A Study of Buckling Analysis of Composite Beam With Different Shear Connecter

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Abstract- *The use of composite structures is increasingly present in civil construction works. Steel-concrete composite beams, particularly, are structures consisting of two materials, a steel section located mainly in the tension region and a concrete section, located in the compression cross sectional area , both connected by metal devices known as shear connectors. The main functions of these connectors are to allow for the joint behavior of the beam-slab, to restrict longitudinal slipping and uplifting at the elements interface and to take shear forces. This paper presents 3D numerical models of steel-concrete composite beams to simulate their structural behavior, with emphasis on the beam-slab interface. Simulations were carried out using version 14.0 ANSYS code, based on the Finite Element Method. The results obtained were compared with those provided either by Standards, experimental work or found in the literature, and such comparison demonstrated that the numerical approach followed is a valid tool in analyzing steel concrete composite beams performance.*

Keywords- ANSYS 16, composite beams, shear connectors, numerical modeling, finite element

I. INTRODUCTION

1.1 General

Composite steel–concrete construction, particularly for multi-storey steel frames, has achieved a high market share in several European countries, the USA, Canada and Australia. This is mainly due to a reduction in construction depth, to savings in steel weight and to rapid construction programmers. Composite action enhances structural efficiency by combining the structural elements to create a single composite section. Composite beam designs provide a significant economy through reduced material, more slender floor depths and faster construction. Moreover, this system is well recognized in terms of the stiffness and strength improvements that can be achieved when compared with Non-composite solutions. A fundamental point for the structural behavior and design of composite beams is the level of connection and interaction between the steel section and the concrete slab. The term “full shear connection” relates to the case in which the connection between the components is able to fully resist the forces applied to it. This is possibly the most common situation; however, over the last two decades the use of beams in building construction has led to many instances when the interconnection cannot resist all the forces applied (partial shear connection). In this case, the connection may fail in shear before either of the other components reaches its own failure state.

In the case of the serviceability limit state of composite beams, the condition when the connection between the components is considered as infinitely stiff is said to comprise “full interaction”. Whilst this is often assumed in design, it is theoretically impossible and cases where the connection has more limited stiffness (partial interaction) often need to be considered. In this case, the connection itself may deform, resulting in relative movement along the steel–concrete interface and the effect of increased shear deformation in the beam as a whole. Therefore, partial interaction occurs to some

Extent in all beams whether fully connected or Not. However, studies have shown that any flexibility in the connection may be ignored for beams designed for full connection. The use of partial connection provides the opportunity to achieve a better match of applied and resisting moment and some economy in the provision of connectors. Generally, the effects of partial interaction,which are increased by the use. The composite steel-concrete systems were first used in the middle of the last century. They involve the joint work of concrete elements and steel sections, interacting mechanically by means of connectors, dents or bumps, either by friction or adhesion. Generally, composite beams are made out of a combination of a steel section (commonly “I” shaped), located on predominantly tensioned region, with a concrete slab, positioned in predominantly compressed area. The mechanical binding is provided by metal devices called shear connectors. The main functions of the shear connectors are to allow for the joint work of the slab-beam new material, restricting longitudinal slip and vertical displacements of the interface elements, and to take shear forces. By combining steel and concrete this way, it is possible to obtain the advantages of both materials working together. Therefore, from the materials strength point of view, it is possible to take advantage of the steel section to take tension stresses and of the concrete in order to withstand compressive stresses. This combination results in high stiffness and smaller structural sections, lighter foundation design, gains in materials performance and reduced costs. In addition, composite systems allow for the occasional elimination of formwork and shoring, and may reduce steel protection against fire and corrosion, due to the presence and adequate behavior of concrete in the system. In Brazil, the first structures making use of composite systems were built in the 50s. However, in the last twenty years, a growth in steel production, as Noticed by a bigger supply of steel sections in the domestic market, caused composite systems to increase drastically. Having this picture in mind, this article focus numerical analysis of composite beams. The main idea is to make use of the computer program ANSYS, which is based on the Finite Element Method. Of partial shear connection, will result in reduced strength and stiffness, and potentially enhanced ductility of the overall structural system. It is widely known that laboratory tests require a great amount of time, are very expensive and, in some cases, can even be impractical. On the other hand, the finite element method has become, in recent years, a powerful and useful tool for the analysis of a wide range of engineering problems. According to Abdullah a comprehensive finite element model permits a considerable reduction in the number of experiments. Nevertheless, in a complete investigation of any structural system, the experimental phase is essential. Taking into account that numerical models should be based on reliable test results, experimental and numerical/theoretical analyses complement each other in the investigation of a particular structural phenomenon. Previous numerical studies have been conducted to investigate the behaviour of composite beams. Nevertheless, most of them are based on two-dimensional analytical models (e.g., Gattesco and are thus Not able to simulate more complex aspects of behaviour, which are intrinsic for three-dimensional studies; for instance: full distribution of stresses and strains over the entire section of the structural components (steel beam and concrete slab), evolution of cracks and local deformations in the concrete slab. In addition, in the particular case of the model developed by., it was assumed that the shear connectors were uniformly distributed along the length of a composite member. A three-dimensional finite element model has been developed by El-Lobody and Lam using the package ANSYS in which the mode of failure of the beams is detected by a manual check of the compressive concrete stress and stud forces for each load step. Nevertheless, just two beams were used to validate the proposed model for composite beams with solid slabs. All these studies were focused just on the presentation and validation of their corresponding models, but these models were Not used to investigate in more detail either the effect of particular structural parameters or other aspects of the system behaviour. It is only very recently that papers on finite element analyses of composite systems have started to contain parametric studies (e.g., investigations related to the behaviour of individual shear connectors In order to obtain reliable results up to failure, finite element models must properly represent the constituent parts, adopt adequate elements and use appropriate solution techniques. As the behaviour of composite beams presents significant Nonlinear effects, it is fundamental that the interaction of all different components should be properly modeled, as well as the interface behaviour. Once suitably validated, the model can be utilized to investigate aspects of behaviour in far more detail than is possible in laboratory work. For instance, it permits the study of the sensitivity of response to variability of key component characteristics, including material properties and shear stud layout. Consequently, different spacing in distinct parts of the beam can be adopted, allowing the investigation of partial interaction effects. The present investigation focuses on the modeling of composite beams with full and partial shear connection using the software ANSYS. A three-dimensional model is proposed, in which all the main structural parameters and associated Nonlinearities are included (concrete slab, steel beam and shear connectors). Test and numerical data available in the literature are used to validate the model, which is able to deal with simply supported systems with I-beams and solid flat slabs.

1.2 Objective

The objective is to verify the influence of the amount, diameter and height of shear connectors in composite beams for buckling. These verifications were made by means of the analysis of longitudinal slip in the slab-beam interface, the vertical displacement at mid-span and the bearing capacity of composite beams. The results were compared to those provided by standards and to other data found in the consulted literature.

II. NUMERICAL MODELING

This paper uses models for composite beams, particularly the “A3”, extracted from experimental tests and numerical applications. The tested model here presented, developed by a researcher, and uses the same geometry, parameters, material properties and Nomenclature of the composite beam defined in the referenced work. Despite the methodology applied here is broad and general, the “A3” model simulated in this paper refers solely to the simply supported composite beam (Figure 1). It was defined as having solid web, full interaction between the slab and the steel section provided a by number of shear connectors calculated to prevent slipping between the surfaces, flat concrete slab with two way reinforcement (transverse and longitudinal), shear connectors with pin-type head (stud bolt) and subjected to a point load in the mid span. The model was based on the finite element method, also used by other researchers. The model implementation started with the definition of the geometry of the composite beam (Figure 1). Secondly, finite elements available in the ANSYS computer program library were chosen to represent the composite materials. Thirdly, the properties and constitutive relations of the materials involved were introduced. Finally, the mesh, couplings and linkages between the elements were added, taking into consideration the symmetry condition and the consequent restriction of degrees of freedom, and also the beam support conditions and the applied load. The first simulation was done vis-à-vis the unique characteristic of the A3 beam, to validate the model. Then, to analyze the connectors influence on the structural behavior of the composite beam, several alternatives for connectors were analyzed, with diameters ranging from 16 mm, 19 mm and 22 mm and heights from 76 mm, 88 mm and 102 mm. Lastly ,the number of connectors recommended by the standard was used.

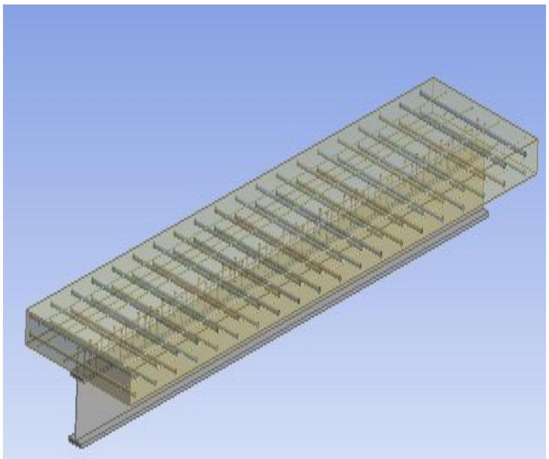


Fig No. 01 Geometry of Composite beam model

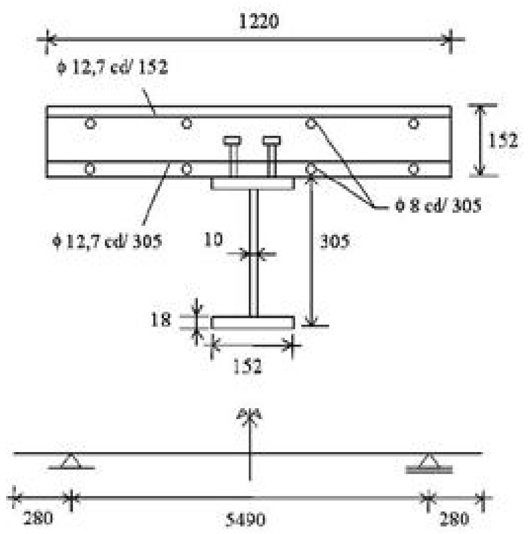


Fig No. 02 Dimensions of Composite beam model

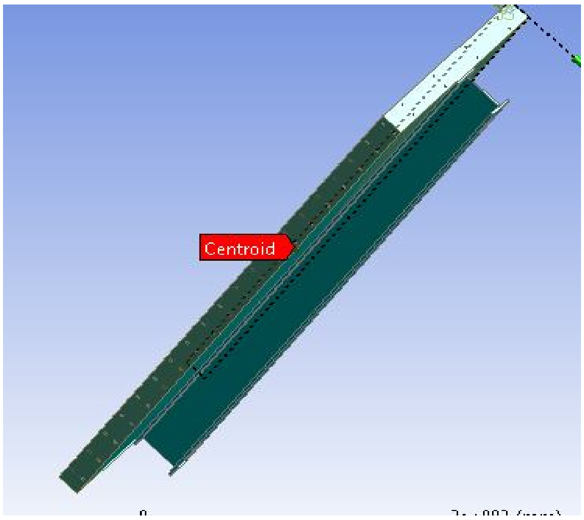


Fig No. 03 Dimensions of Composite beam

2.1 Finite Elements

The definition of the proposed numerical model was made by using finite elements available in the ANSYS code default library. The three-dimensional elements SOLID 186 were adopted to discretize the concrete slab, which are also able to simulate cracking behavior of the concrete under tension (in three orthogonal directions) and crushing in compression, to evaluate the material Non-linearity and also to enable the inclusion of reinforcement (reinforcement bars scattered in the concrete region). The representation of the steel section was made by the SHELL 43 elements, which allow for the consideration of Non-linearity of the material and show linear deformation on the plane in which it is present. The modeling of the shear connectors was done by the BEAM 189 elements, which allow for the configuration of the cross section, enable consideration of the Non-linearity of the material and include bending stresses. The TARGE 170 and CONTA 173 elements were used to represent the contact slab-beam interface. These elements are able to simulate the existence of pressure between them when there is contact, and separation between them when there is not. The two material contacts also take into account friction and cohesion be-tween the parties.

Table No. 01 Characteristics of the beam and material properties

Number of connectors	68
Diameters of connectors body	19
Total Height of connectors body	102
External load	Concentrated at mid span
E (kN/cm2)	19,456
	20.064
F_y (kN/cm^2)	30,2
	25,2
F_u (kN/cm^2)	44,4
	44,7
E (kN/cm^2)	20,500
F_u (kN/cm^2)	51,4
f_y(kN/cm^2)	42,1
E (kN/cm^2)	20.500
F_y (kN/cm^2)	32.0

2.2 Materials properties

The characteristics of the "A3" beam and the real properties of materials are presented in Table 1. It is noteworthy mentioning that this study also considered other configurations for the connectors, as number, height and diameter.

2.3 Constitutive relations

It was considered that the steel section has a multilinear elastic plastic constitutive relationship with an isotropic hardening consideration, associated with the von Mises' plasticity criterion. The stress-strain curve followed the constitutive model presented in and it was used in and, as shown in Figure 6. The adopted model for the steel connectors is a bi-linear isotropic hardening, also associated with von Mises' plasticity criterion. Figure 7 shows the stress-strain diagram for the steel connectors. The constitutive relationship for the steel reinforcement follows a perfect elastoplastic model and it is also associated with von Mises' plasticity criterion, based on the relationship between uniaxial tensions and their respective plastic deformations, as shown in the stress-strain diagram in Figure 4. For the concrete slab, the constitutive tension relationship followed the CONCRETE model, provided by ANSYS, which is based on the Willam-Warnke solution and allows for the material cracking. This model was also used in and. For the concrete in compression, on the other hand, von Mises' laminating criterion was adopted. The model represents the behavior of a multilinear isotropic concrete hardening, given by the stress-strain diagram in Figure 5. The solution for the contact between the concrete slab, the steel section and the connectors made use of the Pure Lagrange Multiplier method, also provided by ANSYS. This method assumes that there is No interpenetration between the two materials when the contact is closed and also that the slip is null, as long as it does Not reach the shear stress limit .The parameters that define if the contact is open or closed are set by FTOLN, which refers to a minimum value of penetration as to presume that the contact is closed and TNOP, which refers to a minimum value of Normal tension to the contact surface, so that the status changes to open. The absolute value adopted for FTOLN was -0.01 cm. For the TNOP the value adopted was 0.18 kN/cm^2. The established value of the friction coefficient between steel and concrete was 0.4 and, for cohesion, an estimated number of 0.18 kN/cm^2 was taken from average values of adhesion tension related to the initial slip of the interface

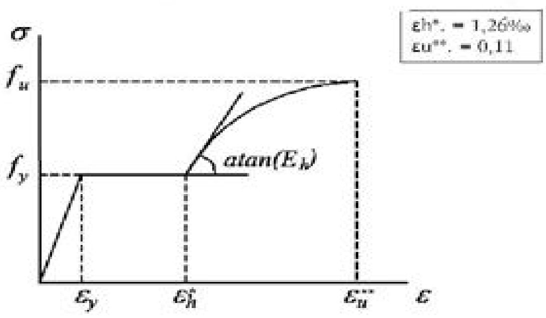


Fig No. 06 Constitutive relation for steel profile (8)

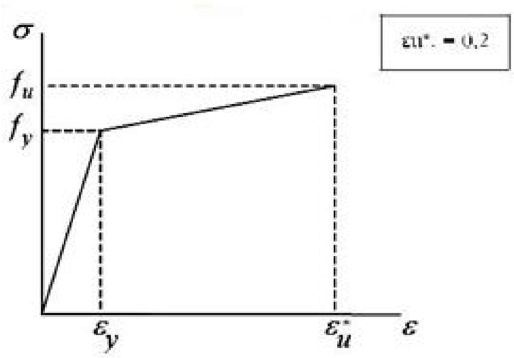


Fig No. 07 Constitutive relation for the shear connectors (10)

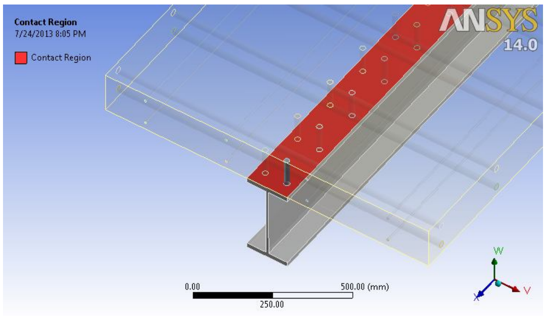


Fig.No.08 couplings connecting the elements

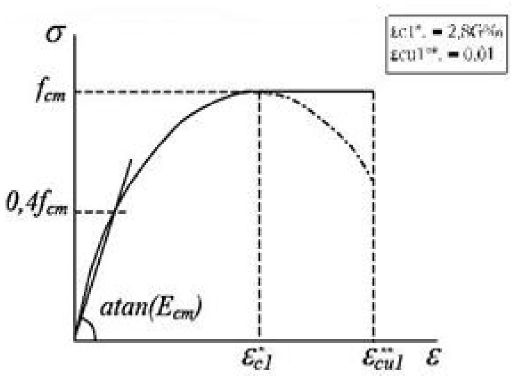


Fig.No.05 Constitutive relation for the concrete

2.4 Finite elements mesh

The model designed for the numerical analysis was defined by four types of elements that form the concrete slab with added reinforcements, such as steel beam, shear connectors and the pair of contact at the slab-beam interface. The elements were established separately, but the Nodes were one by one coupled on the interface between them. The finite element mesh developed for all elements followed the same methodology and degree of refinement presented in Figure 10 and Figure 11 shows the finite element mesh for the components cited, where (a) corresponds to the concrete slab, (b) to the steel beam, (c) to the shear connectors and (d) to the pair.

2.5 Couplings and linkages

The couplings connecting the elements consider the Nodes superposition, with the degrees of freedom adapted, as illustrated in Figure 8. The contact between the slab and the beam was established by the CONTA 173 elements, attached to the section web, and TARGE 170, attached to the inferior surface of the slab. The beam-connector link was considered as a clamped metal pin in the steel section, with rotations and translations made compatible. On the slab-connector interface, translational referring to the X and Z axis were also made compatible and, at the Node below the pin head, there was a consideration of coupling in the Y direction to represent the mechanical anchoring between the head of the connector and the concrete slab. Attempting to reproduce a movable type support, the degrees of freedom related to the translation in X and the rotation in Z were Not restricted at referred Nodes of the composite beam support. At the Nodes of the central section of the composite beam, a symmetry condition was applied, also provided by ANSYS and, consequently, a restriction of degrees of freedom. Figure 8 shows the symmetry condition, the binding of the composite support beam in detail, and also the coupling between the materials. When applying mixed beams loading without shoring, it was assumed that the steel section would support its dead weight and that the recently set concrete on the table would Not have joint between the two materials. The behavior as a composite beam would only occur after the concrete curing, when it would be possible to apply an external load, because the composite beam would have reached the expected resistance as set in the project. Thus, by the time it would start acting as a composite beam, the structure would already be deformed.
In this context, to simulate the loading application in beam A3, the Birth & Death's technique, available in ANSYS , was adopted.

This technique, which allows for elements activation and inactivation of a discretized mesh, consists of the multiplication of the value of the inactivated entity in the stiffness matrix and a reduction factor, which practically blocks the effects of the results of such entity. In this paper the adopted reduction factor was 10^{-6}. Firstly, the concrete slab and the shear connectors were inactivated and the structure dead weight was applied to the steel section. Secondly, the concrete slab was activated and the applied load was used in regard of the solidarity slab-beam work. The structure dead weight was inputted into the modeling according to the unit weight of the materials, which were: 24 kN/m³ for the concrete and 77 kN/m³ for the steel girder, connectors and reinforcements. The applied load was incrementally and monotonically included immediately after the action of the dead weight of the composite beam. Although concentrated in the middle of the span, the load was considered

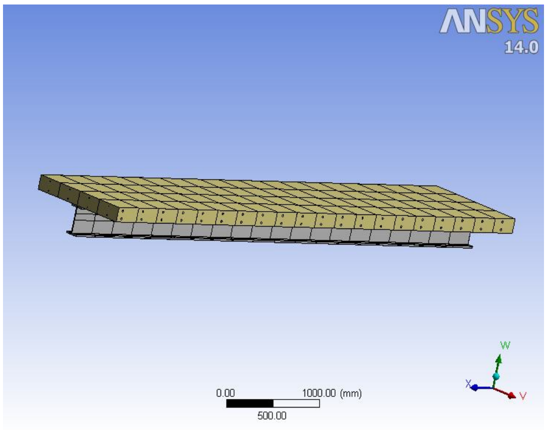


Fig No.10 Finite Elements mesh

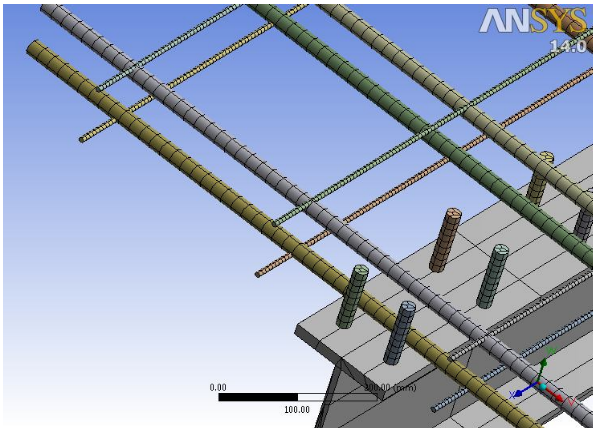


Fig No. 11 Finite Elements mesh

as spread throughout a small area, applied at the Nodes of the upper surface of the concrete slab, centered on the axis of the beam, according to the experimental model presented in. Both the structure dead load and the applied load were included incrementally in the model to take into account the nonlinear behavior of the materials that form the composite beam. Figure 9 shows the composite beam with an applied load concentrated on the mid-span.

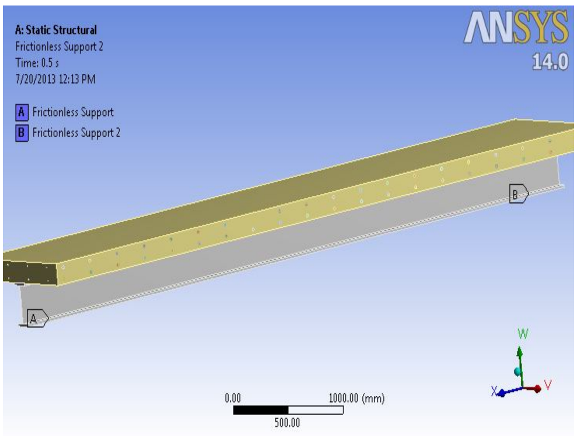
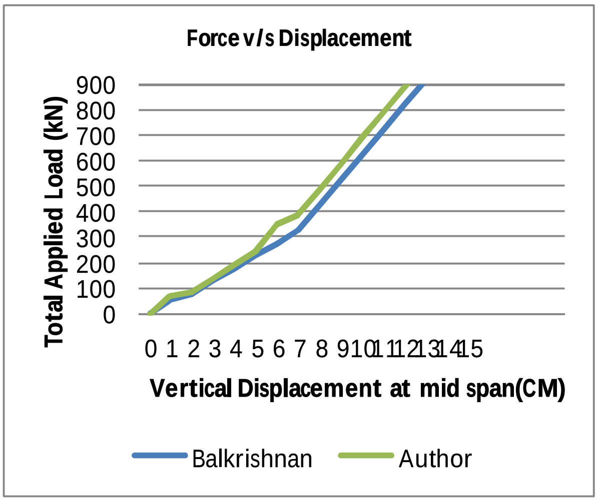


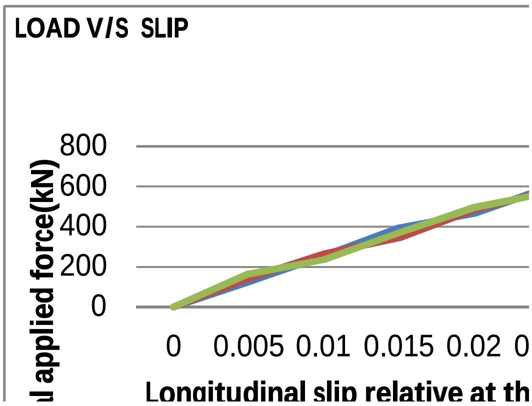
Fig.No.09 Boundary condition and support linkages

III. RESULTS AND DISCUSSIONS

Graph 01 shows comparative results of vertical displacements at mid-span with the increment of the applied load. These results refer to the first stage of the simulation and compare well with values experimentally obtained and numerically presented in and in this work. It is Note-worthy that the computational model developed in took into consideration shored composite beams, while this work and the experimental tests shown in deal with Non-shored composite beams Figure 11 shows that, under the elastic range, results for the composite beams are similar for both the experimental and numerical models.



Graph No. 01(a): Graphical Force v/s Vertical Displacement



Graph No. 01(b): Force v/s Slide of A3 Beam

Table No.02: Summary of the results considering variations of H

Parameter	Φ(mm)	F_{max} (kN)	U_{max}(cm)	d_{max} (cm)
H=76mm		506.9	9.24	0.0188
H=88mm	19	481.3	6.48	0.0143
H=102mm		481.4	6.54	0.0149

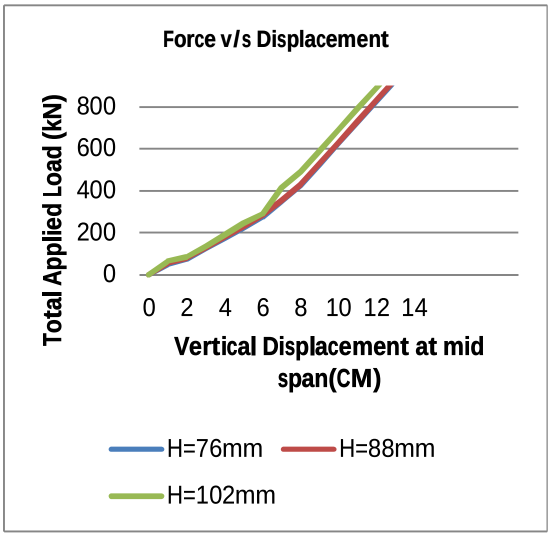
Table No.03: Summary of results considering the influence of changes in Φ

Parameter	Φ(mm)	Fmax (kN)	Umax(cm)	dmax (cm)
Φ=16mm	102	437.68	3.84	0.0133
Φ=19mm	102	481.46	6.54	0.0149
Φ=22mm	102	506.28	9.29	0.0151

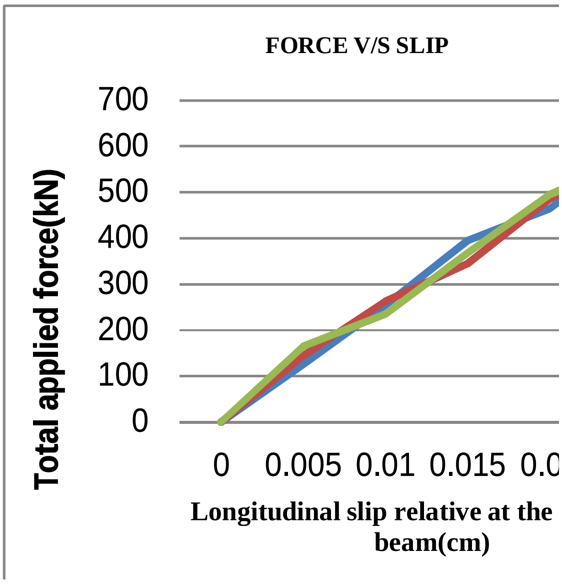
Regarding the vertical displacement at the center of the span of the beams in the limit load, the value of the numerical model is 27% lower than the experimental one exposed in. This suggests a more rigid behavior of the model developed in this work. The analyzed slip, on the other hand, did Not show the same behavior. At the limit load, the experimental and numerical model presented similar sliding, while the experimental model resulted in a sliding 20% lower. Thus, it can be.

3.1 Influence of connectors

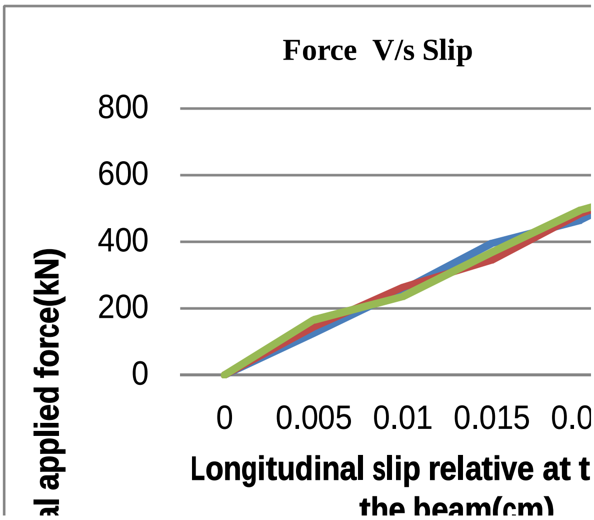
Table 2 displays the result of the influence of the connector height (H) in the limit load (F_{max}), in the vertical displacement at mid-span (u_{max}) and in the average relative longitudinal slip (d_{max}) (between the slab and the steel section), at the end of the beam for the second stage of simulations. Maximum loading occurs for the connector with H=76mm. This solution was also the one which showed greater vertical displacement and longitudinal sliding; this suggests a more ductile behavior than others. Thus, it appears that increasing the height of the connector does Not necessarily increases the load limit, the vertical displacement or the longitudinal sliding. It is presented in Table 3 the result of the influence of the diameter of the connector (Ø) in the limit load (F_{max}), in the vertical displacement at mid-span (u_{max}) and in the average relative longitudinal slip (d_{max}) (between the slab and the profile steel), at the end of the beam. Table 3 shows that increasing the diameter of the connector in-creases the limit load, the vertical displacement and the longitudinal slip, whose highest value corresponds to the connector Ø=22 mm. Table 4 shows the comparative result for the second and the third steps of the simulations for the influence of the numbers of connectors (NC), with different heights (H), in the limit load (F_{max}), in the vertical displacement at mid-span (u_{max}) and in the average relative longitudinal slip (d_{max}) (between the slab and the steel profile), at the end of the beam. It may be noted from Table 4 that reducing the number of connectors (↓NC) results in an amplification (↑) of the longitudinal slip but, Not necessarily, in the decrease (↓) of the maximum force and the increase of the vertical displacement. Table 5 shows the comparative results of the second and third steps of the simulation for the influence of the number of connectors (NC), with different diameters (Ø), in the load limit (F_{max}), in the vertical displacement at mid-span (u_{max}) and in the average relative longitudinal slip (d_{max}) (between the slab and the steel section), at the end of the beam.



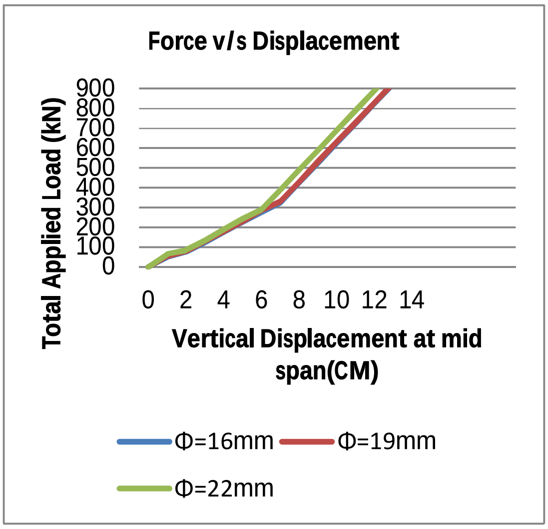
Graph No. 02(a) Graphics Force v/s displacement to connectors with 19mm Φ (AUTHOR)



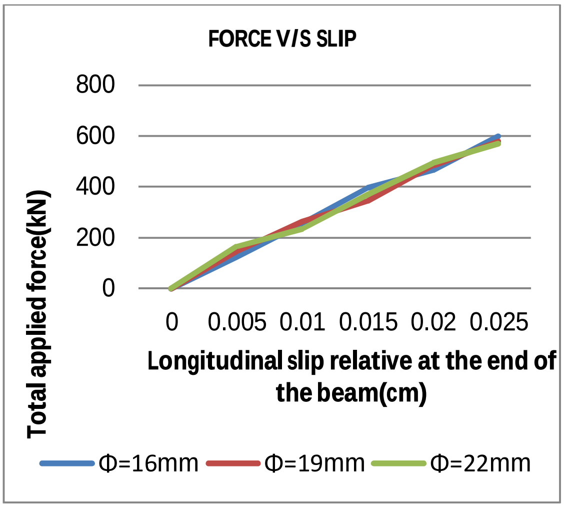
Graph No. 02(b) Force v/s Slide to connectors with 19 mm Φ (Author)



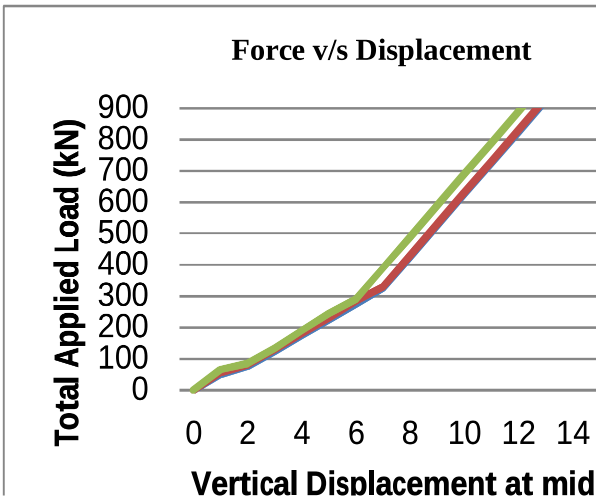
Graph No. 05(a) Graphics Force v/s Displacement to connectors with variation in depth



Graph No. 04(a) Graphics Force v/s Displacement to connectors with 102mm high (Author)

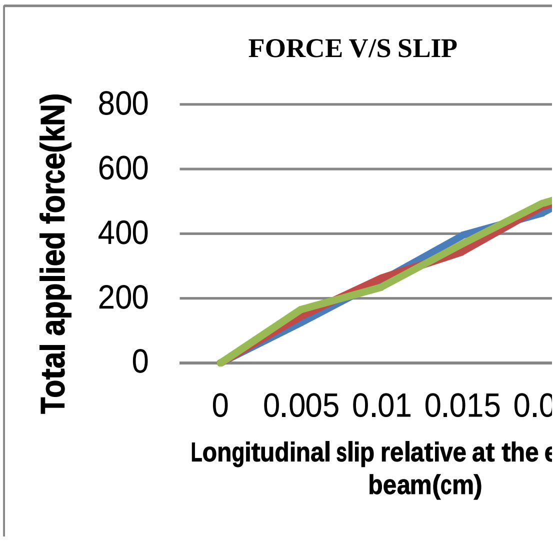


Graph No. 04(b) Force v/s Slide to connectors with 102mm high (Author)



Graph No. 05(a) Graphics Force v/s Displacement to connectors with 102mm high (NBR8800)

It is possible to infer from Table 5 that the decrease (↓) of the number of connectors (NC) implies an increase (↑) of the vertical displacement and of the longitudinal sliding, but not necessarily, in a reduction (↓) of the maximal force.



Graph No. 05 (b) Graphics Force v/s Displacement to connectors with 102mm high (NBR8800)

To better visualize the data in Graph No. 02,03,04 display comparative graphics of steps two and three of the evolution of vertical displacements at mid-span (a) and average relative longitudinal sliding (between the slab and the steel section), at the end of the composite beam (b) with the total force applied to the mixed system. Figure 11 shows the comparative graphics of force versus dis-placement (a) and force versus slip (b) for beams with connectors 19 mm in diameter, and Graph No. 05(a) and Graph No. 05(b) shows the comparative graphics of force versus displacement (a) and force versus slip (b) of composite beams with connectors 102 mm in height. Graphs shows that the behavior of composite beams in the elastic range were similar for all connectors, regardless their number, diameter and height. However, in the nonlinear range, the reduction in the number of connectors implies an increasing of the longitudinal slip and it has. The cracking modes in the slab are due to a strength and stiffness reduction of the concrete in the triaxial compression zone as a consequence of concrete cracking caused by the connector, when it applies a concentrated force in the slab. pan of this figure. The cracking of the concrete slab for the loading step is shown in Figure 12. In the cracking of the concrete slab of the composite beam with connectors 19 mm in diameter and 102 mm in height, it was found that the first cracks are in the slab-beam interface and appear to an, inclined 45 degrees in the elements of the.

Tables 6 and 7 present the results obtained in the third stage of the simulation and in the calculation of composite beams, according to standard recommendations, whose procedure is in for the limit load (F_{max}) and the vertical displacement mid-span (u_{max}). It is Noteworthy that the number of shear connectors and the other parameters (force, displacement) were defined in terms of the lower resistance between the concrete slab and the steel section.

That is, according to standard recommendations, the values obtained for the vertical displacements and forces are independent of the diameter and height of the shear connectors; because, in this case, since the composite section of the adopted beam was the same for all simulated models and since the concrete resistance to rupture is lower than the connector capacity to resist shearing, the force and displacement are the same for both models. Table 6 refers to results of composite beams with 19 mm diameter (Ø) connectors as a function of a variation in height (H), and Table 7 shows results of composite beams with 102 mm high (H) connectors versus the variation in diameter (Ø). It can be seen from Tables 6 and 7 that the maximum force calculated by the NBR 8800 is conservative, because it presents lower values than those obtained in the numerical simulation. The vertical displacements were much higher than those found in numerical simulations, indicating a more rigid behavior of simulated models and a more ductile performance of the calculated ones.

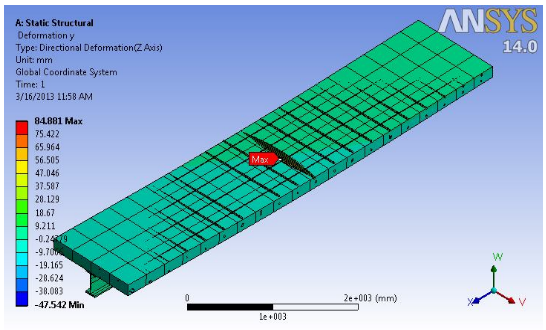


Fig No.12: Cracking on slab due to concentrated force.

Table No.07: Comparative results of the beams with connectors Φ=19mm NBR 8800

Beams with Connectors Φ= 19mm	**H (mm)**	**F_{max} (kN)**	**U_{max}(cm)**
Simulated Value	76	466,78	4,80
Calculated Value	76	394,29	14,38
Simulated Value	88	488,12	6,07
Calculated Value	88	394,29	14,38
Simulated Value	102	502,30	7,22
Calculated Value	102	394,29	14,38

Table No.08: Comparative results of the beams with connectors H=102mm-NBR 8800(1)

Beams with Connectors H=102 mm	**Φ (mm)**	**F_{max} (kN)**	**U_{max}(cm)**
Simulated Value	16	471,54	6,16
Calculated Value	16	394,29	14,38
Simulated Value	19	502,30	7,22
Calculated Value	19	394,29	14,38
Simulated Value	22	487,42	7,50
Calculated Value	22	394,29	14,38

IV. CONCLUSIONS

The behavior of composite beams has been studied experimentally by Chapman and Balkrishnan. Later Qing Quan Liang has attempted computational modeling this problem in ABAQUS. However, the effect of variation in shear connectors was yet to be studied. Therefore in this work the authors have considered variation in height, diameter and shape of shear connectors. By computations, it has been observed that height of the shear connectors does not influence much the deflection of the composite beam. However, increase in diameter of the shear connectors Decreases deflection in composite beam. Further it is observed that shape of cross section of shear connector also matters in behavior of composite beam. The shear connectors having rectangular cross section are found more effective than those with circular cross section for arresting the deflection of composite beam

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