Some Result on Integral Root Labeling of Graphs

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Abstract- *Let $G = (V, E)$ be a graph with p vertices and q edges. Let $f: V \to \{1,2, \ldots q+1\}$ is called an **Integral Root labeling** if it is possible to label all the vertices $v \in V$ with distinct elements from $\{1,2, \ldots q+1\}$ such that it induces an edge labeling $f^+: E \to \{1,2, \ldots q\}$ defined as $f^+(uv) = \left\lceil \sqrt{\frac{(f(u))^2+(f(v))^2+f(u)f(v)}{3}} \right\rceil$ is distinct for all $uv \in E$. (i.e.) The distinct vertex labeling induces a distinct edge labeling on the graph. The graph which admits Integral Root labeling is called an **Integral Root Graph**.*

In this paper, we investigate the some result on Integral Root labeling of graphs like $T_n \odot K_1$, $Q_n \odot K_1$, $D(C_n)$, $TL_n \odot K_1$, $D(T_n)$, $D(T_n) \odot K_1$, $D(Q_n)$, $D(Q_n) \odot K_1$.

Keywords- $T_n \odot K_1$, $Q_n \odot K_1$, $D(C_n)$, $TL_n \odot K_1$, $D(T_n)$, $D(T_n) \odot K_1$, $D(Q_n)$, $D(Q_n) \odot K_1$.

I. INTRODUCTION

The graph considered here will be finite, undirected and simple. The vertex set is denoted by $V(G)$ and the edge set is denoted by $E(G)$. For all detailed survey of graph labeling we refer to Gallian [1]. For all standard terminology and notations we follow Haray[2]. V.L Stella Arputha Mary and N.Nanthini introduced the concept of Integral Root Labeling of graphs in [8]. In this paper we investigate Integral Root labeling of disconnected graphs. The definitions and other informations which are useful for the present investigation are given below.

II. BASIC DEFINITION

Definition: 3.1

A walk in which $u_1, u_2, \ldots u_n$ are distinct is called a **Path**. A path on n vertices is denoted by P_n

Definition: 3.2

The graph obtained by joining a single pendent edge to each vertex of a path is called a **Comb.**

Definition: 3.3

The Cartesian product of two graphs $G_1=(V_1,E_1)$ and $G_2=(V_2,E_2)$ is a graph $G=(V,E)$ with $V=V_1 \times V_2$ and two vertices $u=(u_1u_2)$ and $v=(v_1v_2)$ are adjacent in $G_1 \times G_2$ whenever ($u_1=v_1$and u_2 is adjacent to v_2) or ($u_2=v_2$and u_1 is adjacent to v_1) .It is denoted by $G_1 \times G_2$.

Definition: 3.3

The Corona of two graphs G_1 and G_2 is the graph $G=G_1 \odot G_2$ formed by taking one copy of G_1 and $|(G_1)|$ copies of G_2 where the i^{th} vertex of G_1 is adjacent to every vertex in the i^{th} copy of G_2.

Definition: 3.4

The product graph $P_2 \times P_n$ is called a **Ladder** and it is denoted by L_n

Definition: 3.5

The union of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is a graph $G = G_1 \cup G_2$ with vertex set $V = V_1 \cup V_2$ and the edge set $E = E_1 \cup E_2$.

III. MAIN RESULT

Theorem: 4.1

$T_n \odot K_1$ is a Integral Root graph $n \geq 2$.

Proof:

Let $G = T_n \odot K_1$.

Let $u_1, u_2, \ldots, u_n$ be a path of length n.

Let v_i, $1 \leq i \leq n-1$ be the new vertex joined to u_i and u_{i+1}.

Let x_i be the vertex which is joined to u_i, $1 \leq i \leq n$.

Let y_i be the vertex which is joined to v_i, $1 \le i \le n-1$.

Define a function $f: V(G) \to \{1, 2, \dots, q+1\}$ by

$f(u_i) = 5i - 3; \quad 1 \le i \le n;$
$f(v_i) = 5i - 2; \quad 1 \le i \le n-1;$
$f(x_i) = 5i - 4; \quad 1 \le i \le n;$
$f(y_i) = 5i - 1; \quad 1 \le i \le n-1.$

Then the edge labels are

$f^+(u_i u_{i+1}) = 5i - 1; \quad 1 \le i \le n-1;$
$f^+(u_i v_i) = 5i - 3; \quad 1 \le i \le n-1;$
$f^+(v_i u_{i+1}) = 5i; \quad 1 \le i \le n-1;$
$f^+(u_i x_i) = 5i - 4; \quad 1 \le i \le n;$
$f^+(v_i y_i) = 5i - 2; \quad 1 \le i \le n-1.$

Then the edges labels are distinct.

Hence $T_n \odot K_1$ is a Integral Root graph.

Example: 4.2

The Integral Root labeling of $T_4 \odot K_1$ is given below.

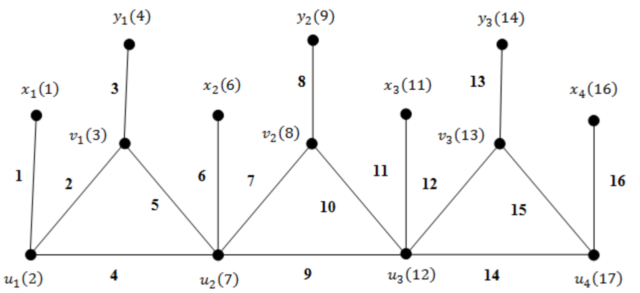

Figure: 1

Theorem: 4.3

$Q_n \odot K_1$ is an Integral Root graph

Proof:

Let $G = Q_n \odot K_1$.

Let $u_1, u_2, \dots, u_n$ be a path.

Let v_i and w_i be two vertices joined to u_i and u_{i+1} respectively and then join v_i and w_i, $1 \le i \le n-1$.

Let y_i be the new vertex joined to v_i, $1 \le i \le n-1$.

Let z_i be the new vertex joined to w_i, $1 \le i \le n-1$.

Let x_i be the new vertex joined to u_i, $1 \le i \le n$.

Define a function $f: V(G) \to \{1, 2, \dots, q+1\}$ by

$f(u_i) = 7i - 6; \quad 1 \le i \le n;$
$f(v_i) = 7i - 4; \quad 1 \le i \le n-1;$
$f(x_i) = 7i - 5; \quad 1 \le i \le n;$
$f(y_i) = 7i - 3; \quad 1 \le i \le n-1;$
$f(w_i) = 7i; \quad 1 \le i \le n-1;$
$f(z_i) = 7i - 1; \quad 1 \le i \le n-1.$

Then the edge labels are

$f^+(u_i u_{i+1}) = 7i - 3; \quad 1 \le i \le n-1;$
$f^+(u_i v_i) = 7i - 5; \quad 1 \le i \le n-1;$
$f^+(w_i u_{i+1}) = 7i; \quad 1 \le i \le n-1;$
$f^+(u_i x_i) = 7i - 6; \quad 1 \le i \le n;$
$f^+(v_i y_i) = 7i - 4; \quad 1 \le i \le n-1;$
$f^+(v_i w_i) = 7i - 2; \quad 1 \le i \le n-1;$
$f^+(w_i z_i) = 7i - 1; \quad 1 \le i \le n-1.$

It is found all the edges labels are distinct.

Hence $Q_n \odot K_1$ is a Integral Root graph.

Example: 4.4

The Integral Root labeling of $Q_4 \odot K_1$ is given below.

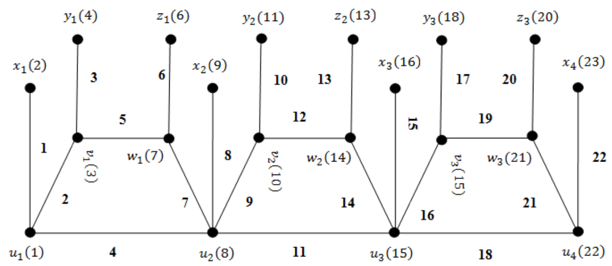

Figure: 2

Theorem: 4.5

Double Comb $D(C_n)$ is a Integral Root graph.

Proof:

Let $G = D(C_n)$

Let u_i, v_i, and w_i, $1 \le i \le n$ be the new vertices of G.

Define a function $f: V(G) \to \{1, 2, 3, \dots, q+1\}$ by

$f(u_i) = 3i - 1; \quad 1 \le i \le n;$
$f(v_i) = 3i; \quad 1 \le i \le n;$
$f(w_i) = 3i - 2; \quad 1 \le i \le n.$

Then we find the edge labels

$f^+(u_iu_{i+1}) = 3i;\quad 1 \le i \le n-1;$
$f^+(u_iv_i) = 3i-1;\quad 1 \le i \le n;$
$f^+(u_iw_i) = 3i-2;\quad 1 \le i \le n.$

Then the edge labels are distinct.

Hence G is an Integral Root graph.

Example: 4.6

The Integral root labeling of D(C_6)is given below.

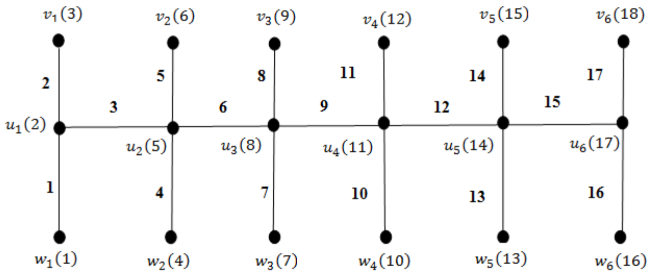

Figure: 3

Theorem: 4.7

$TL_n \odot K_1$ is an Integral Root graph.

Proof:

Let u_i and v_i, $1 \le i \le n$ be the vertices of TL_n.
Let x_i, $1 \le i \le n$ be the vertex which is attached to u_i.
Let y_i, $1 \le i \le n$ be the vertex which is attached to v_i.
Let $G = TL_n \odot K_1$.

Define a function $f : V(G) \to \{1,2,\dots,q+1\}$ by

$f(u_i) = 6i-5;\quad 1 \le i \le n;$
$f(v_i) = 6i-3;\quad 1 \le i \le n;$
$f(x_i) = 6i-4;\quad 1 \le i \le n;$
$f(y_i) = 6i-2;\quad 1 \le i \le n.$

Then we find the edge labels

$f^+(u_iu_{i+1}) = 6i-2;\quad 1 \le i \le n-1;$
$f^+(u_ix_i) = 6i-5;\quad 1 \le i \le n;$
$f^+(u_iv_i) = 6i-4;\quad 1 \le i \le n;$
$f^+(v_iv_{i+1}) = 6i;\quad 1 \le i \le n-1;$
$f^+(v_iy_i) = 6i-3;\quad 1 \le i \le n;$
$f^+(u_iv_{i+1}) = 6i-1;\quad 1 \le i \le n-1.$

Then the edge labels are distinct.

Hence G is an Integral Root graph.

Example: 4.8

The Integral Root labeling of $TL_5 \odot K_1$ is given below.

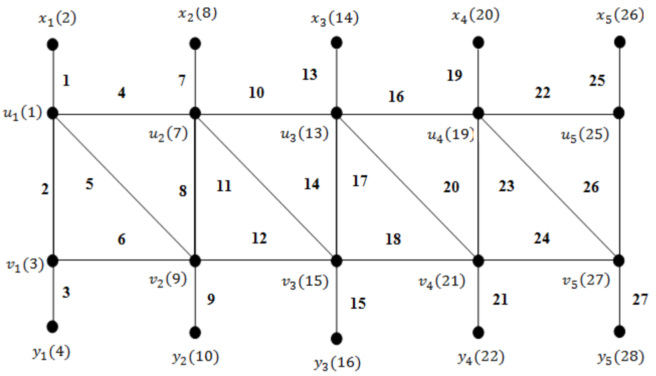

Figure: 4

Theorem: 4.9

A Double Triangular $D(T_n)$ is a Integral Root graph.

Proof:

Let $D(T_n)$ be the Double Triangular.
Consider a Path $u_1, u_2, \dots, u_n$.
Join u_iu_{i+1} with two new vertices v_i and w_i, $1 \le i \le n-1$.

Define a function $f : V(D(T_n)) \to \{1,2,\dots,q+1\}$ by
$f(u_i) = 5i-4;\quad 1 \le i \le n;$
$f(v_i) = 5i-3;\quad 1 \le i \le n-1;$
$f(w_i) = 5i-1;\quad 1 \le i \le n-1.$

Then the edge labels are

$f^+(u_iv_i) = 5i-4;\quad 1 \le i \le n-1;$
$f^+(u_iu_{i+1}) = 5i-2;\quad 1 \le i \le n-1;$
$f^+(u_{i+1}v_i) = 5i-1;\quad 1 \le i \le n-1;$
$f^+(u_iw_i) = 5i-3;\quad 1 \le i \le n-1;$
$f^+(u_{i+1}w_i) = 5i;\quad 1 \le i \le n-1.$

Then the edges labels are distinct.

Hence $D(T_n)$ is Integral Root graph.

Example: 4.10

An Integral Root labeling of $D(T_4)$ is given below.

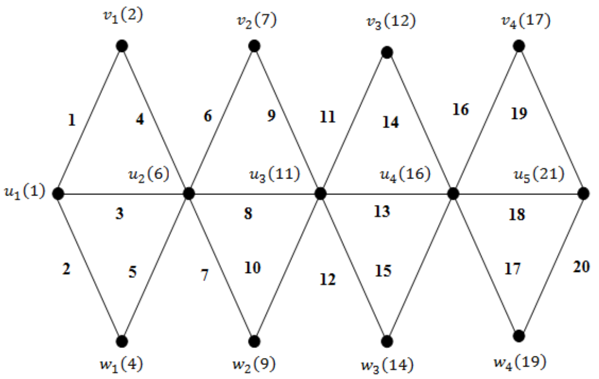

Figure: 5

Theorem: 4.11

$D(T_n)\odot K_1$ is an Integral Root graph.

Proof:

Let $G = D(T_n)\odot K_1$

Let $u_1, u_2, \dots, u_n$ be the path of length n.

Let v_i and w_i, $1 \le i \le n-1$ be the two vertices which are joined to u_i and u_{i+1}.

Let x_i and y_i, $1 \le i \le n$ be two new vertices which are attached to u_i.

Let t_i be the vertex attached to v_i, $1 \le i \le n-1$ and s_i be the vertex attached to w_i, $1 \le i \le n-1$.

Define a function $f: V(G) \to \{1,2,\dots,q+1\}$ by

$f(u_i) = 9i-5;\ \ 1 \le i \le n;$

$f(v_i) = 9i-8;\ \ 1 \le i \le n-1;$

$f(w_i) = 9i-4;\ \ 1 \le i \le n-1;$

$f(s_i) = 9i-3;\ \ 1 \le i \le n-1;$

$f(t_i) = 9i-7;\ \ 1 \le i \le n-1;$

$f(x_i) = 9i-1;\ \ 1 \le i \le n;$

$f(y_i) = 9i-6;\ \ 1 \le i \le n.$

Then the edge labels are

$f^+(u_iv_i) = 9i-7;\ \ 1 \le i \le n-1;$

$f^+(u_iw_i) = 9i-5;\ \ 1 \le i \le n-1;$

$f^+(u_{i+1}v_i) = 9i-2;\ 1 \le i \le n-1;$

$f^+(u_{i+1}w_i) = 9i;\ \ 1 \le i \le n-1;$

$f^+(u_iu_{i+1}) = 9i-1;\ 1 \le i \le n-1;$

$f^+(u_iy_i) = 9i-6;\ \ 1 \le i \le n;$

$f^+(u_ix_i) = 9i-3;\ \ 1 \le i \le n;$

$f^+(v_it_i) = 9i-8;\ \ 1 \le i \le n-1;$

$f^+(w_is_i) = 9i-4;\ \ 1 \le i \le n-1.$

Then the edge labels are distinct.

Hence $D(T_n)\odot K_1$ is an Integral Root graph.

Example: 4.12

The Integral Root labeling of $D(T_4)\odot K_1$ is given below.

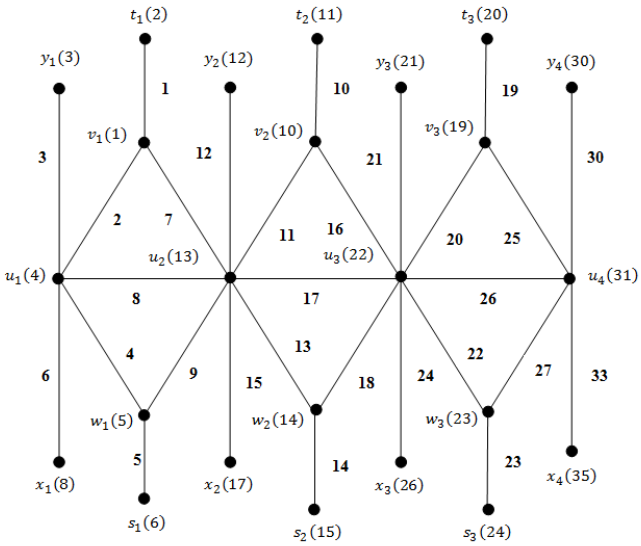

Figure: 6

Theorem: 4.13

A Double Quadrilateral Snake $D(Q_n)$ is a Integral Root graph.

Proof:

Let $D(Q_n)$ be the Double Quadrilateral Snake.

Let P_n be the path $u_1, u_2, \dots, u_n$.

Join u_i and u_{i+1} to four new vertices v_i, w_i, x_i and, y_i $1 \le i \le n-1$.

Define a function $f: V(D(T_n)) \to \{1,2,\dots,q+1\}$ by

$f(u_i) = 7i-6;\ \ 1 \le i \le n;$

$f(v_i) = 7i-5;\ \ 1 \le i \le n-1;$

$f(w_i) = 7i-4;\ \ 1 \le i \le n-1;$

$f(x_i) = 7i-2;\ \ 1 \le i \le n-1;$

$f(y_i) = 7i; \quad 1 \le i \le n-1.$

Then the edge labels are

$$
\begin{aligned}
&f^+(u_iv_i) = 7i - 6; && 1 \le i \le n-1;\\
&f^+(u_iu_{i+1}) = 7i - 3; && 1 \le i \le n-1;\\
&f^+(u_{i+1}w_i) = 7i - 2; && 1 \le i \le n-1;\\
&f^+(v_iw_i) = 7i - 5; && 1 \le i \le n-1;\\
&f^+(u_{i+1}y_i) = 7i; && 1 \le i \le n-1;\\
&f^+(u_ix_i) = 7i - 4; && 1 \le i \le n-1;\\
&f^+(x_iy_i) = 7i - 1; && 1 \le i \le n-1.
\end{aligned}
$$

Then the edges labels are distinct.

Hence $D(Q_n)$ is an Integral Root graph.

Example: 4.14

An Integral Root labeling of $D(Q_4)$ is given below.

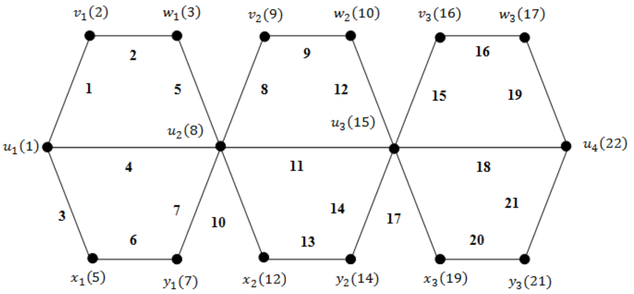

Figure: 7

Theorem: 4.15

$D(Q_n) \odot K_1$ is an Integral Root graph.

Proof:

Let $G = D(Q_n) \odot K_1$

Join u_i and u_{i+1} to four new vertices v_i, w_i, x_i, y_i by the edges u_iv_i, $u_{i+1}w_i$, u_ix_i, v_iw_i, $u_{i+1}y_i$, x_iy_i, $1 \le i \le n-1$.

Let v_i and w_i, be the two vertices joined to a_i and b_i, $1 \le i \le n-1$ respectively.

Let x_i and y_i be two new vertices joined to c_i and d_i, $1 \le i \le n-1$ respectively.

Let s_i and t_i, be two vertices joined to u_i, $1 \le i \le n-1$.

Define a function $f: V(G) \to \{1, 2, \dots, q+1\}$ by

$$
\begin{aligned}
&f(u_i) = 13i - 9; && 1 \le i \le n;\\
&f(v_i) = 13i - 7; && 1 \le i \le n-1;\\
&f(w_i) = 13i - 3; && 1 \le i \le n-1;\\
&f(x_i) = 13i - 12; && 1 \le i \le n-1;\\
&f(y_i) = 13i - 6; && 1 \le i \le n-1;\\
&f(a_i) = 13i - 5; && 1 \le i \le n-1;\\
&f(b_i) = 13i; && 1 \le i \le n-1;\\
&f(c_i) = 13i - 11; && 1 \le i \le n-1;\\
&f(d_i) = 13i - 1; && 1 \le i \le n-1;\\
&f(s_i) = 13i - 10; && 1 \le i \le n;\\
&f(t_i) = 13i - 4; && 1 \le i \le n.
\end{aligned}
$$

Then the edge labels are

$$
\begin{aligned}
&f^+(u_iv_i) = 13i - 8; && 1 \le i \le n-1;\\
&f^+(v_iw_i) = 13i - 5; && 1 \le i \le n-1;\\
&f^+(u_ix_i) = 13i - 11; && 1 \le i \le n-1;\\
&f^+(u_{i+1}w_i) = 13i; && 1 \le i \le n-1;\\
&f^+(u_iu_{i+1}) = 13i - 3; && 1 \le i \le n;\\
&f^+(u_{i+1}y_i) = 13i - 1; && 1 \le i \le n-1;\\
&f^+(v_ia_i) = 13i - 6; && 1 \le i \le n-1;\\
&f^+(x_iy_i) = 13i - 9; && 1 \le 1 \le n-1;\\
&f^+(w_ib_i) = 13i - 2; && 1 \le i \le n-1;\\
&f^+(x_ic_i) = 13i - 12; && 1 \le i \le n-1;\\
&f^+(y_id_i) = 13i - 4; && 1 \le i \le n-1;\\
&f^+(u_is_i) = 13i - 10; && 1 \le 1 \le n;\\
&f^+(u_it_i) = 13i - 7; && 1 \le i \le n.
\end{aligned}
$$

Then the edge labels are distinct.

Hence $D(Q_n) \odot K_1$ is an Integral Root graph.

Example: 4.16

The Integral Root labeling of $D(Q_4) \odot K_1$ is given below

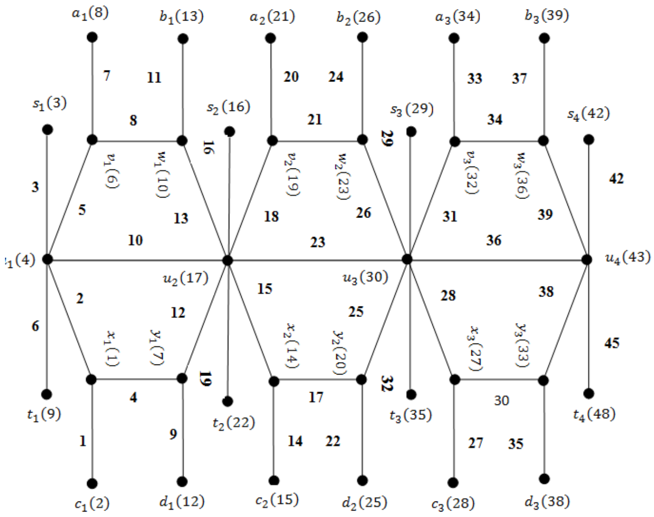

Figure: 8

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