Explication of Dio 3-Tuples From Truncated Octahedral Number-II

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Abstract- *We attempt to explicate dio 3-tuples from truncated octahedral number of different ranks. Also we present 4 sets of dio 3-tuples under 3 cases with some numerical examples.*

Keywords- Dio 3-tuples, Truncated octahedral number.

I. INTRODUCTION

In Mathematics, a diophantine equation is a polynomial equation usually in 2 or more unknowns such that only integer solutions are sought. Diophantine problems have fewer equations than unknowns and involve finding integers that work correctly for all equations. In more technical language they define an algebraic curve, algebraic shape or more general object and ask about lattice points on it. While individual equations present a kind of puzzle and have been considered throughout history, the formulation of general theories of diophantine equations was an achievement of twentieth century.

[1-3] has been referred for various ideas on number theory. [4-20] has been studied for plentiful ideas on diophantine triples. In this paper we attempt to explicate dio 3-tuples from truncated octahedral number. We have also presented 4 sets of dio 3-tuples under 3 cases with some numerical examples.

Notations:

$$TO_n = Truncated\ Octahedral\ number\ of\ rank\ n$$

II.METHOD OF ANALYSIS

Case(i)

Let $a = TO_{n-1} = 16n^3 - 81n^2 + 138n - 79$

$b = TO_n = 16n^3 - 33n^2 + 24n - 6$

We then have

$$ab + 544n^4 - 686n^3 - 2057n^2 + 5604n - 150 = \alpha^2$$

Where $\alpha = 16n^3 - 57n^2 + 80n + 18$ (1)

Let c be any non-zero integer such that

$$ac + 544n^4 - 686n^3 - 2057n^2 + 5604n - 150 = \beta^2 \quad (2)$$

$$bc + 544n^4 - 686n^3 - 2057n^2 + 5604n - 150 = \gamma^2 \quad (3)$$

Introducing the linear transformation

$$\beta = a - \alpha \qquad \gamma = b - \alpha \quad (4)$$

Using some algebra between (2) , (3) and (4) we've

$$c = a + b - 2\alpha$$
$$\therefore c = 2n - 121$$

Hence (a, b, c) is a dio 3-tuple with the property

$$D(544n^4 - 686n^3 - 2057n^2 + 5604n - 150).$$

Some numerical examples satisfying the above mentioned property is listed below in table1

Table1

n	(a, b, c)	D(n)
1	(-6,1,-119)	3255
2	(-262,38,-238)	10005
3	(-383,201,-154)	77179
4	(-474,586,-70)	286045
5	(-439,1289,14)	740595

Below we present dio 3-tuples with their corresponding property in table2

Table2

S. No	(a, b, c)	D(n)
1	$(TO_{n-1}, TO_n, 2n-119)$	$544n^4 - 718n^3 - 1943n^2 + 5444n - 185$
2	$(TO_{n-1}, TO_n, 2n-123)$	$544n^4 - 654n^3 - 2171n^2 + 5764n - 113$
3	$(TO_{n-1}, TO_n, 2n-125)$	$544n^4 - 622n^3 - 2285n^2 + 5924n - 74$

Case (ii):

Here we take

$$a = TO_{n-2} = 16n^3 - 129n^2 + 220n - 314$$
$$b = TO_n = 16n^3 - 33n^2 + 24n - 6$$

Proceeding as in case (i) we've $c = 84n - 406$

$\therefore$ {a, b, c} is a dio 3-tuple with the property $D(960n^4 + 3892n^3 - 16982n^2 + 15736n - 35)$.

Some numerical examples satisfying the above mentioned property is listed below in table

Table3

n	(a, b, c)	D(n)
1	(-207,1,-322)	3571
2	(-262,38,-238)	10005
3	(-383,201,-154)	77179
4	(-474,586,-70)	286045
5	(-439,1289,14)	740595

Below we present dio 3-tuples with their corresponding property in table4

S. No	(a, b, c)	D(n)
1	$(TO_{n-2}, TO_n, 84n-408)$	$960n^4 + 3924n^3 - 17144n^2 + 15896n + 52$
2	$(TO_{n-2}, TO_n, 84n-410)$	$960n^4 + 3956n^3 - 17306n^2 + 16056n + 141$
3	$(TO_{n-2}, TO_n, 84n-412)$	$960n^4 + 3988n^3 - 17468n^2 + 16216n + 232$

Case (iii):

Here we take

$$a = TO_{n-2} = 16n^3 - 129n^2 + 220n - 314$$
$$b = TO_{n-1} = 16n^3 - 81n^2 + 138n - 79$$

Proceeding as in earlier cases we've $c = 278n - 693$

Hence {a, b, c} is a dio 3-tuple with the property $D(-3872n^4 + 38310n^3 - 95885n^2 + 72712n - 2306)$. Some numerical examples satisfying the above mentioned property is listed below in table5

Table5

n	(a, b, c)	D(n)
1	(207,-6,-415)	8959
2	(-262,1,-137)	4106
3	(-383,38,141)	73603
4	(-474,201,419)	214990
5	(-439,586,697)	332879

Below we present some dio 3-tuples with their corresponding property in table6

Table6

S. No	(a, b, c)	D(n)
1	$(TO_{n-2}, TO_{n-1}, 278n-695)$	$-3872n^4 + 38342n^3 - 96095n^2 + 72792n - 2005$
2	$(TO_{n-2}, TO_{n-1}, 278n-697)$	$-3872n^4 + 38374n^3 - 96305n^2 + 72872n - 1702$
3	$(TO_{n-2}, TO_{n-1}, 278n-699)$	$-3872n^4 + 38406n^3 - 96515n^2 + 72952n - 1397$

III. CONCLUSION

In this paper we have explicated dio 3-tuples from truncated octahedral number. Also we have presented 4 sets of dio 3-tuples under 3 cases with some numerical examples. One may also try to explicate such type of dio 3-tuples from other numbers with suitable property.

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