

Stability of Rivlin-Ericksen Elastico Viscous Fluids In A Uniform Horizontal Magnetic Field

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Abstract- This paper analyzes the stability of a plane interface separating two superposed visco-elastic fluids in a two dimensional horizontal magnetic field. The dispersion relation is derived employing the normal mode technique and has been solved numerically for different values of the physical parameters involved. It is found that viscosity and visco-elasticity all have a stabilizing influence on the growth rate of unstable mode of disturbance.

Keywords- Rayleigh Taylor Stability, Viscosity, Visco Elasticity, Magnetic Field

I. INTRODUCTION

The Rayleigh–Taylor instability, or RT instability (after Lord Rayleigh and G. I. Taylor), is an instability of an interface between two fluids of different densities which occurs when the lighter fluid is pushing the heavier fluid. The growth rate of the instability and the rate of mixing between the two fluids depend on the effective viscosity of the two fluids.

Rayleigh-Taylor instability problems have been the centre of research of various researchers under varying assumption. Chandrasekhar (1961) has given a detailed account of various investigations in hydrodynamics and hydromagnetics of the Rayleigh-Taylor instability which arises from the character of equilibrium of a layer of hetrogenous fluid and of which the two superposed layers of homogeneous fluids is a particular case. Bhatia (1974) has studied the Rayleigh-Taylor instability of two superposed electrically conducting viscous fluids in a horizontal magnetic field. Pradeep Kumar (2000) has studied the Rayleigh-Taylor instability of Rivlin Ericksen elastico viscous fluids in the presence of suspended particles through Porous Medium. Bhatia and Chhonkar (1985) have studied the stability of superposed viscous rotating plasma in the presence of finite larmor radius (FLR) effects while Hooper and Grimshaw (1985) have examined the nonlinear instability of the interface between two fluids. Gupta and Bhatia (1991) have studied the stability of plane interface between two viscous, superposed, partially ionized plasmas of uniform densities in a uniform two dimensional horizontal magnetic field. Srivastava and

Khare (1996) have investigated the Rayleigh-Taylor instability of two viscous, superposed conducting fluids in a vertical magnetic field. Allah (1998) has investigated the effect of surface tension and heat and mass transfer on the instability of two streaming superposed fluids. Sharma and Kumar (1998) have studied the Rayleigh-Taylor instability of two superposed, conducting Walters B'elastico viscous fluids in two dimensional magnetic field. MH Obied Allah (1998) has studied the stability of fluid flows in a porous medium with spherical symmetry. Nidhi Bansal, Jaimala and S.C. Agarwal (1999) have studied the shear flow instability of an incompressible, viscoelastic fluid in a porous medium in the presence of weak magnetic field. MD. Abdul Sattar and Mahmud Alam (1995) have studied the MHD free convective heat and mass transfer flow with Hall current and constant heat flux through a porous medium.

Recently Khan and Bhatia (2001) have studied the stability of two superposed viscoelastic fluids in a horizontal magnetic field through porous medium. The investigation for the stability of two superposed viscoelastic fluids in a horizontal magnetic field with the effect of surface Tension has been done by Mahesh Bohra, S.L. Maheshwari and P.K. Bhatia (2004). More recently Mehta and Singh (2006) have investigated the stability of two superposed ,viscoelastic ,Walter B' elastico viscous fluids in a horizontal magnetic field through porous medium. Viscoelastic fluids play an important role in industrial applications, hence it would be of interest to investigate the stability of two superposed viscoelastic fluids through porous medium. The purpose of this work is to examine the effect of kinematic viscosity on the Rayleigh Taylor instability of Rivlin-Ericksen elastico viscous fluid layer of varying densities . The fluid is assumed to be permeated by a uniform two dimensional horizontal magnetic field.

II. FORMULATION OF THE PROBLEM AND PERTURBATION EQUATION

We consider the motion of an incompressible, infinitely conducting Rivlin-Ericksen viscous fluid of variable density through a porous medium. The fluid is assumed to be

immersed in a uniform two dimensional horizontal magnetic field, $\vec{H} = (H_x, H_y, 0)$

The relevant linearized perturbation equation :-

$$\rho \frac{\partial \vec{v}}{\partial t} = -\nabla \delta p + \vec{g} \delta \rho + \rho \left(\nu + \nu' \frac{\partial}{\partial t} \right) \nabla^2 \vec{v} + (\nabla \times \vec{h}) \times \vec{H} + \left(\frac{d\mu}{dz} - \frac{\partial}{\partial t} \frac{d\mu'}{dz} \right) \left(\frac{\partial w}{\partial x} + \frac{\partial \vec{v}}{\partial z} \right) \tag{1}$$

$$\nabla \cdot \vec{v} = 0 \tag{2}$$

$$\nabla \cdot \vec{h} = 0 \tag{3}$$

$$\frac{\partial}{\partial t} (\delta \rho) + (\vec{v} \cdot \nabla) \rho = 0 \tag{4}$$

$$\frac{\partial}{\partial t} \delta z_s = w_s \tag{5}$$

$$\frac{\partial \vec{h}}{\partial t} = \nabla \times (\vec{v} \times \vec{H}) \tag{6}$$

where $\vec{v} (u, v, w)$, $\delta \rho$, δp and $\vec{h} (h_x, h_y, h_z)$ denote the perturbation in velocity, density, pressure and magnetic field \vec{H} respectively. Here $\mu, \mu', \vec{g} = (0, 0, -g)$ are respectively the co-efficient of viscosity, co-efficient of visco-elasticity and acceleration due to gravity. Equation (4) ensures that the density of every particle remains unchanged as we follow it with its motion.

Analyzing in terms of normal modes, we assume that the perturbed quantities have the space (x,y,z) and time (t) dependence of the form

$$f(z) \exp(ik_x x + ik_y y + nt) \tag{7}$$

where f(z) is some function of z, k_x and k_y are the horizontal numbers $(k^2 = k_x^2 + k_y^2)$ and n is the growth rate of harmonic disturbance.

Making use of equation (7) in equations (1) to (6), we get

$$\rho n u = -ik_x \delta p + \rho (\nu + n\nu') (D^2 - k^2) u + H_y (ik_y h_x - ik_x h_y) + (D\mu - nD\mu') (ik_x w + Du) \tag{8}$$

$$\rho n v = -ik_y \delta p + \rho (\nu + n\nu') (D^2 - k^2) v + H_x (ik_x h_y - ik_y h_x)$$

$$+ (D\mu - nD\mu') (ik_y w + Dv) \tag{9}$$

$$\rho n w = -D\delta p - g\delta \rho + \rho (\nu + n\nu') (D^2 - k^2) w + H_y (ik_y h_z - Dh_y) + H_x (ik_x h_z - Dh_x) + (D\mu - nD\mu') (2Dw) \tag{10}$$

$$ik_x u + ik_y v + Dw = 0 \tag{11}$$

$$ik_x h_x + ik_y h_y + Dh_z = 0 \tag{12}$$

$$n\delta \rho = -wD\rho \tag{13}$$

$$n\vec{h} = (ik_x H_x + ik_y H_y) \vec{v} \tag{14}$$

where $D \equiv \frac{d}{dz}$

Eliminating some of the variables from above equations, we get following equation in w :

$$n(D(\rho Dw) - k^2 \rho w) + \frac{gk^2(D\rho)w}{n} - D\{\rho(\nu + n\nu')(D^2 - k^2)Dw\} + k^2 \rho(\nu + n\nu')(D^2 - k^2)w - D(D\mu - nD\mu')(D^2 + k^2)w + 2k^2(D\mu - nD\mu')Dw + \frac{(k_x H_x + k_y H_y)^2}{n} (D^2 - k^2)w = 0 \tag{15}$$

III. TWO SUPERPOSED RIVLIN-ERICKSEN FLUIDS SEPARATED BY A HORIZONTAL BOUNDARY

We consider the case when two superposed Rivlin-Ericksen fluids of uniform densities ρ_1 and ρ_2 , uniform viscosities μ_1 and μ_2 and uniform visco-elasticities μ_1' and μ_2' occupy the regions $z < 0$ and $z > 0$ and are separated by a horizontal boundary at $z = 0$. The subscripts 1 and 2 distinguish the lower and upper fluids, respectively. Therefore in both the regions $z < 0$ and $z > 0$, equation (3.15) becomes:

$$(D^2 - k^2)(D^2 - q^2)w = 0 \tag{16}$$

where

$$q^2 = \left\{ k^2 + \frac{n}{(\nu + n\nu')} \left(1 + \frac{(k_x H_x + k_y H_y)^2}{n^2 \rho} \right) \right\} \tag{17}$$

where $\nu = \frac{\mu}{\rho}$ and $\nu' = \frac{\mu'}{\rho}$ being the co-efficients of kinematic viscosity and kinematic viscoelasticity.

Since w must vanish when $z \rightarrow \infty$ (for the upper fluid) and $z \rightarrow -\infty$ (for the lower fluid), we can write the solution of equation(16) appropriate to the two regions as :

$$w_1 = A_1 e^{kz} + B_1 e^{q_1 z} \quad (z < 0) \tag{18}$$

$$w_2 = A_2 e^{-kz} + B_2 e^{-q_2 z} \quad (z > 0) \tag{19}$$

where A_1, B_1, A_2, B_2 are constants of integration and q_1 and q_2 are the positive square roots of equation (17) for the two regions. It is assumed here that q_1 and q_2 are so defined that their real parts are positive.

IV. BOUNDARY CONDITIONS

The solutions (18) and (19) must satisfy four boundary conditions. The three conditions to be satisfied at the interface $z=0$ are that

$$w, Dw, (\mu + n\mu') (D^2 + k^2)w \quad \text{must be continuous.} \tag{20}$$

If we integrate (15) across the interface, we obtain the required another condition as:

$$\begin{aligned} & \left(1 + \frac{\nu}{n} \right) (\rho_2 Dw_2 - \rho_1 Dw_1)_{z=0} + \frac{gk^2}{n^2} (\rho_2 - \rho_1) (w)_0 \\ & - \frac{1}{n} \{ (\mu_2 + n\mu'_2) (D^2 - k^2) Dw_2 \\ & - (\mu_1 + n\mu'_1) (D^2 - k^2) Dw_1 \}_{z=0} \\ & + \frac{2k^2}{n} (\mu_2 - n\mu'_2 - \mu_1 + n\mu'_1) (Dw)_0 \\ & + \frac{(\vec{k} \cdot \vec{H})^2}{n^2} (Dw_2 - Dw_1)_{z=0} = 0 \end{aligned} \tag{21}$$

where $(w)_0$ and $(Dw)_0$ are unique values of these quantities at $z=0$. On applying the boundary conditions (20)–(21) to the solutions (18) and (19), we obtain :

$$A_1 + B_1 = A_2 + B_2 \tag{22}$$

$$kA_1 + q_1 B_1 = -kA_2 - q_2 B_2 \tag{23}$$

$$\begin{aligned} & (\mu_1 + n\mu'_1) (2A_1 k^2 + (q_1^2 + k^2) B_1) \\ & = (\mu_2 + n\mu'_2) (2A_2 k^2 + (q_2^2 + k^2) B_2) \end{aligned} \tag{24}$$

$$\left(1 + \frac{\nu}{n} \right) (-k\rho_2 A_2 - \rho_2 q_2 B_2 - k\rho_1 A_1 - \rho_1 q_1 B_1)$$

$$+ \frac{gk^2}{2n^2} \left\{ (\rho_2 - \rho_1) - \frac{k^2 T}{g} \right\} (A_1 + B_1 + A_2 + B_2)$$

$$- \frac{1}{n} \{ (\mu_2 + n\mu'_2) (-q_2 (q_2^2 - k^2) B_2)$$

$$- (\mu_1 + n\mu'_1) q_1 (q_1^2 - k^2) B_1 \}$$

$$+ \frac{k^2}{n} (\mu_2 - n\mu'_2 - \mu_1 + n\mu'_1) (kA_1 + q_1 B_1 - kA_2 - q_2 B_2)$$

$$+ \frac{(\vec{k} \cdot \vec{H})^2}{n^2} (-kA_2 - q_2 B_2 - kA_1 - q_1 B_1) = 0$$

$$\tag{25}$$

On eliminating the constants A_1, B_1, A_2, B_2 and evaluating the determinant of the given matrix of the coefficients in equations (22) to (25), we obtain following characteristic equation:

$$\begin{aligned} & \left[(q_1 - k) \left[-2k^2 C \left\{ \frac{k}{n} C(q_2 - k) + \alpha_2 \left(1 + \frac{\nu}{n_1} \right) + \frac{(\vec{k} \cdot \vec{V}_A)^2}{n^2} \right\} + \right. \right. \\ & \left. \left. (v_2 \alpha_2 + n v_2^1 \alpha_2) (q_2^2 - k^2) \left\{ R - \left(1 + \frac{\nu}{n_1} \right) - \frac{2(\vec{k} \cdot \vec{V}_A)^2}{n^2} \right\} \right] \right. \\ & - 2k \left[(v_1 \alpha_1 + n v_1^1 \alpha_1) (q_1^2 - k^2) \left\{ \frac{-k}{n} C(q_2 - k) + \frac{(\vec{k} \cdot \vec{V}_A)^2}{n^2} + \alpha_2 \left(1 + \frac{\nu}{n_1} \right) \right\} + \right. \\ & \left. \left. (v_2 \alpha_2 + n v_2^1 \alpha_2) (q_2^2 - k^2) \left\{ \frac{k}{n} C(q_1 - k) + \frac{(\vec{k} \cdot \vec{V}_A)^2}{n^2} + \alpha_1 \left(1 + \frac{\nu}{n_1} \right) \right\} \right] \right. \\ & \left. + (q_2 - k) \left[(v_1 \alpha_1 + n v_1^1 \alpha_1) (q_1^2 - k^2) \left\{ R - \left(1 + \frac{\nu}{n} \right) - \frac{2(\vec{k} \cdot \vec{V}_A)^2}{n^2} \right\} \right. \right. \\ & \left. \left. + 2k^2 C \left\{ \frac{k}{n} C(q_1 - k) + \frac{(\vec{k} \cdot \vec{V}_A)^2}{n^2} + \alpha_1 \left(1 + \frac{\nu}{n} \right) \right\} \right] \right] = 0 \end{aligned} \tag{26}$$

where $C = (v_2\alpha_2 + nv'_2\alpha_2 - v_1\alpha_1 - nv'_1\alpha_1)$

$$(\vec{k} \cdot \vec{V}_A)^2 = \frac{(\vec{k} \cdot \vec{H})^2}{(\rho_1 + \rho_2)} = \frac{(k_x H_x + k_y H_y)^2}{(\rho_1 + \rho_2)}$$

$$= \frac{k^2 (H_x \cos \theta + H_y \sin \theta)^2}{(\rho_1 + \rho_2)}$$

$$R = \frac{gk}{n^2} (\alpha_2 - \alpha_1),$$

$$\alpha_{1,2} = \frac{\rho_{1,2}}{\rho_1 + \rho_2}$$

$$v_{1,2} = \frac{\mu_{1,2}}{\rho_{1,2}}, v'_{1,2} = \frac{\mu'_{1,2}}{\rho_{1,2}}$$

\vec{V}_A being Alfvén velocity vector, θ being angle which k makes with x-axis.

The dispersion relation (26) is quite complex, particularly as q_1 and q_2 involve square roots. We therefore carry out the stability analysis for highly viscous and highly visco elasticity, for then we can write q_1 and q_2 as :

$$q_{1,2} - k = \left[\frac{n}{2k(v_{1,2} + nv'_{1,2})} \left\{ 1 + \frac{(\vec{k} \cdot \vec{V}_{A1,2})^2}{n^2 \alpha_{1,2}} + \frac{v_{1,2}}{n} \right\} \right] \quad (27)$$

Substituting the values of q_1 and q_2 in (3.26), we get the dispersion relation which involves the values of the parameters α, v, v', \vec{V}_A corresponding to two fluids. We set $v_1 = v_2 = v, v'_1 = v'_2 = v'$, to obtain qualitatively the influence of these effects on the instability of the system. For mathematical simplicity $\vec{V}_{A1} = \vec{V}_{A2} = \vec{V}_A$.

The dispersion relation is then :

$$\begin{aligned} & \left[n^6 (2k^2 v^1 - 1) \alpha_1 \alpha_2 + n^5 (-2k^2 v) \alpha_1 \alpha_2 \right. \\ & + n^4 \left(\alpha_1 \alpha_2 (R_1 - R_2) - 2(\vec{k} \cdot \vec{V}_A)^2 (\alpha_1 \alpha_2) \right. \\ & \quad \left. \left. - (\vec{k} \cdot \vec{V}_A)^2 + 2k^2 (\vec{k} \cdot \vec{V}_A)^2 v' \right) \right. \\ & + n^3 (-2k^2 (\vec{k} \cdot \vec{V}_A)^2 v) \\ & \left. + n^2 \left((\vec{k} \cdot \vec{V}_A)^2 R_1 - 3(\vec{k} \cdot \vec{V}_A)^4 \right) \right] \end{aligned}$$

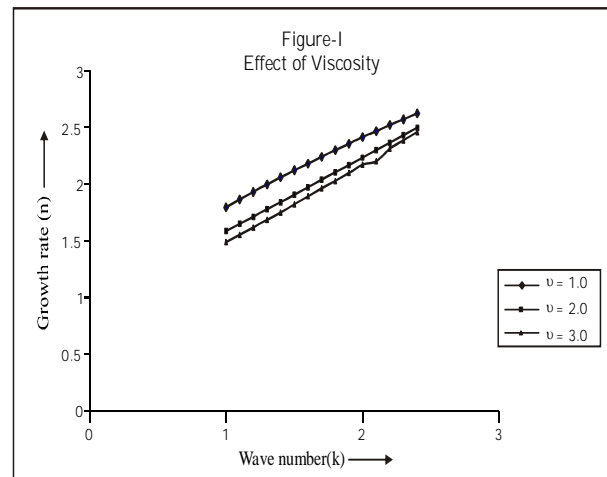
$$\begin{aligned} & + 2k^2 (\vec{k} \cdot \vec{V}_A)^4 v' + \\ & + n \left(-2k^2 (\vec{k} \cdot \vec{V}_A)^4 v \right) \\ & + \left(\vec{k} \cdot \vec{V}_A \right)^4 R_1 - 2(\vec{k} \cdot \vec{V}_A)^6 = 0 \end{aligned} \quad (28)$$

where $R_1 = gk(\alpha_2 - \alpha_1)$,

V. DISCUSSION

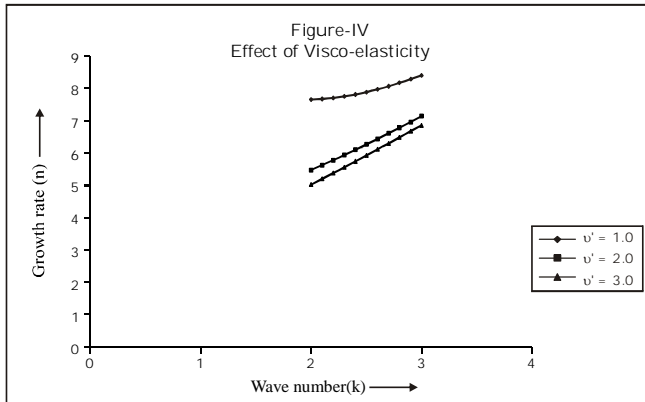
The dispersion relation (28) has been solved numerically to depict the roots of growth rate as a function of wave number k . The growth rate (positive part of n) is plotted against the wave number to ascertain the effects of various physical parameters on the growth rate of unstable modes. The numerical calculations are presented in figures I and II, where we have taken a potentially unstable arrangement by taking

$\alpha_1=0.25, \alpha_2=0.75$ for fixed $\vec{V}_A=0.5, \theta=45^\circ$.



Variation of growth rate (n) against the wave number k for different values of viscosity taking $\alpha_1=0.25, \alpha_2=0.75$ & $T=0.5, v'=0.2, \epsilon=0.1, \vec{V}_A=0.5, k_1=1, \theta=45^\circ$

In Figure I, we plot the growth rate against wave number for parameter viscosity by taking $v = 1.0, 2.0, 3.0$. We observe that as growth rate decreases as viscosity increase showing thereby stabilizing influence of kinematic viscosity.



Variation of growth rate (n) against the wave number k for different values of Visco-Elasticity taking $\alpha_1=0.25$

$\alpha_2=0.75$ & $\nu=3.0, T=0.5, \epsilon=0.1, \vec{V}^A=0.5, k_1=1, \theta=45^\circ$

In Figure IV, we have given the variation of the growth rate against the wave number for the values of visco-elasticity $\nu'=1.0, 2.0, 3.0$. We observe that as growth rate decreases as visco elasticity increase showing thereby stabilizing influence of visco elasticity.

We may thus conclude that viscosity and visco elasticity have a stabilizing influence on the unstable configuration.

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The Rayleigh-Taylor instability occurs when a light fluid is accelerated into a heavy fluid, and is a fundamental fluid-mixing mechanism. Any perturbation along the interface between the two fluids will grow. The width of the mixing layer at a given time is reduced by the development of turbulence. The growth rate of the instability and the rate of mixing between the two fluids depends on the effective viscosity of the two fluids.

This notion for a fluid in a gravitational field was first discovered by Lord Rayleigh [not Raleigh or Reyleigh] in the 1880s and later applied to all accelerated fluids by Sir Geoffrey Taylor in 1950.

Understanding the rate of mixing caused by Rayleigh-Taylor instabilities is important to a wide variety of applications, including inertial confinement fusion, nuclear weapons explosions and stockpile management, and supernova explosions. Avoiding Rayleigh-Taylor instability in inertial confinement fusion applications requires both very high precision in the target manufacture, and very high uniformity in the heating of the outside of the capsule--that is, very high symmetry.

Rayleigh-Taylor instabilities emerge in the implosion of both a fission primary device and a fusion secondary device. In the case of the fission primary, the instability arises when the shock wave from the lighter high explosive detonation reaches the much denser tamper. In the case of a fusion secondary, the instability arises when the lighter radiation implosion plasma in the hohlraum reaches the metallic core of the secondary.

Hydrodynamic instabilities play a major role in determining the efficiency and performance of inertial confinement fusion implosions. In laser-driven implosions, high-performance capsules require high aspect ratios (the ratio of the radius to the shell thickness). These capsules are susceptible to hydrodynamic instabilities of the Rayleigh-Taylor, Richtmyer-Meshkov, and Kelvin-Helmholtz varieties, which can in principle severely degrade capsule performance. Rayleigh-Taylor instabilities develop behind the supernova blast wave on a time scale of a few hours. The importance of the Rayleigh-Taylor (RT) instability and turbulence in accelerating a thermonuclear flame in Type Ia supernovae (SNe Ia) is well recognized. Flame instabilities play a dominant role in accelerating the burning front to a large fraction of the speed of sound in a Type Ia supernova. The Kelvin-Helmholtz instabilities accompanying the RT instability in SNe Ia drives most of the turbulence in the star, and, as the flame wrinkles, it will interact with the turbulence generated on larger scales.

The early nonlinear phase of Rayleigh- Taylor (RT) growth is typically described in terms of the classic model of Layzer [1955] in which bubbles of light fluid rise into the heavy fluid at a constant rate determined by the bubbleradius and the gravitational acceleration. However, this model is strictly valid only for planar interfaces and hence ignores any effects which might be introduced by the spherically converging interfaces of interest in inertial confinement fusion. The work of G.I. Bell [1951] and M.S. Plesset [1954] introduced the effects of spherical convergence on RT growth but only for the linear regime.

A generalization of the Layzer nonlinear bubble rise rate is given for a spherically converging flow of the type studied by Kidder [1974]. Kidder's self-similar (homogeneous) spherical implosion provided a simple formula for the bubble amplitude. This showed that, while the bubble initially rises with a constant velocity similar to the Layzer result, during the late phase of the implosion, an acceleration of the bubble rise rate occurs. The bubblerise rate is verified by comparison with full, 2-D hydrodynamics simulations.

Initially calculations were limited to two spatial dimensions in order to achieve maximum resolution. By the late 1990s sufficiently powerful massively parallel computers became available so that fully three-dimensional simulations can be carried out at comparable resolution. LLNL has employed the MIRANDA code to conduct several very large simulations (including 720 x 720 x 1620 and 1152 x 1152 x 1152 meshes) of Rayleigh-Taylor flows. These large-eddy simulations achieved unprecedented development in the flow, including observations of a mixing transition and early stages of what was likely the first simulation of the truly asymptotic behavior of the instability.

While fire-polishing keeps the small features suppressed in two dimensions, turbulence wrinkles the fusion flame on far smaller scales in the three-dimensional case, suggesting that the transition to the distributed thermonuclear burning regime occurs at higher densities in three dimensions