Fuzzy Translation And Fuzzy Multiplication In B–Algebras

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Abstract- *In this paper, we introduced the concept of fuzzy translation and fuzzy multiplication on B-algebras and discussed some of its properties in detail by using the concepts of fuzzy B-ideal and fuzzy B-sub-algebra.*

Keywords- B–algebra, B–Ideal, Fuzzy B–Ideal, Fuzzy α-translation, Fuzzy α- multiplication.

I. INTRODUCTION

After the introduction of fuzzy subsets by L.A.Zadeh[8], several researchers explored on the generalization of the notion of fuzzy subset.Y. Imai and K. Iseki[1,2] introduced two classes of abstract algebras: BCK-algebras and BCI-algebras . It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras.H.K.Park and H.S.Kim[4] introduced the notion of B-Ideals. Sun ShinAhn and KeumseongBang[6] have discussed the fuzzy subalgebra in B-algebra.J.Neggers and H. S. Kim[3] introduced a new notion, called a B- algebra which is related to several classes of algebras of interest such as BCH/BCI/BCK-algebra.C. Yamini and S. Kailasavalli[7] introduced the notion of Fuzzy B-ideals on B- algebras. T.Priya and T.Ramachandran[5] introduced the notion of Fuzzy translation and Fuzzy multiplication on PS-Algebras. In this paper, we introduced the concept of Fuzzy translation and Fuzzy multiplication on B-Algebras and established some of its properties in detail.

II. PRELIMINARIES

Definition 2.1:[J.Neggers and H. S. Kim [3]]

A B-algebra is a non-empty set X with a constant 0 and a binary operation '*' satisfying the following axioms:

(i) $x * x = 0$
(ii) $x * 0 = x$
(iii) $(x * y) * z = x * (z * (0 * y)), for\ all\ x, y, z \in X$

For brevity we also call X a B-algebra. In X we can define a binary relation "$\leq$" by $x \leq y$ if and only if $x * y = 0$.

Definition 2.2:[Sun ShinAhn and Keumseong Bang [6]]

A non-empty subset *M* of a B-algebra X is called a sub-algebra of X if $x * y \in M$ for any $x, y \in M$.

Definition 2.3:[Sun ShinAhn and Keumseong Bang [6]]

Let γ be a fuzzy set in a B-algebra. Then γ is called a fuzzy subalgebra of X if $\gamma(x * y) \geq \gamma(x) \wedge \gamma(y)\ for\ all\ x, y \in X$.

Definition 2.4:[H. K. Park and H.S Kin [4]]

A non-empty subset M of a B-algebra X is called a B-ideal of X if it satisfies for $x, y, z \in X$

(i) $0 \in M$
(ii) $(x * y) \in M and\ (z * x) \in M$ implies $(y * z) \in M$

Definition 2.5:[C. Yamini and S. Kailasavalli [7]]

Let $(X, *, 0)$ be a B-algebra, a fuzzy set γ in X is called a fuzzy B-ideal of X if it satisfies the following axioms

(i) $\gamma(0) \geq \gamma(x)$
(ii) $\gamma(y * z) \geq \gamma(x * y) \wedge \gamma(z * x), for\ all\ x, y, z \in X$

*	0	1	2
0	0	2	1
1	1	0	2
2	2	1	0

The fuzzy set γ given by $\gamma(0) = 0.8, \gamma(1) = 0.5, \gamma(2) = 0.2$ is a fuzzy B-ideal.

III. FUZZY TRANSLATION AND FUZZY MULTIPLICATION IN B-ALGEBRAS

Let X be a B-algebra. For any fuzzy set γ of X, we define $T = 1 - \sup\{\gamma(x)/x \in X\}$, unless otherwise we specified.

Definition 3.1:

Let γ be a fuzzy subset of X and $\alpha \in [0,T]$. A mapping $\gamma_\alpha^T : X \to [0,1]$ is said to be a fuzzy $\alpha -$ translation of γ if it satisfies $\gamma_\alpha^T = \gamma(x) + \alpha, \forall x \in X$.

Definition 3.2:

Let γ be a fuzzy subset of X and $\alpha \in [0,1]$. A mapping $\gamma_\alpha^M : X \to [0,1]$ is said to be a fuzzy $\alpha -$ multiplication of γ if it satisfies $\gamma_\alpha^M = \alpha\, \gamma(x), \forall x \in X$.

Examples 3.3:

Let X=$\{0,1,2\}$ be the set with the following table.

*	0	1	2
0	0	2	1
1	1	0	2
2	2	1	0

Then (X, *, 0) is a B-algebra

We define a fuzzy set γ of X by

$$\gamma(x) = \begin{cases} 0.4, & if\ x \neq 1 \\ 0.3, & \text{if } x = 1 \end{cases}$$

Then γ is a fuzzy B- sub algebra of X.

Here,

T= 1- sup$\{\gamma(x)/x \in X\}$

= 1-0.4=0.6

Choose α = 0.5$\in [0,T]$ and $\beta = 0.7 \in [0,1]$

Then the mapping $\gamma_{0.5}^T : X \to [0,1]$ is defined by

$$\gamma_{0.5}^T = \begin{cases} 0.4 + 0.5=0.9, & \text{if } x \neq 1 \\ 0.3+ 0.5=0.8, & \text{if } x = 1 \end{cases}$$

which satisfies $\gamma_{0.5}^T(x) = \gamma(x) + 0.5$, $\forall x \in X$, is a fuzzy 0.5-translation and the mapping.

$\gamma_{0.7}^M : X \to [0,1]$ is defined by

$$\gamma_{0.7}^M = \begin{cases} (0.7)\ (0.5) =0.28, & \text{if } x \neq 1 \\ (0.7)(0.3)=0.21, & \text{if } x = 1 \end{cases}$$

which satisfies $\gamma_{0.7}^M(x) = (0.7)\ \gamma(x)$, $\forall x \in X$, is a fuzzy 0.7-multilplication.

Theorem: 3.4

If γ of X is a fuzzy B-Sub algebra and α ϵ [0, T] then the fuzzy α- translation $\gamma_\alpha{}^T(x)$ of γ is also a fuzzy B- sub algebra of X.

Proof:

Let $x, y \in X$ and $\alpha \epsilon [0,T]$

Then, $\gamma(x * y) \geq \gamma(x) \wedge \gamma(y)$

Now,

$\gamma_\alpha{}^T(x * y) = \gamma(x * y) + \alpha$

$\geq [\gamma(x) \wedge \gamma(y)\ ] + \alpha$

$=(\gamma(x) + \alpha) \wedge (\gamma(y) + \alpha)$

$= \gamma_\alpha{}^T(x) \wedge \gamma_\alpha{}^T(y)$.

Hence the proof.

Theorem: 3.5

Let γ be a fuzzy subset of X such that the fuzzy α-translation $\gamma_\alpha{}^T(x)$ of γ is a fuzzy sub algebra of X, for some $\alpha\epsilon[0,T]$, then γ is a fuzzy sub algebra of X.

Proof:

Assume that $\gamma_\alpha{}^T(x)$ is a fuzzy sub algebra of X for some $\alpha\epsilon[0,T]$

Let $x, y \in X$, we have

$\gamma(x*y)+\alpha=\gamma_\alpha{}^T(x*y)$

$\geq \gamma_\alpha{}^T(x)\wedge\gamma_\alpha{}^T(y)$

$=(\gamma(x)+\alpha)\wedge(\gamma(y)+\alpha)$

$=[\gamma(x)\wedge\gamma(y)\ ]+\alpha$

$\Rightarrow \gamma(x*y)\geq \gamma(x)\wedge\gamma(y)\ ,\forall x,y\in X$

Hence, γ is fuzzy sub algebra of X.

Theorem: 3.6

For any fuzzy B- sub algebra γ of X and $\alpha\epsilon[0,1]$, if the fuzzy α- multiplication $\gamma_\alpha{}^M(x)$ of γ is a fuzzy B-sub algebra of X.

Proof:

Let $x, y \in X$ and $\alpha\epsilon[0,T]$

Then $\gamma(x*y)\geq \gamma(x)\wedge\gamma(y)$

Now,

$\gamma_\alpha{}^M(x*y)=\alpha\gamma(x*y)$

$\geq\alpha\ [\gamma(x)\wedge\gamma(y)\ ]$

$\geq\alpha\ \gamma(x)\wedge\alpha\,\gamma(y)$

$=\gamma_\alpha{}^M(x)\wedge\gamma_\alpha{}^M(y)$

$\Rightarrow \gamma_\alpha{}^M(x*y)\geq\gamma_\alpha{}^M(x)\wedge\gamma_\alpha{}^M(y)$

Therefore, $\gamma_\alpha{}^M$ is a fuzzy B- sub algebra of X.

Theorem: 3.7

For any fuzzy subset γ of X and $\alpha\epsilon[0,1]$, if the fuzzy α-multiplication $\gamma_\alpha{}^M(x)$ of γ is a fuzzy B-sub algebra of X, then so in γ.

Proof:

Assume that $\gamma_\alpha{}^M(x)$ of γ is a fuzzy B- sub algebra of X for some $\alpha\epsilon[0,T]$

Let $x, y \in X$, we have

$\alpha\gamma(x*y)=\gamma_\alpha{}^M(x*y)$

$\geq\gamma_\alpha{}^M(x)\wedge\gamma_\alpha{}^M(y)$

$=\alpha\ \gamma(x)\wedge\alpha\,\gamma(y)$

$=\alpha\ [\gamma(x)\wedge\gamma(y)\ ]$

$\Rightarrow \gamma(x*y)\geq\gamma(x)\wedge\gamma(y)$

Hence, γ is a fuzzy B- sub algebra of X.

Theorem: 3.8

If γ is a fuzzy B- ideal of X, then the fuzzy α- translation $\gamma_\alpha{}^T(x)$ of γ is a fuzzy B- ideal of X, for all $\alpha\epsilon[0,T]$.

Proof:

Let γ be a fuzzy B-ideal of X and let $\alpha\epsilon[0,T]$

Then,

$\gamma_\alpha{}^T(o)=\gamma(o)+\alpha$

$\geq\gamma(x)+\alpha$

$=\gamma_\alpha{}^T(x)$

$\gamma_\alpha{}^T(y*z)=\gamma(y*z)+\alpha$

$\geq\{\gamma(x*y)\wedge\gamma(z*x)\}+\alpha$

$\geq[\gamma(x*y)+\alpha]\wedge[\gamma(z*x)+\alpha]$

Hence, $\gamma_\alpha{}^T$ of γ is a fuzzy B- ideal of X, $\forall\alpha\epsilon[0,T]$.

Theorem: 3.9

Let γ is a fuzzy subset of X such that the fuzzy α- translation $\gamma_\alpha{}^T(x)$ of γ is a fuzzy B- ideal of X, for some $\alpha\epsilon[0,T]$, then γ is a fuzzy B- ideal of X.

Proof:

Assume that $\gamma_\alpha{}^T$ is a fuzzy B- ideal of X for some $\alpha\epsilon[0,T]$

Let $x, y \in X$

Then,

$\gamma(o)+\alpha=\gamma_\alpha{}^T(o)$

$\geq\gamma_\alpha{}^T(x)$

$=\gamma(x)+\alpha$

and so,

$\gamma(o) \geq \gamma(x)$
also,
$\gamma(y * z) + \alpha = \gamma_\alpha{}^T(y * z)$
$\geq \gamma_\alpha{}^T(x * y) \wedge \gamma_\alpha{}^T(z * x)$
$= [\gamma(x * y) + \alpha] \wedge [\gamma(z * x) + \alpha]$
$= \{\gamma(x * y) \wedge \gamma(z * x)\} + \alpha$
and so
$\gamma(y * z) \geq \gamma(x * y) \wedge \gamma(z * x)$
Hence, γ is a fuzzy B-ideal of X.

Theorem: 3.10

Let γ be a fuzzy subset of X such that the fuzzy α-multiplication $\gamma_\alpha{}^M$ of γ is a fuzzy B-ideal of X for some $\alpha \epsilon [0,1]$, then γ is a fuzzy B-ideal of X.

Proof:

Assume that $\gamma_\alpha{}^M$ is a fuzzy B- ideal of X for some $\alpha \epsilon [0,1]$.
Let $x, y \in X$
Then, $\alpha\, \gamma(o) = \gamma_\alpha{}^M(o)$
$\geq \gamma_\alpha{}^M(x)$
$= \alpha\, \gamma(x)$
and so,
$\gamma(o) \geq \gamma(x)$
Now,
$\alpha \gamma(y * z) = \gamma_\alpha{}^M(y * z)$
$\geq \gamma_\alpha{}^M(x * y) \wedge \gamma_\alpha{}^M(z * x)$
$= \alpha\, \gamma(x * y) \wedge_\alpha \gamma(z * x)$
$= \alpha[\gamma(x * y) \wedge \gamma(z * x)]$
and so,
$\gamma(y * z) \geq \gamma(x * y) \wedge \gamma(z * x)$.
Hence, γ is a fuzzy B-ideal of X.

Theorem: 3.11

If γ is a fuzzy B-ideal of X, then the fuzzy α-multiplication $\gamma_\alpha{}^M$ of γ is a fuzzy B-ideal of X, for all $\alpha \epsilon [0,T]$.

Proof:

Let γ be a fuzzy B- ideal of X and let $\alpha \epsilon [0,T]$
Then,
$\gamma_\alpha{}^M(0) = \alpha\, \gamma(0)$
$\geq \alpha\, \gamma(x)$
$= \gamma_\alpha{}^M(x)$
$\gamma_\alpha{}^M(y * z) = \alpha\, \gamma(y * z)$
$\geq \alpha\{\gamma(x * y) \wedge \gamma(z * x)\}$
$= \alpha\, \gamma(x * y) \wedge_\alpha \gamma(z * x)$
$= \gamma_\alpha{}^M(x * y) \wedge \gamma_\alpha{}^M(z * x)$
Hence, $\gamma_\alpha{}^M$ of γ is a fuzzy B-ideal of X, for all $\alpha \epsilon [0,T]$.

IV. CONCLUSION

In this paper we have discussed Fuzzy Translation and Fuzzy Multiplication on B-Algebras through B- sub algebras and B- Ideals. And also proved some theorems on them. This concept can further be generalized to bipolar fuzzy set , Intuitionistic fuzzy set, Interval valued fuzzy sets for new result in our future work.

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