# Mathematical Modeling & Control Techniques Of Quadcopter-UAV: A Review

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***Abstract***- *Since last two-three decades, a lot of experimental, analytical and computational work is being carried out in a robotic system such as quadcopter. Various models and controllers have been applied to control the dynamics of Quadcopter. Further simplifications of expressions and their solutions are done either in MATLAB or similar software's.*

*It is a dynamic vehicle controlled by four input forces (one for each rotor) and six degrees of freedom (6DOF). The frequency of the four rotors changes by the controller to achieve required motion of quadcopter, hence change the lift force and rotational force. In this paper the review of mathematical modeling of quadcopter is represented.*

***Keywords***- Quadcopter, Control, Mathematical modeling.

## I. INTRODUCTION

A Quad Copter is unmanned aerial vehicle with four-rotor to control its motion in six degrees of freedom (6DOF). The pairs of rotors rotate in clockwise - anticlockwise direction. To control the yaw produced due to the drag force on propellers. Most of the practicing engineers and researchers have been using quadcopter for an incredible growth in applications and its simple mechanical design. They have demand in the market because of their ability to replace conventional operations in standard and complicated tasks, which thus gives an economical model of various aerial operations. These quadcopters are useful for different applications like military missions, climate monitoring, news media, and for safety authorization offices. For rolling and Pitching, Quadcopter tilts toward the direction of the slow spinning motor. Linear Motion is achieved by dividing the thrust into two directions which are done by Roll and pitch angles. A mathematical model has been developed by using kinematics and dynamics of a quadcopter for trajectory control.

## II. HISTORY OF QUADCOPTER

The first quadcopter was developed in 1924 by French engineer Étienne Oehmichen. He flew his quadcopter for a distance of 360m (1,181ft) setting a world record. In the same year he flew a 1km (0.62miles) circle in 7m and 40s [18]. The aircraft exhibited a considerable degree of stability and controllability for its time, and made more than a thousand test flights. After Oehmichen, Dr. George de Bothezat and Ivan Jerome developed this quadcopter with six bladed rotors at the end of an X-shaped structure in October 1922 as shown in fig. 1. The design featured two engines driving four rotors with wings added for additional lift in forward flight. No tail rotor was needed and control was obtained by varying the thrust between rotors. Flown successfully many times in the mid-1950s, this helicopter proved the quadrotor design and it was also the first four-rotor helicopter to demonstrate successful forward flight [8].

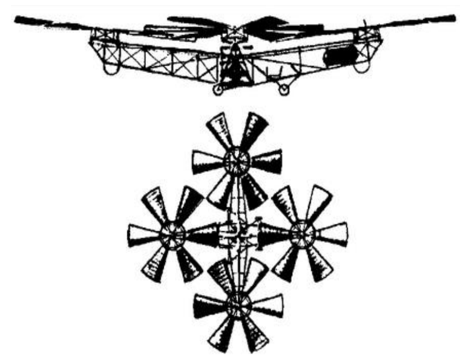

Fig.1 The de Bothezat Quadrotor [8]

## III. LITERATURE SURVEY

A quadcopter has four motors mounted at the ends of cross arm which are labelled as 1 through 4. The motor 1 & 3 rotate in counter clockwise whereas motor 2 & 4 rotates in clockwise direction. The center of mass and the origin is defined at the center of the body-fixed frame $\{B\}$, which is attached to the vehicle. Rotor 1, 2, 3 and 4 produces upward thrust T1, T2, T3 and T4 respectively, and $d$ is the length between the vehicle's center and each rotor shown in fig.2[19]. A mathematical model of a quadrotor dynamics had derived, using Newton's and Euler's laws. The behavior of the quadrotor had been observed in computer-generated simulation by using the Simulink 3D Animation toolbox [8]. The classical mechanics theories on rigid body motion are described by using the modern mathematical formulation. This modern mathematical formulation is based on the differential manifold and differential Topology method. A mathematical model has been developed for trajectory control by using kinematics and dynamics of a quadcopter and algorithms [1]. A mathematical model of a quadrotor dynamics differential equations were derived by using Newton's and Euler's-Lagrange equations [8,11]. There were two feedback linearization control schemes has been derived. In first control scheme the dynamic inversion with zero dynamics stabilization, based on static feedback linearization obtaining a partial linearization of the mathematical model and second one is the exact linearization and non-interacting control via dynamic feedback, based on dynamic feedback linearization obtaining a total linearization of the mathematical model [8]. Mathematical model derived for position & altitude control of quadcopter provides efficient stability at desired altitude and attitude with use of PD controller [11]. The dynamic model described for control of a tilting rotor quadcopter. The relationship between the tilting-rotor angles and the quadcopter orientation derived using the dynamic model [5].

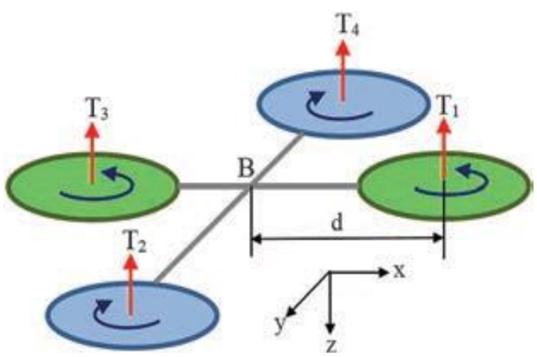


Fig. 2 Quadcopter notation showing the four motors [19].

The physical model is obtained by applying the dynamic equations from classical and Lagrangian mechanics to bring focus on the dynamics of a quadcopter UAV and analytical models for the system [4]. The mathematical model is develop for comparing experimental identification and the computation of model parameters like thrust coefficient, drag coefficient, inertia matrix, translational and rotational drag coefficients [3]. The main feature of designed quadcopter model is a fully-controlled system which can track any arbitrary trajectory with controlled pitch and roll angle, and motion with desired orientation [5]. The improvement can be done in mathematical modeling by developing a model for the brushless DC electric motor and the propeller by considering the effects of air as a medium, modeling ground effect and possible disturbances [2].

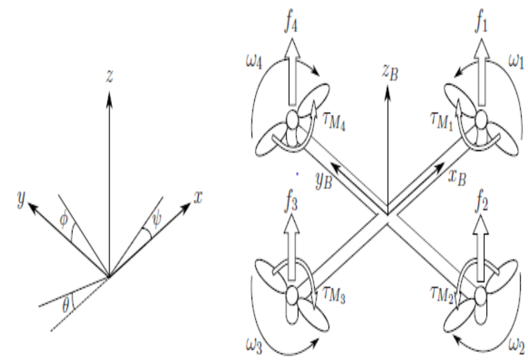


Fig. 3 The inertial and body frame of a quadcopter[11].

A quadrotor orientates within six degrees of freedom, three are the x, y and z-axes with rotation considered about each axis. The angular velocity of one rotor is changed with respect to the other three rotors for flight control of a quadrotor. The total yaw force will be zero if two opposite rotors rotate at the same speed in opposite directions. The pitch & roll changes wit change in speed of opposite pairs [5, 19].

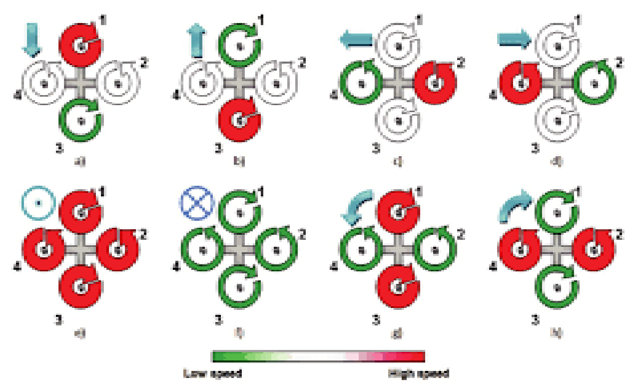


Fig. 4 Movement of Quadcopter with change in speed of rotor[5]

A dynamic model of a quadcopter developed for comparing Classic and cascade PID controller to demonstrate the effectiveness of the designed controller using a robust cascade PID control algorithm [6]. A unique mathematical model of quadcopter UAV in 6 DOF has been derived from basic Newtonian equations for representing a set of input, output and state variables related to first-order differential equations which is easy to understand dynamics of quadcopter for the readers at a root level [7]. The modeling of the system developed by concerning the reference frames, coordinates, transformations and equations of motion which built in the form of modules assigned to each process within the Quadcopter system [10].

## IV. CONTROL TECHNIQUES FOR QUADCOPTER

In order to maintain stability of quadcopter different controllers are designed for a robust control law. There are many different algorithms designed to implement this control, which sometimes uses model-free controllers employing fuzzy logic algorithms, neural networks and genetic algorithms, proportional integral derivative (PID) controllers, Linear Quadratic Regulator (LQR) for controlling mobile robots [9.11,19]. A PD controller is used for control theory & flight control algorithms for stabilization of the quadcopter attitude and newly method developed to control the trajectory of the quadcopter. The PD controller was integrated into a new method for better response to disturbances in the flight conditions of the quadcopter [1, 11]. Quadcopter behavior is observed for different speed of rotors for the roll, pitch and yaw angles with respect to time, also the behavior can be observed in computer generated simulation by using the Simulink 3D Animation toolbox [2, 8]. The different control strategy for controlling a quadcopter was tested in Simulink and then implemented on the system [4]. For simulation purpose the developed mathematical model is programmed within the MATLAB environment for different engagement scenarios and different sources of uncertainties. The performance of designed model is observed in MATLAB or other similar software using different control techniques like PD, PID, LQR, and H1 controller [6, 10, 15, 16]. It is observed that the PID controller gives better performance in both attitude and altitude as compare to fuzzy and the original controller, also the efforts required to design PID controller is less than that of the FUZZY and the original controller [10].

A sliding mode controller or H1 controller could be used for observing the performance of quadcopter under the condition of disturbance and uncertainty, and then the derived model can be used for designing control laws for the 6DOF stability of the quadcopter [7]. The focus on recent advantages in the development of the output control approach is “consecutive compensator”. This algorithm has extended the theory of output feedback control of nonlinear MIMO systems. Output controller is designed by decomposition of the mathematical model on two parts. The first one is a static MIMO transformation and second one is a few SISO channels such trick allows to design a control law in two steps. The further work will associate with the output controlling by considering unknown biased sinusoidal disturbance for an internally unstable plant with the input delay [9]. There is a need to develop a dynamic mathematical model to allow the desired orientation to be achieved in any arbitrary combination of pitch and roll angles with total control, on rotational axes x, y, z of the quadcopter and parameters like angular velocity, torque, & angular acceleration[4,5].

## V. CONCLUSION

This paper reviewed the research work had been done regarding methods of mathematical modelling & control of Quadcopter. The mathematical model is derived by using Newton-Euler method and Lagrangian method. Different controllers like PD, PID, and LQR are used to control the trajectory of quadcopter and balance it at desired altitude. The derived mathematical model simulation is done using MATLAB Simulink. Further the development in the mathematical model of quadcopter should be done, so that the robustness and performance of quadcopter system against parameter uncertainties and external disturbances due to torque & angular acceleration can be improved. As the applications of quadcopter is widely increases in number and complexity, so the control techniques involved must be improve in order to provide better performance and increased versatility.

## REFERENCES

[1] Qikai Huang, Applied Mathematical Modelling-158, Mathematical Modeling of Quadcopter Dynamics, 103–315, 2016.

[2] Petar Piljek, IEEE, 14(1), Mathematical Modelling Of Unmanned Aerial Vehicles With Four Rotors, 88-100, 2016.

[3] Anežka Chovancová, Procedia Engineering, Mathematical Modelling and Parameter Identification of Quadrotor, 96(2014) 172 – 181.

[4] Daniel Gheorghiţă, IEEE, Quadcopter Control System Modelling and Implementation, 978-1-4799-8481-7/15, 2015.

[5] Nemati, AACC, 978-1-4799-3274, Modeling and Control of a Single-Axis Tilting Quadcopter, 2014.

[6] Pengcheng Wang, IEEE, 978-1-5090-5346, Dynamics Modelling and Linear Control of Quadcopter, 2016.

[7] Zaid Tahir, Vol 9(27), State Space System Modeling of a Quad Copter UAV, DOI: 10.17485/ijst/2016.

[8] Francesco Sabatino, “Quadrotor Control: Modeling, Nonlinear Control Design, And Simulation”, International Journal of Control, Automation and Systems, 7(3):419–428, 2009.

[9] Anton A. Pyrkin, Alexey A. Bobtsov, Sergey A. Kolyubin,” Output Controller for Quadcopters with Wind Disturbance Cancellation”, 2014 IEEE Conference on Control Applications, 978-1-4799-7409-2/14, October 8-10, 2014.

[10] Shady. A. Kader, Prof.Dr. Adel-Ezzat EL-Henawy, Dr.Oda, A.N “Quadcopter System Modeling and Autopilot Synthesis”, Electronics and Communications Department-Ain Shams University Egypt, Nov 2014.

[11] T. Luukkonen, Aalto University,"Modelling and control of quadcopter"School of Science, Espoo, August 22, 2011.
[12] Jing- jing Xiong, En-HuiZheng," Position and attitude tracking control for a quadrotor UAV", College of Mechanical and Electrical Engineering, China Jiliang University, Hangzhou 310018, PR China, ISA Transaction 53, 2014, 725-731.
[13] Jimoh O. Pedro, Muhammed Dangor, Praneet J. Kala, "Differential Evolution-Based PID Control of a Quadrotor System for Hovering Application", School of Mechanical, Aeronautical and Industrial Engineering University of the Witwatersrand, IEEE 978-1-5090-0623-6/16/2016, 2791-2798.
[14] A Ram Kim, Prasanth Vivekanandan, Patrick McNamee, Ian Sheppard, "Dynamic Modeling and Simulation of A Quadcopter with Motor Dynamics", AIAA Modeling and Simulation Technologies Conference, Grapevine, Texas, AIAA SciTech Forum, 9 - 13 January 2017.
[15] Ernesto Paiva, Juan Soto, Julio Salinas and William Ipanaqué, "Modeling, Simulation and Implementation of a modified PID Controller for stabilizing a Quadcopter", IEEE 9781509011476/16/2016.
[16] Atheer L. Salih, M. Moghavvemi, Haider A. F. Mohamed and Khalaf Sallom Gaeid,"Modelling and PID Controller Design for a Quadrotor Unmanned Air Vehicle", Centre for Research in Applied Electronics (CRAE), University of Malaya, Jan 1, 2011.
[17] Dario Brescianini, Markus Hehn, and RaffaelloD'Andrea "Quadro copter Pole Acrobatics", 2013.
[18] [http://www.krossblade.com/history-of-quadcopters-and-multirotors](http://www.krossblade.com/history-of-quadcopters-and-multirotors).
[19] Omar I. Dallal Bashi1, W. Z. Wan Hasan1, N. Azis1, S. Shafie2, and Hiroaki Wagatsuma3, Journal of Computational and Theoretical Nanoscience, Vol. 14, 5663–5675, 2017