A Game Theory Approach on Winning Probability of Two Person Zero Sum Game

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Abstract- *Game theory is a mathematical system for analyzing and predicting how humans behave in strategic situations. Game theory can present a strategic tool for decision making that offers perspectives on how players may act under various circumstances. A game is said to be zero-sum if for any outcome, the sum of the payoffs to all players is zero. In a two-player zero-sum game, one player's gain is the other player's loss, so their interests are diametrically opposed. The extreme case of players with fully opposed interests is embodied in the class of two player zero-sum (or constant-sum) games. A game is called zero-sum if the sum of payoffs equals zero for any outcome. That means that the winnings of the winning players are paid by the losses of the losing players. This paper offers an introduction to game theory for winning probability on two person zero sum game. If a person always doubles increase their bet amount and continuously repeat the game. Whenever game will be his favor then he will always earn same profit of their principal amount. A fact is common knowledge if all players know it, and know that they all know it, and so on. The structure of the game is often assumed to be common knowledge among the players.*

Keywords- Game theory, Player, Zero sum game, Payoff, Strategy. MSC: 91A60, 60G40

I. INTRODUCTION

Game theory is the formal study of conflict and cooperation. Game theoretic concepts apply whenever the actions of several agents are interdependent. These agents may be individuals, groups, firms, or any combination of these. The concepts of game theory provide a language to formulate structure, analyze, and understand strategic scenarios. Game theory has been developed as a general framework for decision making in uncertainty when payoffs depend on the actions taken by other players. as a method, game theory helps individuals and firms to study rational behavior in interactive decision problems. More accurate predictions help in designing more effective mechanisms and policies which ultimately should make the coordination of efforts and the allocation of resources more efficient. When someone overreacts, we sometimes say "it's just a game." Games are often not serious. Mathematical games, which are the subject of this book, are different. It was the purpose of game theory from its beginnings in 1928 to be applied to serious situations in economics, politics, business, and other areas. The complete set of rules describes a game. A play is an instance of the game. In certain situations, called positions, a player has do make a decision, called a move or an action. This is not the same as strategy. a strategy is a plan that tells the player what move to choose in every possible position. Rational behavior is usually assumed for all players. That is, players have preferences, beliefs about the world (including the other players), and try to optimize their individual payoffs. Moreover, players are aware that other players are trying to optimize their payoffs. a game is called zero-sum if the sum of payoffs equals zero for any outcome. That means that the winnings of the winning players are paid by the losses of the losing players. For zero-sum two-player games, the bimatrix representation of the game can be simplified: the payoff of the second player doesn't have to be displayed, since it is the negative of the payoff of the first player. The extreme case of players with fully opposed interests is embodied in the class of two player zero-sum (or constant-sum) games. a classic case of a zero-sum game, which was considered in the early days of game theory by von Neumann, is the game of poker.

II. HISTORICAL BACKGROUND

The origins of game theory date as early back as the 18th century but had been established as a respectable branch of mathematics in the 1920s through the work of Emile Borel and John von Neumann. The true framework for game theory as we know it today was established in the 1944s when John von Neumann and Oskar Morgenstern published their book Theory of Games and Economic Behaviors. In 1950 John F. Nash reasoned that a player must determine their action profile while considering what all other players will decide to do. Until that point, decisions were based solely on maximizing one's own utility. Depending on the situation, this would lead to constant conflict between each player. - The earliest example of a formal game-theoretic analysis is the study of a duopoly by Antoine Cour not in 1838. The mathematician Emile Borel suggested a formal theory of games in 1921, which was furthered by the mathematician John von Neumann in 1928 in a "theory of parlor games." Game theory was established as a field in its own right after the 1944 publication of the monumental volume Theory of Games and Economic Behavior by von Neumann and the economist Oskar Morgenstern. This book provided much of the basic terminology and problem setup that is still in use today. In 1950, John Nash demonstrated that finite games have always have an equilibrium point, at which all players choose actions which are best for them given their opponents' choices. This central concept of noncooperative game theory has been a focal point of analysis since then. In the 1950s and 1960s, game theory was broadened theoretically and applied to problems of war and politics. Since the 1970s, it has driven a revolution in economic theory. Additionally, it has found applications in sociology and psychology, and established links with evolution and biology. Game theory received special attention in 1994 with the awarding of the Nobel Prize in economics to Nash, John Harsanyi, and Reinhard Selten. At the end of the 1990s, a high-profile application of game theory has been the design of auctions. Prominent game theorists have been involved in the design of auctions for al locating rights to the use of bands of the electromagnetic spectrum to the mobile telecom communications industry. Most of these auctions were designed with the goal of allocating these resources more efficiently than traditional governmental practices, and additionally raised billions of dollars in the United States and Europe.

III. DEFINITIONS

The object of study in game theory is the game, which is a formal model of an interactive situation. It typically involves several players; a game with only one player is usually called a decision problem. The formal definition lays out the players, their preferences, their information, the strategic actions available to them, and how these influence the outcome. The object of study in game theory is the game, which is a formal model of an interactive situation. It typically involves several players; a game with only one player is usually called a decision problem. The formal definition lays out the players, their preferences, their information, the strategic actions available to them, and how these influence the outcome. Games can be described formally at various levels of detail. A coalitional (or cooperative) game is a high-level description, specifying only what payoffs each potential group, or coalition, can obtain by the cooperation of its members. What is not made explicit is the process by which the coalition forms. As an example, the players may be several parties in parliament. Each party has a different strength, based upon the number of seats occupied by party members. The game describes which coalitions of parties can form a majority, but does not delineate, for example, the negotiation process through which an agreement to vote en bloc is achieved. Cooperative game theory investigates such coalitional games with respect to the relative amounts of power held by various players, or how a successful coalition should divide its proceeds. This is most naturally applied to situations arising in political science or international relations, where concepts like power are most important. For example, Nash proposed a solution for the division of gains from agreement in a bargaining problem which depends solely on the relative strengths of the two parties' bargaining position. The amount of power a side has is determined by the usually inefficient outcome that results when negotiations break down. Nash's model fits within the cooperative frame work in that it does not delineate a specific timeline of offers and counteroffers, but rather focuses solely on the outcome of the bargaining process.

Some definition-

Game

A game is a formal description of a strategic situation.

Game theory

Game theory is the formal study of decision-making where several players must make choices that potentially affect the interests of the other players.

Mixed strategy

A mixed strategy is an active randomization, with given probabilities, that determines the player's decision. As a special case, a mixed strategy can be the deterministic choice of one of the given pure strategies.

Nash equilibrium

Nash equilibrium, also called strategic equilibrium, is a list of strategies, one for each player, which has the property that no player can unilaterally change his strategy and get a better payoff.

Payoff

A payoff is a number, also called utility, that reflects the desirability of an outcome to a player, for whatever reason. When the outcome is random, payoffs are usually weighted with their probabilities. The expected payoff incorporates the player's attitude towards risk.

Perfect information

A game has perfect information when at any point in time only one player makes a move and knows all the actions that have been made until then.

Player

A player is an agent who makes decisions in a game.

Rationality

A player is said to be rational if he seeks to play in a manner which maximizes his own payoff. It is often assumed that the rationality of all players is common knowledge.

Strategic form

A game in strategic form, also called normal form, is a compact representation of a game in which players simultaneously choose their strategies. The resulting payoffs are presented in a table with a cell for each strategy combination.

Strategy

In a game in strategic form, a strategy is one of the given possible actions of a player. In an extensive game, a strategy is a complete plan of choices, one for each decision point of the player.

Zero-sum game

A game is said to be zero-sum if for any outcome, the sum of the payoffs to all players is zero. In a two-player zero-sum game, one player's gain is the other player's loss, so their interests are diametrically opposed.

IV. REPETITION IN TWO PERSON ZERO SUM GAME

The theory of repeated games explores how mutual help and cooperation are sustained through repeated interaction, even when economic agents are completely self-interested beings. This thesis analyzes two models that involve repeated interaction in an environment where some information is private. we characterize the equilibrium set of the following game. Two players interact repeatedly over an infinite horizon and occasionally, one of the players has an opportunity to do a favor to the other player. The ability to do a favor is private information and only one of the players is in a position to do a favor at a time. The cost of doing a favor is less than the benefit to the receiver so that, always doing a favor is the socially optimal outcome. Intuitively, a player who develops the ability to do a favor in some period might have an incentive to reveal this information and do a favor if she has reason to expect future favors in return. We show that the equilibrium set expands monotonically in the likelihood that someone is in a position to do a favor. It also expands with the discount factor. How-ever, there are no fully efficient equilibrium for any discount factor less than unity. We find sufficient conditions under which equilibrium on the Pareto frontier of the equilibrium set are supported by efficient payoffs. We also provide a partial characterization of payoffs on the frontier in terms of the action profiles that support them. These inner and outer monotone approximations are found by looking for boundary points of the relevant sets and then connecting these to form convex sets. Working with eight boundary points gives us estimates that are coarse but still capture the comparative statics of the equilibrium set with respect to the discount factor and the other parameters. By increasing the number of boundary points from eight to twelve, we obtain very precise estimates of the equilibrium set. With this tightly approximated equilibrium set, the properties of its inner approximation provide good indications of the properties of the equilibrium set itself. We find a very specific shape of the equilibrium set and see that payoffs on the Pareto frontier of the equilibrium set are supported by current actions of full favors. This is true so long as there is room for full favors, that is, away from the two ends of the frontier.

In two person zero sum game if a player applies this rules, then he will always get profit. Let X and Y are two player and they are playing 2 person zero sum game. They used to play cards which play by playing card every playing card bundle has total 52 cards. And every card no. has total 4 types. Two person play this game. One person mix all card then another person cut some cards. And a single number arise from the card. Then first person distribute the left card. After cutting a single card is open but three card left under the bundle. Then a person which get same type of card firstly he is a winner. They bet ru.100. X cut the cards and Y mix the cards. If X has to win always then he will repeat his principal amount in 2n-1*p. ratio. Where N=2n-1*p. Where N= number of opportunities, (N=1,2,3,4,5,……...) n = natural number (n=1,2,3,4,5,…………) p= principal amount, if principal amount is 100. The following chart is given below:

V. CONCLUSION

Table 1.

N =	$N=2^{n-1}*p$.	If loss	Finally profit
1.	$2^{1-1}*100=$ 100	100	100
2.	$2^{2-1}*100=$ 200	100+200 =300	400-300 =100
3.	$2^{3-1}*100=$ 400	300+400 =700	800-700 =100
4.	$2^{4-1}*100=$ 800	700+800 =1500	1600-1500 =100
5.	$2^{5-1}*100=$ 1600	1500+1600 =3100	3200-3100 =100
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With the help of the mathematical formula a player can increase the probability of winning, but it required a lot of money, passion, continuity, and patience. And a player can always get same profit of their principal amount. In a two person model, the meaning of exchanging favors is very clear. With more than two players, when in a position to do a favor, it is not so clear whom a player will provide a favor to. This will require careful modelling with respect to the values of favors from different opponents and the cost of doing favors to different opponents. If we assume that the benefit and cost are identical for all players, we will still have to incorporate in the strategies some rules on how favors are done. For example, a player might do one favor for each opponent before doing any second favors. With an appropriate generalization to the n-player case, it is reasonable to still expect the comparative statics results for the equilibrium set that we see in the current model. It is harder to say what the equilibrium strategies for the Pareto frontier will look like.

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