P-CLOSED M-SETS IN MULTI-SET TOPOLOGICAL SPACES

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Abstract- *The purpose of this paper is to introduce a new class of sets called ρ -open M-sets in multiset topological spaces and also we introduce a ρ-continuous M-set functions in multi-set topology. Also some of its interesting properties are discussed.*

I. INTRODUCTION

The concept of generalized closed sets in a topological space was introduced by Levine N[8].GanambalY[2] ,introduced by On Generalized Pre regular-closed sets in Topological Spaces. Girish K P ,Sunil Jacob John[3], introduced the concept of On Multi-set.. Jafari S, Noiri T, Rajesh N and Thivagarn ML[5]., introduced the Another Generalization of closed sets.James MunkersR[6], introduced the notation on Topology. LevineN[7]., introduced the Semi open sets, Semi-continuity in Topological spaces. Mashour A.S, Abd El-Monsef M E and El-deep S N[9], discussed the On pre-continuous and weak pre-continuous mapping . Noiri T, Maki H and UmeharaJ[10], introduced the Generalized pre-closed functions. Sundaram P and Sheik John M[11], discussed the On ω-closed sets in Topology.Devamanoharan C,Pious Missier S and JafariS[4],introduced the notions of ρ -closed sets and ρ-open sets in topological spaces.Devamanoharan C, Pious Missier S[1] , introduced the notions of ρ-continuous functions.In this paper we introduce new class of M-sets called ρ-closed M-sets in multi-set topological spaces.

Key words and Phrases:

ρ –closed, ρ –open, ρ –closed M-Sets, ρ –continuous, ρ –continuous M-set function

II. PRELIMINARIES

Definition : 2.1

Let X be any non-empty set.A family τ of subsets of X is said to be a topology on X if and only if τ satisfies the following axioms:

(i) $\emptyset$ and X are in τ
(ii) The union of the elements of any sub-collection of τ is in τ.
(iii) The finite intersection of the elements of any sub collection of τ is in τ. Then τ is a topology on X.The ordered pair (x,τ)is called a topological space.

Definition: 2.2

Let (x, τ) be a topological space.A subset A of x is said to be a preopen set if $A \subseteq \inf(\mathrm{cl}(A))$and pre-closed set if $cl(\inf(A)) \subseteq A.$

Definition: 2.3

Let (x, τ) be a topological space. A subset A of X is said to be Semi Open set if $A \subseteq cl(\inf(A))$ and Semi closed set if $\inf(\mathrm{cl}(A)) \subseteq A.$

Definition: 2.4

Let (x,τ) be a topological space. A Subset A of X is said to be regular open set if $A = \inf(\mathrm{cl}(A))$ and regular closed set if $A = \mathrm{cl}(\inf(A))$.

Thought-out this paper (x, τ), (y, δ) and(z,η) will always denote topological spaces and (M, τ), (N, δ) and (P, η) denote Multiset topological spaces. Then $\inf(A), cl(A)$ denote the interior and closure of the set A, respectively.

Definition: 2.5

Let (x, τ) be a topological space. A subset set if A⊆X is said to begeneralised pre-closed (briefly gp-closed) if whenever A⊆U and U is open in X.

Definition: 2.6

Let (x, τ) be a topological space. A subset set if A⊆X is said to be generalised pre-regular closed (briefly gpr-closed) if whenever A⊆U and U is regular open in X.

Definition: 2.7

Let(x, τ) be a topological space. A subset A⊆X is said to be $\hat{g}$ -closed set if cl(A)⊆U whenever A⊆U and U is a $\hat{g}$ −closed set if cl(A)⊆U whenever A⊆U and U is a $\hat{g}$-open set in(x, τ).The complement of a $\hat{g}$ −closed set is said to be a $\hat{g}$ − open set.

Definition: 2.8

Let (x, τ) be a topological space. A subset A⊆X is said to be $^*g$ -closed set if cl(A)⊆U whenever A⊆U and U is semi open set in (x, τ) .The complement of a $\hat{g}$-closed set is said to be a g˄ open set.

Definition: 2.9

Let (X,τ) be a topological space. A subset $A \subseteq X$ is said to be a ≠gs-closed set if $Scl(A) \subseteq U$ whenever $A \subseteq U$ and U is a *g-open set in (X,τ). The complement of a ≠gs-closed set is said to be ≠gs-closed set is said to be ≠gs-open set.

Definition: 2.10

Let (X,τ) be a topological space. A subset $A \subseteq X$ is said to be a $\tilde{g}$-closed set if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is a ≠gs-open set in (X,τ). The complement of a $\tilde{g}$-closed set is said to be a $\tilde{g}$- open set.

Definition: 2.11

Let (X,τ)be a topological space. A subset A of X is said to be ρ-closed set if $\rho cl(A) \subseteq int(U)$ whenever $A \subseteq U$ and U is $\tilde{g}$-open in (X,τ).

Definition: 2.12

Let (X,τ) and (N,δ) be any two topological spaces. A function $f:(X,\tau) \to (Y,\delta)$ is said to be ρ-continuous if $f^{-1}(V)$ is ρ-closed in (x,τ) for every ρ-closed set V of (Y,δ).

Definition: 2.13

Let (X,τ) and (N,δ) be any two topological spaces. A function $f:(X,\tau) \to (Y,\delta)$ is said to be ρ-irresolute if $f^{-1}(V)$ is ρ-closed in (x,τ) for every ρ-closed set V of (Y,δ).

Definition: 2.14

A M-set M drawn from the set X is represented by a function count M or C_M defined as $C_M: X \to W$ where W represents the set of whole numbers.

Example: 2.14.1

Let $X = \{x, y, z\}$be any set and $W = 3$. Then $M = \left\{\frac{3}{x}, \frac{2}{y}, \frac{3}{z}\right\}$ is an M-set drawn from X. Clearly, a set is a special case of a M-set.

Definition: 2.15

A sub M-set N of M is a whole sub M-set of M with each element in N having full multiplicity as in M.

i.e., $C_N(x) = C_M(x)$ for every $x$in N.

Definition: 2.16

A sub M-set N of M is a partial whole sub M-set of M with atleast one element in N having full multiplicity as in M.

i.e., $C_N(x) = C_M(x)$ for some x in N.

Definition: 2.17

A sub M-set N of M is a full sub M-set of M if each element in M is an element in N with the same or lesser multiplicity as in M.

i.e., $M^* = N^*$ with $C_N \leq C_M(x)$ for every x in N.

Example: 2.17.1

Let $M = \left\{\frac{2}{x}, \frac{3}{y}, \frac{5}{z}\right\}$ be an M-set. Following are the some of the sub M-set of M which are whole sub M-sets, partial whole sub M-sets and full sub M-sets.

a) A Sub M-set $\left\{\frac{2}{x}, \frac{3}{y}\right\}$ is a whole sub M-set a partial whole sub M-set of M but it is not full sub M-set of M.

b) A sub M-set $\left\{\frac{1}{x}, \frac{3}{y}, \frac{2}{z}\right\}$ is a partial whole sub M-set and full sub M-set of M but it is not a whole sub M-set of M.

c) A sub M-set $\left\{\frac{1}{x}, \frac{3}{y}\right\}$ is partial whole sub M-set of M which neither whole sub M-set nor full sub M-set of M.

Definition: 2.18

A sub M-set R of $M \times M$ is said to be an M-set relation on M if for every member $\left(\frac{m}{x}, \frac{n}{y}\right)$ of R has a count, product of $c_1(x,y)$ and $c_2(x,y)$. We denote $\frac{m}{x}$ related to $\frac{n}{y}$ by $\frac{m}{x} R \frac{n}{y}$.

Definition: 2.19

A M-set relation f is called an M-set function if for every element $\frac{m}{x}$ in Dom F, there is exactly one Ran f such that $\left(\frac{m}{x}, \frac{n}{y}\right)$ is in f with the pair occurig as the product of $c_1(x,y)$ and $c_2(x,y)$.

Example: 2.19.1

Let $M_1 = \left\{\frac{8}{x}, \frac{6}{y}\right\}$ and $M_2 = \left\{\frac{2}{a}, \frac{5}{b}\right\}$ be two M-sets. Then an M-set function from M_1 to M_2 may be defined as $f = \left\{\frac{\left(\frac{8}{x}\frac{2}{a}\right)}{16}, \frac{\left(\frac{6}{y}\frac{5}{b}\right)}{30}\right\}$.

Definition: 2.20

Let $M \in [x]$ and $\tau \subseteq \rho^*(M)$. Then τ is called Multiset topology of M if τ satisfies the following properties

a) A M-set M and the empty M-set φ are in τ.

b) The M-set union of the elements of any sub collection of τ is in τ.

c) The M-set intersection of the elements of any finite subcollection of τ is in τ.

Definition: 2.21

A sub M-set N of an M-topological space M in $[x]^W$ is said to be closed if the M-set $M \ominus N$ is open. i.e., $N^c = M \ominus N$.

Example: 2.21.1

Let $X = \{x, y, z\}, W = 2$ and $M = \left\{\frac{2}{x}, \frac{1}{y}, \frac{1}{z}\right\}$ be a M-set drawn from x. Let $\tau = \left\{M, \varphi, \left\{\frac{1}{x}\right\}, \left\{\frac{2}{x}\right\}, \left\{\frac{1}{x}, \frac{1}{y}\right\}, \left\{\frac{2}{x}, \frac{1}{y}\right\}\right\}$. Clearly, τ is an M-topology and (M, τ) is an M-topological space.

Then the complement of any sub M-set N is a M-topological space (M, τ) is shown as:

a). If $N = \left\{\frac{2}{x}, \frac{1}{y}\right\}$, then $N^c = \left\{\frac{1}{z}\right\}$ and b). If $N = \left\{\frac{1}{x}\right\}$ then $N^c = \left\{\frac{1}{x}, \frac{1}{y}, \frac{1}{z}\right\}$

Definition: 2.22

Given a sub M-set A of an M-topological space M in $[x]^W$, the interior of A is defined as the M-set union of all open M-sets contained in A and is denoted by Int (A).

i.e., $Int\ (A) = \cup\ \{G \subseteq M : G$ is open M-set and $G \subseteq A\}$ and $C_{int(A)}(x) = \max\{C_G(x) : G \subseteq A, G \in \tau\}$

Definition: 2.23

Given a sub M-set A of a M-topological space M in $[x]^W$, the closure of A is defined as the M-set intersection of all closed M-sets containing A and is denoted by $cl(A)$.

i.e.
$cl(A) = \cap \{K \subseteq M: K \text{ is a closed } M - \text{set and } A \subseteq K\}$
and $C_{cl(A)}(x) = \min\{C_K(x): A \subseteq K,\ K \in \tau^c\}$.

Example: 2.23.1

Let $X = \{a, b, c\}, W = 3$ and $M = \left\{\frac{2}{a}, \frac{2}{b}, \frac{1}{c}\right\}$ be the M-set on X. Let $\tau = \left\{M, \varphi, \left\{\frac{2}{a}\right\}, \left\{\frac{2}{b}\right\}, \left\{\frac{2}{a}, \frac{2}{b}\right\}\right\}$. Clearly, τ is a M-topology and the ordered pair (M, τ) is an M-topological space. Then $\tau^c = \left\{\varphi, M, \left\{\frac{2}{b}, \frac{1}{c}\right\}, \left\{\frac{2}{a}, \frac{1}{c}\right\}, \left\{\frac{1}{c}\right\}\right\}$.
Let $A = \left\{\frac{2}{b}, \frac{1}{c}\right\}$ be the sub M-set of M. Then $Int\ (A) = \left\{\frac{2}{b}\right\}$ and $cl(A) = \left\{\frac{2}{a}, \frac{1}{c}\right\}$.

Definition: 2.24

Let M and N be two M-topological spaces. The M-set function $f: M \to N$ is said to be continuous if for each open sub M-set V of N, the M-set $f^{-1}(V)$ is an open sub M-set of M, where $f^{-1}(V)$ is the M-set of all points $\frac{M}{x}$ in M for which $f\left(\frac{m}{x}\right) \in^n V$ for some n.

Example: 2.24.1

Let $M = \left\{\frac{5}{a}, \frac{4}{b}, \frac{4}{c}, \frac{3}{d}\right\}$ and $N = \left\{\frac{7}{x}, \frac{5}{y}, \frac{6}{z}, \frac{4}{w}\right\}$ be two M-sets, then $\tau = \left\{M, \varphi, \left\{\frac{5}{a}\right\}, \left\{\frac{5}{a}, \frac{4}{b}\right\}, \left\{\frac{5}{a}, \frac{4}{b}, \frac{4}{c}\right\}\right\}$ and $\sigma = \left\{N, \varphi, \left\{\frac{7}{x}\right\}, \left\{\frac{5}{Y}\right\}, \left\{\frac{7}{x}, \frac{5}{y}\right\}, \left\{\frac{5}{y}, \frac{6}{z}, \frac{4}{w}\right\}\right\}$ be two M-topologies on M and N respectively.

Consider two M-set function $f: M \to N$ and $g: M \to N$ are given by $f = \left\{\frac{\left(\frac{5}{a}, \frac{5}{y}\right)}{25}, \frac{\left(\frac{4}{b}, \frac{6}{z}\right)}{24}, \frac{\frac{4}{c}, \frac{4}{w}}{16}, \frac{\frac{3}{d}, \frac{6}{x}}{18}\right\}$ and $g = \left\{\frac{\left(\frac{5}{a}, \frac{7}{x}\right)}{35}, \frac{\left(\frac{4}{b}, \frac{7}{x}\right)}{28}, \frac{\left(\frac{4}{c}, \frac{6}{x}\right)}{24}, \frac{\left(\frac{3}{d}, \frac{4}{w}\right)}{12}\right\}$. The M-set function $'f'$ is continuous since the inverse of each member of the M-topology σ on N is a member of the M-topology τ on M.

The M-set function g is not continuous. Since $\left\{\frac{5}{y}, \frac{6}{z}, \frac{4}{w}\right\} \in 6$. i.e., an open M-se of N, but its inverse image $g^{-1}\left(\left\{\frac{5}{y}, \frac{6}{z}, \frac{4}{w}\right\}\right) = \left\{\frac{4}{c}, \frac{3}{d}\right\}$ is not an open sub M-set of M, because the M-set $\left\{\frac{4}{c}, \frac{3}{d}\right\}$ does not belong to τ.

Definition: 2.25

Let M and N be two M-sets drawn from a set X. Then, the following are defined

a) $M = N \text{ if } C_M(x) = C_N(x)$ for all $x \in X$
b) $M \subseteq N$ if $C_M(x) \leq C_N(x)$ for all $x \in X$
c) $P = M \cup N$ if $C_P(x) = Max\ \{C_M(x), C_N(x)\}$ for all $x \in X$
d) $P = M \cap N$ if $C_P(x) = Min\ \{C_M(x), C_N(x)\}$ for all $x \in X$
e) $P = M \oplus N$ if $C_P(x) = C_M(x) + C_N(x)$ for all $x \in X$
f) $P = M \ominus N$ if $C_P(x) = Max\{C_M(x) - C_N(x), 0\}$ for all $x \in X$

where $\oplus$ and $\ominus$ represents M-set addition and M-set subtraction respectively.

Definition: 2.26

A domain X is defined as a set of elements from which M-sets are constructed. The M-set space $[x]^W$ is the set of all M-sets whose elements are in X such that no elements in the M-set occurs more than W times.

The set $[x]^\infty$ is the set of all M-sets over a domain X such that there is no limit on the number of occurrences of an element in an M-set.

III. BASIC PROPERTIES OF ρ-CLOSED M-SETS

Throughout this paper X denote a non – empty set, $M \in [x]^W$ and $C_M: X \to W$ where W is the set of all whole numbers.

Definition: 3.1

Let (M, τ) be an M-topological space. A sub M-set A of M is said to be a pre open M-set if

$A \subseteq int\left(cl(A)\right)$ with $C_A(x) \leq C_{int(cl(A))}(x)$, for all $x \in X$. The complement of the pre open M-set is said to be a pre closed M-set if $A \supseteq cl\left(int(A)\right)$ with $C_A(x) \geq C_{cl(int(A))}(x)$, for all $x \in X$.

Definition: 3.2

Let (M,τ) be a M-topological space. Then the preclosure of a M-set is denoted by $\rho cl(A)$ and defined as $\rho cl(a) = \bigcap\{B: B \supseteq A, each\ B \subseteq M\ is\ a\ preclosed\ M - set\}$, for all $x \in X$.

Example: 3.2.1

Let $X = \{x, y\}, W = 2$ and $M = \left\{\frac{2}{x}, \frac{1}{y}\right\}, \tau = \left\{M, \varphi, \left\{\frac{1}{x}\right\}, \left\{\frac{1}{y}\right\}, \left\{\frac{1}{x}, \frac{1}{y}\right\}\right\}$. Clearly, τ is M-topology and the ordered pair (M,τ) is a M-topological space. Now, the preopen M-sets are: $M, \varphi, \left\{\frac{1}{x}\right\}, \left\{\frac{1}{y}\right\}, \left\{\frac{1}{x}, \frac{1}{y}\right\}$. Let $A = \left\{\frac{2}{x}\right\}$ be a sub M-set of M. Then $pcl(A) = \left\{\frac{2}{x}\right\}$.

Definition: 2.3

Let (x, τ) be a topological space. A subset A of X is said to be Semi Open set if $A \subseteq cl(\inf(A))$ and Semi closed set if $\inf(cl(A)) \subseteq A$.

Definition: 3.4

Let (M,τ) be M-topological space. A sub M-set A of M is said to be semi open M-set if $A \subseteq cl\left(int(A)\right)$ with $C_A(x) \leq C_{cl(int(A))}(x)$ for all $x \in X$. The complement of the semi-open M-set is said to be a semi closed M-set if $A \supseteq int\left(cl(A)\right)$ with $C_A(x) \geq C_{int(cl(A))}(x)$ for all $x \in X$.

Example: 3.4.1

Let $X = \{x, y\}, W = 2$ and $M = \left\{\frac{2}{x}, \frac{2}{y}\right\}, \tau = \left\{M, \varphi, \left\{\frac{1}{x}\right\}, \left\{\frac{1}{y}\right\}, \left\{\frac{2}{y}\right\}, \left\{\frac{1}{x}, \frac{2}{y}\right\}, \left\{\frac{1}{x}, \frac{1}{y}\right\}\right\}$. Clearly, τ is a M-topology and the ordered pair (M,τ) is a M-topological space. Now, the semi open M-sets are: $M, \varphi, \left\{\frac{1}{x}\right\}, \left\{\frac{2}{y}\right\}, \left\{\frac{1}{y}\right\}, \left\{\frac{1}{x}, \frac{2}{y}\right\}, \left\{\frac{1}{x}, \frac{1}{y}\right\}$ and the semi closed M-sets are $\varphi, M, \left\{\frac{1}{x}, \frac{2}{y}\right\}, \left\{\frac{2}{x}\right\}, \left\{\frac{2}{x}, \frac{1}{y}\right\}, \left\{\frac{1}{x}\right\}, \left\{\frac{1}{x}, \frac{1}{y}\right\}$.

Definition: 3.5

Let (M,τ) be a M-topological space. A sub M-set A of M is said to be a regular open M-set if $A = int\left(cl(A)\right)$ with $C_A(x) = C_{int(cl(A))}(x)$ for all $x \in X$. The complement of the regular open M-set is said to be a regular closed M-set if $A = cl\left(int(A)\right)$ with $C_A(x) = C_{cl(int(A))}(x)$, for all $x \in X$.

Example: 3.5.1

Let $X = \{x, y, z\}, W = 2$ and $M = \left\{\frac{1}{x}, \frac{1}{y}, \frac{2}{z}\right\}, \tau = \left\{M, \varphi, \left\{\frac{1}{x}\right\}, \left\{\frac{1}{y}\right\}, \left\{\frac{1}{x}, \frac{1}{y}\right\}\right\}$. Clearly, τ is a M-topology and the ordered pair (M,τ) be a M-topological space. Now, the closed M-sets are: $M, \varphi, \left\{\frac{1}{y}, \frac{2}{z}\right\}, \left\{\frac{1}{x}, \frac{2}{z}\right\}, \left\{\frac{2}{z}\right\}$. Let $A = \left\{\frac{1}{x}\right\}$ be a sub M-set of M. Then $int\left(cl(A)\right) = \left\{\frac{1}{x}\right\} A = int\left(cl(A)\right)$ with $C_A(x) = C_{int(cl(A))}(x)$, for all $x \in X$. Hence A is a regular open M-set. Its complement $A^c = M \oplus A = \left\{\frac{1}{y}, \frac{2}{z}\right\}$ is a regular closed M-set, since $A = cl\left(int(A)\right)$ with $C_A(x) = C_{int(cl(A))}(x)$.

Definition: 3.6

Let (M,τ) be a M-topological space. A sub M-set $A \subseteq M$ is said to be a generalized pre closed (briefly gp-closed) if $\rho cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open M-set in (M,τ) with $C_{\rho cl(A)}(x) \leq C_U(x)$, for all $x \in X$. The complement of gp-closed M-set is said to be a gp-open M-set.

Example: 3.6.1

Let $X = \{x, y\}, W = 2$ and $M = \left\{\frac{1}{x}, \frac{1}{y}\right\}, \tau = \left\{M, \varphi, \left\{\frac{1}{x}\right\}, \left\{\frac{1}{y}\right\}, \left\{\frac{1}{x}, \frac{1}{y}\right\}\right\}$. Clearly, τ is a M-topological space. Here, the pre closed M-sets are: $M, \varphi, \left\{\frac{1}{y}\right\}, \left\{\frac{2}{y}\right\}, \left\{\frac{1}{x}, \frac{1}{y}\right\}$ and the gp-closed M-sets are: $M, \varphi, \left\{\frac{1}{y}\right\}, \left\{\frac{2}{y}\right\}, \left\{\frac{1}{x}, \frac{1}{y}\right\}$.

Definition: 3.7

Let (M, τ) be a M-topological space. A sub M-set $A \subseteq M$ is said to be a generalized pre regular closed M-set (briefly gpr-closed) if $\rho cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open M-set in (M, τ). Then complement of gpr-closed M-set is said to be a gpr-open M-set.

Example: 3.7.1

Let $X = \{x, y\}, W = 2$ and $M = \left\{\frac{1}{x}, \frac{2}{y}\right\}, \tau = \left\{M, \varphi, \left\{\frac{1}{x}\right\}, \left\{\frac{1}{y}\right\}, \left\{\frac{1}{x}, \frac{1}{y}\right\}\right\}$. Clearly, τ is a M-topology and the ordered pair (M, τ) is a M-topological space. Here, the pre closed M-sets are: $M, \varphi, \left\{\frac{1}{y}\right\}, \left\{\frac{2}{y}\right\}, \left\{\frac{1}{x}, \frac{1}{y}\right\}$ and the regular open M-sets are: $M, \varphi, \left\{\frac{1}{y}\right\}, \left\{\frac{1}{x}, \frac{1}{y}\right\}$. Hence the gpr-closed M-sets are: $M, \varphi, \left\{\frac{1}{x}\right\}, \left\{\frac{1}{y}\right\}, \left\{\frac{2}{y}\right\}, \left\{\frac{1}{x}, \frac{1}{y}\right\}$ and the gpr-open M-sets are: $M, \varphi, \left\{\frac{2}{y}\right\}, \left\{\frac{1}{x}, \frac{1}{y}\right\}, \left\{\frac{1}{x}\right\}, \left\{\frac{1}{y}\right\}$.

Definition: 3.8

a) Let (M, τ) be an M-topological space. A sub M-set $A \subseteq M$ is said to be a $\tilde{g}$ closed M-set if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open M-set in (M, τ) with $C_{cl(A)} \leq C_U(x)$ whenever $C_A(x) \leq C_U(x)$, for all $x \in X$. The complement of a $\tilde{g}$-closed M-set is said to be a $\tilde{g}$-open M-set.

b) Let (M, τ) be an M-topological space. A sub M-set $A \subseteq M$ is said to be a $*g$-closed M-set if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is a $\hat{g}$-open M-set in (M, τ) with $C_{cl(A)}(x) \leq C_U(x)$ whenever $C_A(x) \leq C_U(x)$ for all $x \in X$. The complement of a *g-closed M-set is said to be a *g-open M-set.

c) Let (M, τ) be an M-topological space. A sub M-set $A \subseteq M$ is said to be a $\neq$gs-closed M-set if $Scl(A) \subseteq U$ whenever $A \subseteq U$ and U is *g-open M-set in (M, τ) with $C_{Scl(A)} \leq C_U(x)$ whenever $C_A(x) \leq C_U(x)$ for all $x \in X$. The complement of a $\neq$gs-closed M-set is said to be a $\neq$gs-open M-set.

d) Let (M, τ) be an M-topological space. A sub M-set $A \subseteq M$ is said to be a $\tilde{g}$-closed M-set if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is a $\neq$gs-open M-set in (M, τ) with $C_{cl(A)}(x) \leq C_U(x)$ whenever $C_A(x) \leq C_U(x)$, for all $x \in X$. The complement of a $\tilde{g}$-closed M-set is said to be a $\tilde{g}$-open M-set.

e) Let (M, τ) be an M-topological space. A sub M-set $A \subset M$ is said to be a p-closed M-set if $\rho cl(A) \subseteq int\ (U)$ whenever $A \subseteq U$ and U is a $\tilde{g}$-open M-set in (M, τ) with $C_{\rho cl(A)}(x) \leq C_{int\,(U)}(x)$ whenever $C_A(x) \leq C_U(x)$, for all $x \in X$. The complement of a p-closed M-set is said to be a p-open M-set.

Example: 3.8.1

Let $X = \{x, y\}, W = 2$ and $M = \left\{\frac{2}{x}, \frac{2}{y}\right\}, \tau = \left\{M, \varphi, \left\{\frac{1}{x}\right\}, \left\{\frac{1}{y}\right\}, \left\{\frac{2}{y}\right\}, \left\{\frac{1}{x}, \frac{2}{y}\right\}, \left\{\frac{1}{x}, \frac{1}{y}\right\}\right\}$.

Clearly τ is an M-topology and the ordered pair (M, τ) is a M-topological space. Here, the pre closed M-sets are: $M, \varphi, \left\{\frac{2}{x}\right\}, \left\{\frac{1}{x}, \frac{1}{y}\right\}, \left\{\frac{1}{x}\right\}, \left\{\frac{1}{x}, \frac{2}{y}\right\}, \left\{\frac{2}{x}, \frac{1}{y}\right\}$ and the semi open

M-sets are: $M, \varphi, \left\{\frac{1}{x}\right\}, \left\{\frac{1}{y}\right\}, \left\{\frac{1}{x}, \frac{1}{y}\right\}, \left\{\frac{2}{y}\right\}, \left\{\frac{1}{x}, \frac{2}{y}\right\}$ and the $\tilde{g}$-open M-sets are: $M, \varphi, \left\{\frac{1}{x}\right\}, \left\{\frac{1}{y}\right\}, \left\{\frac{1}{x}, \frac{1}{y}\right\}, \left\{\frac{2}{y}\right\}, \left\{\frac{1}{x}, \frac{2}{y}\right\}$ and the *g-open M-sets are: $M, \varphi, \left\{\frac{1}{x}\right\}, \left\{\frac{1}{y}\right\}, \left\{\frac{1}{x}, \frac{1}{y}\right\}, \left\{\frac{1}{x}, \frac{2}{y}\right\}$ and the $\neq$gs-open M-sets are: $M, \varphi, \left\{\frac{1}{x}\right\}, \left\{\frac{1}{y}\right\}, \left\{\frac{1}{x}, \frac{1}{y}\right\}, \left\{\frac{2}{y}\right\}, \left\{\frac{1}{x}, \frac{2}{y}\right\}$ and the $\tilde{g}$-open M-sets are: $M, \varphi, \left\{\frac{1}{x}, \frac{1}{y}\right\}, \left\{\frac{1}{x}\right\}, \left\{\frac{1}{y}\right\}, \left\{\frac{2}{y}\right\}, \left\{\frac{1}{x}. \frac{2}{y}\right\}$. Hence the ρ-closed M-sets are: $M, \varphi, \left\{\frac{2}{x}\right\}, \left\{\frac{1}{x}, \frac{1}{y}\right\}, \left\{\frac{1}{x}\right\}, \left\{\frac{1}{x}, \frac{2}{y}\right\}, \left\{\frac{2}{x}, \frac{1}{y}\right\}$.

Preposition: 3.9

Let (M, τ) be a M-topological space. If a sub M-set A of M is open and pre closed, then A is a ρ-closed M-set.

Proof

Let A be an open and pre closed sub M-set of (M, τ). Let $A \subseteq U$ and U be $\tilde{g}$-open M-set in (M, τ). Then $\rho cl(A) = A = Int\ (A), \rho cl(A) \subseteq int(U)$ with $C_{\rho cl(A)} \leq C_{int}(U)$ for all $x \in X$. Hence A is ρ-closed M-set.

The converse of Proposition 3.9 need not be true as shown from the Example 3.8.1

Example: 3.9.1

Let $X = \left\{\frac{1}{x}, \frac{1}{y}\right\}, W = 2$ and $M = \left\{\frac{2}{x}, \frac{2}{y}\right\}, \tau = \left\{M, \varphi, \left\{\frac{1}{x}\right\}, \left\{\frac{1}{y}\right\}, \left\{\frac{2}{y}\right\}, \left\{\frac{1}{x}, \frac{2}{y}\right\}, \left\{\frac{1}{x}, \frac{1}{y}\right\}\right\}$. Clearly, τ is a M-topology and the ordered pairs (M, τ) is an M-topological space. Here, the open M-sets are = $M, \varphi, \left\{\frac{1}{x}\right\}, \left\{\frac{1}{y}\right\}, \left\{\frac{2}{y}\right\}, \left\{\frac{1}{x}, \frac{2}{y}\right\}, \left\{\frac{1}{x}, \frac{1}{y}\right\}$ and the pre closed M-sets are $M, \varphi, \left\{\frac{2}{x}\right\}, \left\{\frac{1}{x}, \frac{1}{y}\right\}, \left\{\frac{1}{x}\right\}, \left\{\frac{1}{x}, \frac{2}{y}\right\}, \left\{\frac{2}{x}, \frac{1}{y}\right\}$ and the ρ-closed M-sets are $M, \varphi, \left\{\frac{2}{x}\right\}, \left\{\frac{1}{x}, \frac{1}{y}\right\}, \left\{\frac{1}{x}\right\}, \left\{\frac{1}{x}, \frac{2}{y}\right\}, \left\{\frac{2}{x}, \frac{1}{y}\right\}$. Here, the ρ-closed M-set $V = \left\{\frac{2}{x}\right\}$ is pre closed M-set but it is not an open M-set.

Preposition: 3.10

Let (M, τ) be an M-topological space. Every ρ-closed M-set is gpr-closed M-set.

Proof:

Let A be any ρ-closed M-set. Let $A \subseteq U$ and U be regular open M-set. Observe that every regular open M-set is open M-set and every open M-set is $\tilde{g}$-open M-set and therefore A is ρ-closed M-set. It follows that $\rho cl(A) \subseteq int(U) = U$ with $C_{\rho cl(A)} \leq C_U(x)$, for all $x \in X$. Hence A is gpr-closed M-set.

The converse of Proposition 3.10 need not be true as shown from the Example 3.9.1

Example: 3.10.1

Let $x = \{a, b\}, W = 1$ and $M = \left\{\frac{1}{a}, \frac{1}{b}\right\}, \tau = \left\{M, a, \left\{\frac{1}{a}\right\}\right\}$. Clearly, τ is an M-topology and the ordered pairs (M, τ) is an M-topological. Here, the gpr-closed M-sets are $M, \varphi, \left\{\frac{1}{a}\right\}, \left\{\frac{1}{b}\right\}$ and the ρ-closed M-sets are: $M, \varphi, \left\{\frac{1}{b}\right\}$. Here gpr-closed M-set $V = \left\{\frac{1}{a}\right\}$ is not ρ-closed M-set.

Remark:

The union of two ρ -closed M-sets need not be ρ-closed M-set.

Example: 3.10.2

Let $X = \{x, y, z\}, W = 2$ and $M = \left\{\frac{2}{x}, \frac{2}{y}, \frac{1}{z}\right\}, \tau = \left\{m, \varphi, \left\{\frac{2}{x}\right\}, \left\{\frac{2}{y}\right\}, \left\{\frac{2}{x}, \frac{2}{y}\right\}\right\}$. Clearly, τ is an M-topological space. Here, the ρ-closed M-set are: $M, \varphi \left\{\frac{1}{z}\right\}, \left\{\frac{2}{y}, \frac{1}{z}\right\}, \left\{\frac{2}{x}, \frac{1}{z}\right\}, \left\{\frac{1}{x}, \frac{1}{z}\right\}, \left\{\frac{2}{x}, \frac{1}{y}, \frac{1}{z}\right\}, \left\{\frac{1}{x}, \frac{2}{y}, \frac{1}{z}\right\}, \left\{\frac{1}{x}, \frac{1}{y}, \frac{1}{z}\right\}$. Let $U = \left\{\frac{2}{x}, \frac{1}{z}\right\}$ and $V = \left\{\frac{1}{x}, \frac{2}{y}, \frac{1}{z}\right\}$ be ρ-closed M-sets. But $U \cup V = \left\{\frac{2}{x}, \frac{2}{y}, \frac{1}{z}\right\}$ is not ρ-closed M-set.

Proposition: 3.11

Let (M,τ) be an M-topological space. If a sub M-set A of (M,τ) is $\tilde{g}$-open and ρ-closed M-set, then A is pre-closed M-set in (M,τ).

Proof:

If a sub M-set A of (M,τ) is $\tilde{g}$-open M-set and p-closed M-set. Then $\rho cl(A) \subseteq Int(A) \subseteq A$ with $C_{\rho cl(A)}(x) \leq C_A(x)$, for all $x \in X$. Hence A is pre-closed M-set in (M,τ).
The Converseof proposition 3.11 need not be true as shown from the example 3.10.2.

Example: 3.11.1

Let $X = \{x,y\}$, $W = 2$ and $M = \{^2/_x, ^2/_y\}$, $\tau = \{M, \varphi, \{^2/_x\}, \{^1/_x\}, \{^2/_y\}, \{^1/_x, ^2/_y\}\}$. Clearly, τ is an M-topology and the ordered pair (M,τ) is an M-topological space.Here, the preclosed M-sets are $M, \varphi, \{^2/_x\}, \{^1/_x\}, \{^2/_y\}\{^1/_y\}, \{^1/_x, ^1/_y\}, \{^2/_x, ^1/_y\}\{^1/_x, ^2/_y\}$ and the $\tilde{g}$-open M-sets are $M, \varphi, \{^2/_x\}, \{^1/_x\}, \{^2/_y\}, \{^1/_x, ^2/_y\}$ and the ρ-closed M-sets are $M, \varphi, \{^2/_x\}, \{^1/_x\}, \{^2/_y\}\{^1/_y\}, \{^1/_x, ^1/_y\}, \{^2/_x, ^1/_y\}\{^1/_x, ^2/_y\}$. Here, the preclosed M-set $V = \{^1/_x, ^1/_y\}$ is ρ-closed M-sets but it is not $\tilde{g}$-open M-sets.

Proposition: 3.12

Let (M,τ) be an M-topological space. If a sub M-set A of (M,τ) is open and regular closed then A is ρ-closed M-set.

Proof:

Let A be open M-set and regular closed M-set.Since regular closed M-set is pre-closed M-set.Then A is open and pre-closed M-set.By proposition 2.1, A is ρ-closed M-set.
The Converseof proposition 3.12 need not be true as shown from the example 3.11.1.

Example: 3.12.1

Let $X = \{x,y\}$, $W = 2$ and $M = \{^2/_x, ^2/_y\}$, $\tau = \{M, \varphi, \{^1/_x\}, \{^1/_y\}, \{^2/_y\}, \{^1/_x, ^2/_y\}, \{^1/_x, ^1/_y\}\}$. Clearly, τ is an M-topology and the ordered pair (M,τ) is an M-topological space.Here, the regular closed M-sets are $M, \varphi,, \{^1/_x\}, \{^1/_x, ^2/_y\}, \{^1/_x, ^1/_y\}$ and the ρ-closed M-sets are $M, \varphi, \{^2/_x\}, \{^1/_x\}, \{^2/_y\}, \{^1/_x, ^1/_y\}, \{^2/_x, ^1/_y\}, \{^1/_x, ^2/_y\}$. Here,the ρ-closed M-set $V = \{^2/_x, ^1/_y\}$ is not open M-set and is not regular closed M-sets.

Definition:3.13

Let (M,τ) be an M-topological space.Then the semi-closure of an M-set A is denoted by $scl(A)$ and defined as $scl(A) = \cap\{B : B \supseteq A, each\ B \subseteq M\ is\ a\ semiclosed\ M-set\}$ with $C_{scl(A)}(x) = \min\{B : B \supseteq A, each\ B \subseteq M\ is\ a\ semiclosed\ M-set\}$, for all $x \in X$.

Example : 3.13.1

Let $X = \{x,y\}$, $W = 2$ and $M = \{^2/_x, ^2/_y\}$ $\tau = \{M, \varphi, \{^1/_x\}, \{^1/_y\}, \{^2/_y\}, \{^1/_x, ^2/_y\}, \{^1/_x, ^1/_y\}\}$.
Clearly, τ is an M-topology and the ordered pair (M,τ) is an M-topological space.Now the semi open M-set are: $M, \varphi, \{^2/_x\}, \{^1/_x\},, \{^1/_x, ^1/_y\}, \{^2/_x, ^1/_y\}, \{^1/_x, ^2/_y\}$. Let $A = \{^2/_x, ^1/_y\}$ be a sub M-set of (M,τ). Then $scl(A) = \{^2/_x, ^1/_y\}$ and $Sint(A) = \{^1/_x, ^1/_y\}$.

Proposition: 3.14

Let (M,τ) be an M-topological space.If A is ρ-closed M-set and $A \subseteq B \subseteq \rho cl(A)$ with

$C_A(x) \le C_B(x) \le C_{\rho cl(A)}(x)$, for all $x \epsilon X$, then B is ρ-closed M-sets.

Proof:

Let U be a $\tilde{g}$-open M-sets of (M,τ) such that $B \subseteq U$. Then $A \subseteq U$ and since A is ρ-closed, we have $\rho cl(A) \subseteq int(U)$.Now $\rho cl(B) \subseteq \rho cl(\rho cl(A)) = \rho cl(A) \subseteq int(U)$ with $C_{\rho cl(B)}(x) \le C_{int(U)}(x)$, for all $x \epsilon X$.Hence B is ρ-closed M-sets.

IV. . ρ-CONTINUOUS M-SET FUNCTIONS:

Throughout this paper X denote a non – empty set, $M \in [x]^W$ and $C_M: X \to W$ where W is the set of all whole numbers.

Definition: 4.1

Let (M,τ) and (N,δ) be any two M-topological spaces. Any M-set function $f: (M,\tau) \to (N,\delta)$ is called ρ-continuous M-set function of $f^{-1}(V)$ is a ρ-closed M-set in $(M,\tau)$for every closed M-set V in (N,δ).

Example: 4.1.1

Let $X = \{x,y\}, W_1 = 1$and $Y = \{a,b\}, W_2 = 2$. Let $M = \left\{\frac{1}{x}, \frac{1}{y}\right\}$ and $N = \left\{\frac{2}{a}, \frac{1}{b}\right\}$ be two M-sets. Let $\tau = \left\{M, \varphi, \left\{\frac{1}{x}\right\}, \left\{\frac{1}{y}\right\}\right\}$ and $\delta = \left\{\infty, \varphi, \left\{\frac{2}{a}\right\}\right\}$ be two M-topologies on M and N respectively. Then $(M,\tau); (N,\delta)$ the two topological spaces. Now, ρ-closed M-sets of (M,τ) are; $M, \varphi, \left\{\frac{1}{x}\right\}, \left\{\frac{1}{y}\right\}$ and the closed M-sets of $(N,\delta)$are $M, \varphi, \left\{\frac{1}{b}\right\}$.

Let the M-set function $f: (M,\tau) \to (N,\delta)$ be defined as $f = \left\{\frac{\frac{1}{x}\frac{2}{a}}{2}, \frac{\frac{1}{y}\frac{1}{b}}{1}\right\}$. Hence, f is p-continuous M-set function, as the inverse image of every closed M-set in (N,δ) is ρ-closed M-set in (M,τ).

Definition : 4.2

Let $(M,\tau)\ and\ (N,\delta)$ be any two M-topological space.Any M-set function $f: (M,\tau) \to (N,\delta)$ is called an irresolute M-set function if $f^{-1}(V)$ is open M-set in (M,τ) for every open M-set V in (N,δ).

Example: 4.2.1

Let $X = \{x,y\}, W_1 = 2$ and $Y = \{a,b\}, W_2 = 1$,Let $M = \left\{\frac{2}{x}, \frac{1}{y}\right\}$ and $N = \left\{\frac{1}{a}, \frac{1}{b}\right\}$ be two M-sets.

Let $\tau = \left\{M, \varphi, \left\{\frac{2}{x}\right\}, \left\{\frac{1}{y}\right\}\right\}$ and $\delta = \left\{N, \varphi, \left\{\frac{1}{a}\right\}\right\}$ be two M-topologies on M and N respectively. Then $(M,\tau), (N,\delta)$ the two M-topological spaces. Now, the open M-sets of (M,τ) are $M, \varphi, \left\{\frac{2}{x}\right\}, \left\{\frac{1}{y}\right\}$ and the open M-sets of $(N,\delta)$are $N, \varphi, \left\{\frac{1}{a}\right\}$.

Let the M-set function $f: (M,\tau) \to (N,\delta)$ be defined as $f = \left\{\frac{\frac{2}{x}\frac{1}{a}}{2}, \frac{\frac{1}{y}\frac{1}{b}}{1}\right\}$. Here, f is irresolute M-set function, as the inverse image of every open M-set in (N,δ) is open M-set in (M,τ).

Definition: 4.3

Let (M,τ) and (N,δ) be any two M-topological space. Any M-set function $f''(M,\tau) \to (N,\delta)$ is called a ρ-irresolute M-set function if $f^{-1}(V)$ is ρ-closed M-set in $(M,\tau)$for every ρ-closed M-set V in (N,δ).

Example: 4.3.1

Let $X = \{x,y\}, W_1 = 2$and $Y = \{a,b\}, W_2 = 1$. Let $M = \left\{\frac{2}{x}, \frac{2}{y}\right\}$ and $N = \left\{\frac{1}{a}, \frac{1}{b}\right\}$ be any two M-sets. Let $\tau = \left\{M, \varphi, \left\{\frac{1}{x}\right\}, \left\{\frac{1}{y}\right\}, \left\{\frac{2}{y}\right\}, \left\{\frac{1}{x}, \frac{2}{y}\right\}, \left\{\frac{1}{x}, \frac{1}{y}\right\}\right\}$ and $\delta = \left\{\infty, \varphi, \left\{\frac{1}{a}\right\}\right\}$ be two M-topologies on M and N respectively. Then $(M,\tau),(N,\delta)$ the two M-topological spaces. Now, the ρ-closed M-sets of (M,τ) are $M,\varphi,\left\{\frac{2}{x}\right\},\left\{\frac{1}{x},\frac{1}{y}\right\},\left\{\frac{1}{x}\right\},\left\{\frac{1}{x},\frac{2}{y}\right\}$ and the ρ-closed M-sets of (N,δ) are $N,\varphi,\left\{\frac{1}{b}\right\}$.

Let the M-set function $f:(M,\tau)\to(N,\delta)$ be defined as $f:\left\{\frac{\frac{2}{x}\frac{1}{b}}{2},\frac{\frac{2}{y}\frac{1}{a}}{2}\right\}$. Here, f is ρ-irresolute M-set function, as the inverse image of every ρ-closed M-set in (N,δ) is ρ-closed M-set in (M,τ).

Proposition: 4.4

Let $(M,\tau),(N,\delta)$ and (P,η) be any three M-topological spaces. If $f:(M,\tau)\to(N,\delta)$ is a ρ-continuous M-set function and $g:(N,\delta)\to(P,\eta)$ is a continuous M-set function, then $gof:(M,\tau)\to(P,\eta)$ is ρ-continuous M-set function

Proof:

Let V be any closed M-set in (P,η). Since g is a continuous M-set function, $g^{-1}(V)$ is closed M-set in (N,δ). Since f is ρ-continuous M-set function, $f^{-1}(g^{-1}(v))=(gof)^{-1}(v)$ is closed M-set in (M,τ). Therefore gof is a ρ-continuous M-set function.

Proposition: 4.5

Let $(M,\tau),(N,\delta)$ and (P,η) be any three M-topological spaces. If $f:(M,\tau)\to(N,\delta)$ is ρ-irresolute M-set function and $g:(N,\delta)\to(P,\eta)$ is a ρ-irresolute M-set function, then $gof:(M,\tau)\to(P,\eta)$ is ρ-irresolute M-set function.

Proof:

Let V be ρ-closed M-set in (P,η). Since g is ρ-irresolute M-set function, $g^{-1}(V)$ is ρ-closed M-set in (N,δ). As f is ρ-irresolute M-set function, $f^{-1}(g^{-1}(V))=(gof)^{-1}(V)$ is ρ-closed M-set in (M,τ). Therefore gof is ρ-irresolute M-set function.

Preposition: 4.6

Let $(M,\tau),(N,\delta)$ and (P,η) be any three M-topological spaces. If $f:(M,\tau)\to(N,\delta)$ is p-irresolute M-set function and $g:(N,\delta)\to(P,\eta)$ is continuous M-set function, then $gof:(M,\tau)\to(P,\eta)$ is ρ-continuous M-set function.

Proof:

Let V be closed M-set in (P,η). Since g is ρ-continuous M-set function, $g^{-1}(v)$ is ρ-closed M-set in (N,δ). As f is ρ-irresolute M-set function, $f^{-1}g^{-1}(V)=(gof)^{-1}(V)$ is ρ-closed M-set in (M,τ). Therefore gof is ρ-irresolute M-set function.

REPERENCES

[1] Devamanoharan C and Pious Missier S., On $\rho-$ Continuous Functions, International Journal of Mathematical Archive-3(3), 2012, 1102-1112.

[2] Ganambal Y.,On Generalized Preregular-closed sets in Topological Spaces,Indian.J.Pune.Appl.Math.,28(3)(1997),351-360.

[3] GirishK.P.,Sunil Jacob John.,On Multiset Topologies,computer and Mathematics with applications., 2 (2012),37-52.

[4] JafariS.,PiousMisserS.,andDevamanoharan C., ρ-closed sets in Topological Spaces., 5(2012), 554-566.

[5] Jafari S, Noiri T, Rajesh N and Thivagarn M. L., Another Generalization of closed sets, Kochi J. Math. 3 (2008), 25-38

[6] James MunkerR.,Topology, Prentice Hall of Indian Private Limited,NewDelhi

[7] LevineN.,Semi open sets, Semi-continuity in Topological spaces, Amer Math,70(1963),36-41.

[8] Levine N., Generalized closed sets in topology, Rend circ. Math Palermo, 19(2) (1970), 89-96.

[9] Mashour A.S, Abd El-MonsefM.E. and El-deep S.N. ,On pre-continuous and weak pre-continuous mapping Proc.Math.Phys.Soc.egypt.,53(1982),47-53.

[10] Noiri T.,Maki H and Umehara J., Generalized pre-closed functions,Mem.Fac.Sci.Kochi.Univ.ser .Maths.,19(1998), 13-20.

[11] Sundaram P. and Sheik John M.,On ω-closed sets in Topology,Actaciencia Indian. 115 (2017), 1049-1056.