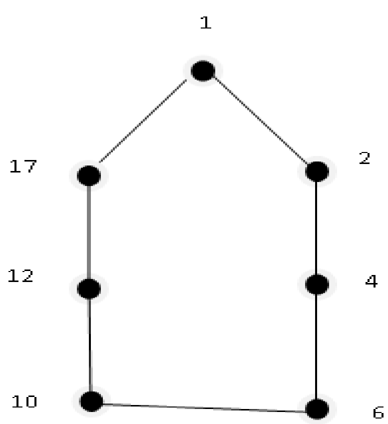
C_{18} (r=2)



C_{15} (r=2)



C_7 (r=1)



THEOREM 3.7

The subdivided star $< K_{1,n} : n >$ is a Triangular cordial graph.

Proof:

Let G = $< K_{1,n} : n >$ be a graph.

Let V(G) = $\{ u, u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n \}$

Let E(G) = $\{ (u, u_i), (u_i, v_i) : 1 \leq i \leq n \}$

Then $|V(G)|$ = 2n + 1 and $|E(G)|$ = 2n

Define f : V(G) $\rightarrow$ $\{1,2,3,\ldots, T_{2n+1}\}$ as follows:

f(u) = T_1

$f(u_i) = T_{i+1} - 1, \text{ for } 1 \leq i \leq n$

$f(v_i) = T_{2n+1} - f(u_i) + 1, \text{ for } 1 \leq i \leq n$

The induced edge labels ar given below

For $1 \leq i \leq n$,

$f(u) + f(u_i) = T_1 + T_{i+1} - 1 = 1 + T_{i+1} - 1 = T_{i+1}$

$f^*(uu_i) = 1$

$f(u_i) + f(v_i) = f(u_i) + T_{2n+1} - f(u_i) + 1 = T_{2n+1} + 1$

(not a triangular number)

$f^*(u_i v_i) = 0$

It is observed as,

$e_f(0)$ = n and $e_f(1)$ = n

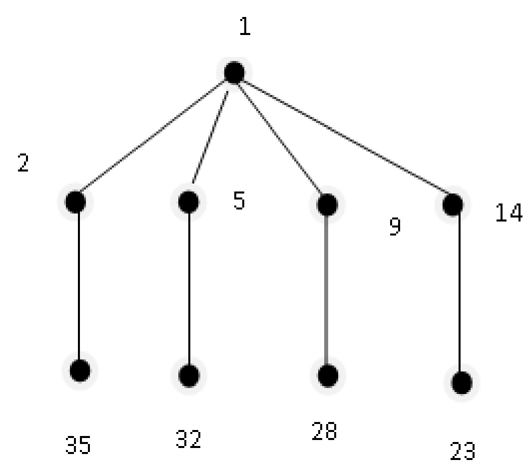
Clearly, $|e_f(0) - e_f(1)| \leq 1$.

Then f is a Triangular cordial labeling.

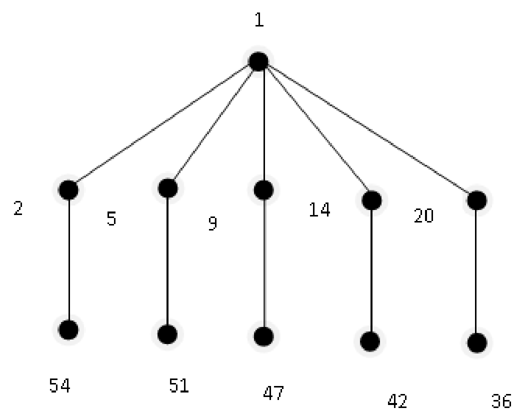
Hence , the subdivided star $< K_{1,n} : n >$ is a Triangular cordial graph.

Example 3.8

$< K_{1,4} : 4 >$



$< K_{1,5} : 5 >$



IV. CONCLUSION

We have introduced here new idea of Triangular cordial labeling. This will add a new dimension to the research work in graph labeling on various numbers. Here we have shown four standard graphs are Triangular cordial graphs. Examples are provided at the end of each theorem for wider grasping of the pattern of labeling defined in each theorem.

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