# Usefulness of P-∆ Analysis of High Rise Buildings in Seismic Zones With Shear Walls at Different Locations

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Abstract- The demand of high rise structure is increasing day by day to accommodate growing population. In view of popularity & less availability of land, tall structure is only solution for overcoming the problems. A tall structure should be designed to resist the lateral load like earthquake force within the permissible limit set by standards. Loads are mainly two types that are gravity loads & lateral loads like earthquake load. Earthquake forces are further two types, Static forces & dynamic forces. It would be linear & non linear also. Linear static analysis can be performed for low rise structure and low earthquake zones only. For tall structure it is necessary to consider nonlinearity, which is generally observed in geometry & materials. Then our study is based on" P- $\Delta$  analysis which incorporates geometric nonlinearity in the analysis. The study will perform structural software E-TABS. The present study seismic analysis of a multi-storeyed with R C building with & without P- $\Delta$  effects is analysed by using E-TABS structurally analysis software. The seismic zone factors 0.16 consider is fall under zone III. From the analysis, the displacement respects to earthquake loads are low when compared with earthquake load with p- $\Delta$  effect.  $P-\Delta$  effect is secondary effect on structure. It is known as geometry nonlinear effect. In this study the P- $\Delta$  effect on high rise building is studied. Linear static analyses (without P- $\Delta$ effect) on high rise building for the analysis of RCC framed are modelled. Earthquake load is applied on model of structure as per IS-1893(2002) for zone-III for E-TABS software. Load combination for analysis is said as per IS-456(2000). Then all analysis is carried out in software E-TABS, story displacement with shear wall and without shear wall P- $\Delta$  effect is calculated. P- $\Delta$  effect found to decrease using shear walls.

*Keywords*- Displacement, Lateral loads, Linear static,  $P-\Delta$  effect, Shear wall, High Rise buildings.

### I. INTRODUCTION

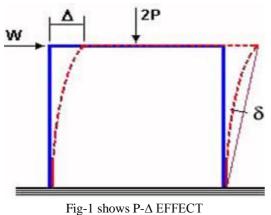
High rise buildings are the most structures that require stability because it consists a lot of frame structure with different width and height. Generally, structural designers are prone to use linear static analysis, which is also known as first order analysis, to compute design forces, moments and displacements resulting from loads acting on the structure. First order analysis is performed by assuming small deflection behaviour, where the resulting forces, moments and displacements take no account of the additional effect due to deformation of the structure under vertical loads prior to imposing lateral loads. In the traditional first order analysis of structures, the effects of change in the structure actions due to structure deformations are neglected. However, when a structure deforms, the applied loads may cause additional actions in the structure that are called second order or p-delta effects. In the case of first order elastic analysis, the deformations and internal forces are proportional to the applied loads. However, in some cases, the deflection of the structure can have a geometric second order effect on the behaviour of the structure, which is not evaluated by the linear first order analysis. This type of geometric non-linearity can be analysed by performing through iterative process which is only practicable by using computer programs. It is generally known as second order analysis. In this type of analysis, the deformations and internal forces are not proportional to the applied loads.

#### A. P-delta effect:

P-Delta is a non-linear second order effect that occurs in every structure where elements are subject to axial loads. It is associated with the magnitude of applied axial load (P) and displacement ( $\Delta$ ). If a P-Delta affected member is subjected to lateral load then it will be prone to more deflection which could be computed by P-Delta analysis not the linear static analysis. The effect of P-Delta is mainly dependent on the applied load and building characteristics. In addition to this it also depends upon the height, stiffness, axial load and asymmetry of the building. The building asymmetry may be unbalanced mass, stiffness, in plane. P-delta effects can be reduce and control by using heavier members and/or stiffer frames. P-Delta effect mainly depends on:

- Axial load on the structure.
- Stiffness of the structure.

- Slenderness ratio of the structure.
- Structure asymmetry.



### **II. LITERATURE REVIEW**

Recently many researchers had been analyzing pdelta effect in seismic analysis of multi story building. Manik Rao, Rajendrakumar S Harsoor (2016) investigated the effect of P-Delta on multi storey buildings: 5, 10, 15 and 20 storey and analyzed using ETABS v. 13. 1software. The non-linear static analysis is performed to account for the P-Delta effect on the four types of building models and is compared with linear static analysis using ETABS v. 13. 1. The variation in the axial forces, storey shears, displacements and bending moments with and without consideration of P-Delta effect is compared. T.J. Sullivan, T.H. Pham (2008), the design of a 45-storey reinforced concrete frame-wall case study structure is used to highlight the significance of the p-delta limit within the modal response spectrum analysis procedure of the Euro code 8. It is found that the strength of the structure is dictated by the P-delta limit for seismic actions, despite anticipated storey drifts and ductility demands being relatively low. A series of non-linear time-history analyses using a suite of spectrum-compatible real and artificial accelerograms indicate that P-delta effects do not have a significant influence on displacements or storey drifts of the tall building. Vijayalakshmi R, Bindu N Byadgi, and Vahini M (2017) analyzed effects of P-Delta on high rise buildings located in seismic zones. They studied effect of lateral load on the structural system for the P-Delta effect. The drift ratio is found for both, earthquake and wind loading, considering with and without P-Delta effect for different number of stories such as G+10, G+20, G+30 and G+40 stories. The load deflection curves and drift ratios have been obtained for different cases and results so obtained have been compared to identify the drift ratios for different stories of the structure. Nikunj Mangukiya, arpit ravani, Yash Miyani and Mehul Bhavsar (2016) studied on P-Delta analysis for reinforced concrete

structure. For tall structure it is necessary to consider nonlinearity, which is generally observed in geometry & materials. Our study is based on "P-Delta" analysis which incorporates geometric nonlinearity in the analysis. The study will be performed on structural software ETABS. Saranya S. Pillai and Namitha Chandran (2015) analyzed effectiveness of P-Delta analysis in the design of tall slender reinforced concrete structures. They studied on the effectiveness of pdelta analysis in the design of tall slender reinforced concrete structures. A study on the stability of tall structures to lateral forces with and without considering p-delta effects is carried out in the present investigation The building models with different storey heights have been analysed to investigate the maximum response in the building in terms of displacement, moment and shear forces. B.J. Davidson, R.C. Fenwick and B.T. Chung (1992) investigated P-Delta effects in multi-storey design. They analyzed on strength required to prevent an increase in ductility demand when p-delta effects are included. That is determined from the performance of a single degree of freedom oscillator. Non-linear numerical integration time history analyses of a series of ductile frame structures indicate that the approach forms the basis of a practical method of design for p-delta effects. Neelapu Ramesh and Shaik Yajdani (2017) Analyzed Effect of P-Delta on Multi-story R.C. Building without and with Shear Wall. They studied on the P-Delta effect on high rise building and the change in P-Delta effect by including shear wall in building. Earthquake load is applied on structure as per IS-18939(2002) for zone V of medium soil condition. Load combinations for analysis is set as per IS-456(2000).All analysis is carried out in software ETABS. Bending moment, story displacement with and without P-Delta effect is calculated and compared for all models.

#### **III. MODEL CONFIGURATION**

The general layout of the building is shown in fig-2 and modelled using E-TABS software. Preliminary sizes of structural components are assumed by experience and general information of building as shown in table-1. In this present analysis shear walls are located at different positions such as mid position of all sides, four corner and combination of both. Shear walls are assumed to be fixed at their base. The 3D views of buildings are shown in fig-3a, 3b, 3c and 3d.

Table-1: Details for Building Models

Table-1: Details for Dunding Wodels				
Details for Building Mo	dels			
Particular	Details			
Plan size	30m X 30m			
Number of Floors	51			
Storey height	153m			
Type of Soil	Medium (II)			
Grade of Concrete	M30			
Grade of Steel	HYSD 415			
Column size	300mm X 600mm			
Beam size	300mm X 600mm			
Slab Thickness	150mm			
Shear Wall thickness	150mm			
Zone	ш			
Region	Bhubaneswar			
Live Load	3KN/sqm			
Roof Load	1.5KN/sqm			
Earthquake Load	As per IS 1893(part-I)-			
	2002			
Response Reduction	5			
Factor				
Zone Factor	0.16			
Building Factor	1			
Damping coefficient	0.05			

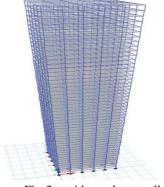


Fig-3a: without shear wall.

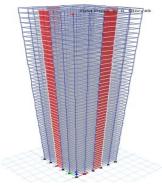


Fig-3b: shear wall at mid-sides.

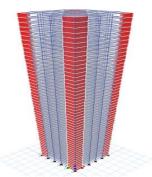


Fig-3c: shear wall at corner.

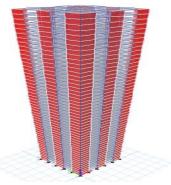


Fig-3d: shear wall at both positions.

# **IV. LOAD CALCULATION**

# **GRAVITY LOAD**:

The loads considered for the following study are as below which are according to IS codes.

- 1. Dead load: The self-weight of the structural members is calculate according to the code provisions and is taken care in the software.
- Live load on floor: 3kN/m2 (Table 1 of IS 875(Part-2):1987)
- Live load on roof: 1.5kn/m2(Table 1 of IS 875(Part-2):1987)



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# EARTHQUAKE LOAD:

As per IS-1893-2002, seismic analysis of the structure is performed. The design horizontal seismic coefficient, Ah for the structure has been computed using the following:

- 1. Zone factor, Z = 0.16 (Zone III)
- 2. Importance factor I = 1.0
- 3. Response Reduction factor, R = 5
- 4. Soil type = Medium Soil
- 5. Damping Coefficient = 0.05

#### LOAD COMBINATION:

As per IS-1893(PART-I):2002,Load combination are considered.

#### V. RESUTS AND DISCUSSIONS

Multi-story buildings are modelled and both linear and nonlinear P-Delta analyzed using E-TABS 16. Lateral displacement parameter analyzed with providing shear wall at various locations such as mid position of all sides, corner and combinations of both. Earthquake load along with dead and live load is considered in analysis.

**Validation of the results** of the present investigation has been made with that of the results given in Earthquake resistant design of structures by S.K. Duggal [1]. The properties of building are as follows.

A 10-storey OMRF building has plan dimensions as shown in fig-4.

Storey height -3.0m Dead Load per unit area-4 KN/m<sup>2</sup> Weight of partition-2 KN/m<sup>2</sup> Live Load floor-3 KN/m<sup>2</sup> Live Load at roof-1.5 KN/m<sup>2</sup> Soil Type –Hard Soil (Delhi)

Numerical results obtained from software and referred results are plotted in table-2. The present results show good agreement with that previous study.

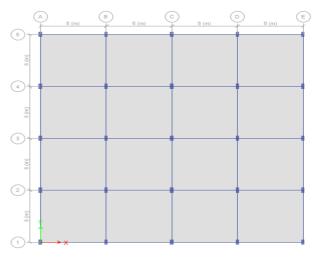


Fig-4: plan dimensions.

	Table	-2	
Storey	Seismic Forces (KN) at different floor le		
	S.K. Duggal [1]	Present	
10	402.0	397.7887	
9	484.6	484.8729	
8	382.9	383.1145	
7	293.3	293.322	
6	215.5	515.5019	
5	149.5	149.6541	
4	95.7	95.7786	
3	54.0	53.8755	
2	24.0	23.9447	
1	6.0	5.9862	

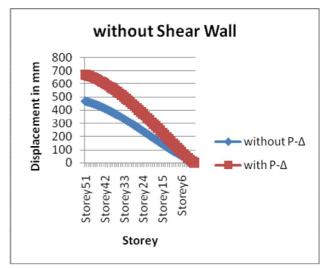
#### B. Displacements without shear wall

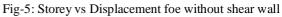
Lateral displacements at each storey level are calculated by linear as well as non-linear analysis due to earthquake load and other building loads. Displacement of without shear wall model is seen as shown in table-3. Results are compared and given in fig-5 below.

Table-3				
		without P-		
Storey	Elevation	Δ	with $\mathbf{P}$ - $\Delta$	
Storey51	153	466.487	670.978	
Storey50	150	462.224	665.356	
Storey49	147	457.528	659.257	
Storey48	144	452.386	652.655	
Storey47	141	446.817	645.56	
Storey46	138	440.838	637.981	
Storey45	135	434.465	629.929	
Storey44	132	427.718	621.414	
Storey43	129	420.612	612.445	
Storey42	126	413.166	603.035	

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Storey41	123	405.396	593.194
Storey40	120	397.319	582.934
Storey39	117	388.954	572.267
Storey38	114	380.317	561.204
Storey37	111	371.424	549.758
Storey36	108	362.292	537.942
Storey35	105	352.939	525.768
Storey34	102	343.38	513.25
Storey33	99	333.632	500.401
Storey32	96	323.71	487.234
Storey31	93	313.632	473.764
Storey30	90	303.411	460.005
Storey29	87	293.064	445.971
Storey28	84	282.605	431.676
Storey27	81	272.051	417.136
Storey26	78	261.414	402.367
Storey25	75	250.711	387.383
Storey24	72	239.955	372.201
Storey23	69	229.16	356.836
Storey22	66	218.339	341.306
Storey21	63	207.507	325.626
Storey20	60	196.677	309.815
Storey19	57	185.86	293.889
Storey18	54	175.071	277.866
Storey17	51	164.32	261.765
Storey16	48	153.622	245.605
Storey15	45	142.986	229.404
Storey14	42	132.426	213.182
Storey13	39	121.951	196.959
Storey12	36	111.575	180.756
Storey11	33	101.306	164.594
Storey10	30	91.156	148.494
Storey9	27	81.136	132.479
Storey8	24	71.255	116.572
Storey7	21	61.524	100.797
Storey6	18	51.953	85.178
Storey5	15	42.55	69.742
Storey4	12	33.326	54.515
Storey3	9	24.291	39.525
Storey2	6	15.456	24.82
Storey1	3	6.881	10.647
Base	0	0	0



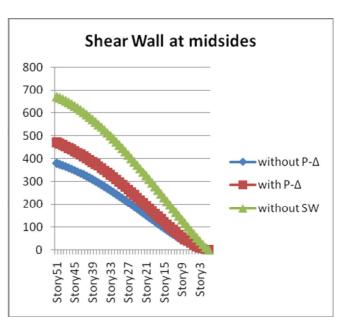


### C. Displacements with shear wall at mid-sides

Series of analyses carried out using ETABS and results obtained are shown in table-3 and fig-5. It was observed that values of lateral displacements are large due to p-delta effects, but suitable as compared to without shear wall.

Table-4				
Story	Elev ation	without P-∆	with P-∧	without SW
Story51	153	382.138	470.61	670.978
Story50	150	377.112	464.71	665.356
Story49	147	371.975	458.69	659.257
Story48	144	366.695	452.5	652.655
Story47	141	361.239	446.12	645.56
Story46	138	355.58	439.5	637.981
Story45	135	349.703	432.62	629.929
Story44	132	343.597	425.48	621.414
Story43	129	337.255	418.06	612.445
Story42	126	330.675	410.35	603.035
Story41	123	323.859	402.36	593.194
Story40	120	316.811	394.07	582.934
Story39	117	309.535	385.5	572.267
Story38	114	302.04	376.65	561.204
Story37	111	294.335	367.53	549.758
Story36	108	286.429	358.14	537.942
Story35	105	278.333	348.49	525.768
Story34	102	270.058	338.59	513.25
Story33	99	261.617	328.46	500.401

Story32	96	253.022	318.11	487.234
Story31	93	244.286	307.54	473.764
Story30	90	235.42	296.78	460.005
Story29	87	226.44	285.82	445.971
Story28	84	217.357	274.7	431.676
Story27	81	208.185	263.43	417.136
Story26	78	198.938	252.01	402.367
Story25	75	189.629	240.46	387.383
Story24	72	180.273	228.82	372.201
Story23	69	170.883	217.08	356.836
Story22	66	161.474	205.27	341.306
Story21	63	152.06	193.41	325.626
Story20	60	142.657	181.52	309.815
Story19	57	133.28	169.62	293.889
Story18	54	123.946	157.74	277.866
Story17	51	114.673	145.9	261.765
Story16	48	105.48	134.13	245.605
Story15	45	96.386	122.47	229.404
Story14	42	87.415	110.94	213.182
Story13	39	78.592	99.596	196.959
Story12	36	69.945	88.467	180.756
Story11	33	61.505	77.607	164.594
Story10	30	53.311	67.073	148.494
Story9	27	45.405	56.926	132.479
Story8	24	37.839	47.241	116.572
Story7	21	30.671	38.101	100.797
Story6	18	23.975	29.603	85.178
Story5	15	17.835	21.862	69.742
Story4	12	12.355	15.009	54.515
Story3	9	7.661	9.197	39.525
Story2	6	3.904	4.608	24.82
Story1	3	1.267	1.451	10.647
Base	0	0	0	0





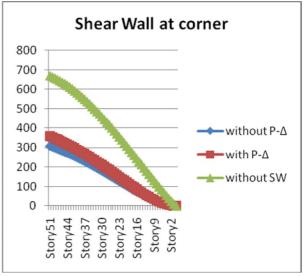
### D. Displacement with shear wall at corner

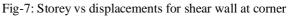
Shear walls are placed at four corner positions as shown in fig-3c. Lateral displacements are less as compared to placing shear wall at mid-sides. The results are shown in table-5 as follow.

Table-5				
storey	Elev ation	without P-∆	with P- $\Delta$	without shear wall
Story51	153	306.863	360.172	670.978
Story50	150	301.899	354.488	665.356
Story49	147	296.874	348.737	659.257
Story48	144	291.776	342.903	652.655
Story47	141	286.584	336.963	645.56
Story46	138	281.284	330.898	637.981
Story45	135	275.863	324.693	629.929
Story44	132	270.312	318.336	621.414
Story43	129	264.624	311.819	612.445
Story42	126	258.794	305.135	603.035
Story41	123	252.821	298.278	593.194
Story40	120	246.704	291.246	582.934
Story39	117	240.442	284.04	572.267
Story38	114	234.038	276.658	561.204
Story37	111	227.497	269.105	549.758
Story36	108	220.822	261.384	537.942

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Story35	105	214.019	253.501	525.768
Story34	102	207.095	245.46	513.25
Story33	99	200.057	237.272	500.401
Story32	96	192.915	228.943	487.234
Story31	93	185.676	220.485	473.764
Story30	90	178.351	211.907	460.005
Story29	87	170.951	203.221	445.971
Story28	84	163.486	194.442	431.676
Story27	81	155.969	185.581	417.136
Story26	78	148.411	176.654	402.367
Story25	75	140.826	167.677	387.383
Story24	72	133.227	158.666	372.201
Story23	69	125.629	149.639	356.836
Story22	66	118.046	140.616	341.306
Story21	63	110.495	131.615	325.626
Story20	60	102.991	122.657	309.815
Story19	57	95.552	113.766	293.889
Story18	54	88.196	104.965	277.866
Story17	51	80.942	96.278	261.765
Story16	48	73.81	87.732	245.605
Story15	45	66.822	79.356	229.404
Story14	42	60.001	71.178	213.182
Story13	39	53.371	63.231	196.959
Story12	36	46.958	55.548	180.756
Story11	33	40.79	48.165	164.594
Story10	30	34.897	41.121	148.494
Story9	27	29.311	34.455	132.479
Story8	24	24.068	28.211	116.572
Story7	21	19.206	22.438	100.797
Story6	18	14.766	17.183	85.178
Story5	15	10.796	12.502	69.742
Story4	12	7.344	8.454	54.515
Story3	9	4.468	5.103	39.525
Story2	6	2.232	2.519	24.82
Story1	3	0.708	0.782	10.647
Base	0	0	0	0





### E. Displacement with combination of both positions

Here shear walls are provided both in corner positions as well as mid-sides and results are obtained. Lateral displacements obtained are good as compared to others may be dueto increase in lateral stiffness.

Table-6				
		without	with	without
Storey	Elevation	<b>P-</b> ∆	<b>P-</b> ∆	SW
Story51	153	170.125	185.72	670.978
Story50	150	167.588	182.99	665.356
Story49	147	165.011	180.22	659.257
Story48	144	162.386	177.41	652.655
Story47	141	159.7	174.52	645.56
Story46	138	156.945	171.57	637.981
Story45	135	154.111	168.52	629.929
Story44	132	151.195	165.39	621.414
Story43	129	148.192	162.17	612.445
Story42	126	145.1	158.84	603.035
Story41	123	141.917	155.42	593.194
Story40	120	138.642	151.89	582.934
Story39	117	135.277	148.27	572.267
Story38	114	131.823	144.54	561.204
Story37	111	128.282	140.72	549.758
Story36	108	124.657	136.8	537.942
Story35	105	120.952	132.8	525.768
Story34	102	117.17	128.7	513.25
Story33	99	113.315	124.52	500.401
Story32	96	109.394	120.26	487.234

Story31	93	105.41	115.92	473.764
Story30	90	101.37	111.52	460.005
Story29	87	97.279	107.06	445.971
Story28	84	93.144	102.54	431.676
Story27	81	88.971	97.978	417.136
Story26	78	84.767	93.374	402.367
Story25	75	80.539	88.737	387.383
Story24	72	76.296	84.077	372.201
Story23	69	72.043	79.402	356.836
Story22	66	67.791	74.721	341.306
Story21	63	63.547	70.045	325.626
Story20	60	59.32	65.383	309.815
Story19	57	55.12	60.747	293.889
Story18	54	50.957	56.148	277.866
Story17	51	46.842	51.599	261.765
Story16	48	42.785	47.113	245.605
Story15	45	38.8	42.704	229.404
Story14	42	34.899	38.388	213.182
Story13	39	31.096	34.18	196.959
Story12	36	27.406	30.099	180.756
Story11	33	23.846	26.163	164.594
Story10	30	20.434	22.393	148.494
Story9	27	17.188	18.811	132.479
Story8	24	14.132	15.441	116.572
Story7	21	11.288	12.311	100.797
Story6	18	8.683	9.449	85.178
Story5	15	6.346	6.887	69.742
Story4	12	4.309	4.662	54.515
Story3	9	2.61	2.811	39.525
Story2	б	1.29	1.381	24.82
Story1	3	0.398	0.422	10.647
Base	0	0	0	0

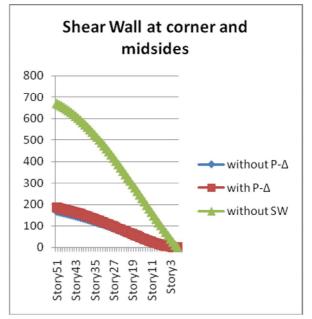


Fig-8: Storey vs displacements for shear wall at both corner as well as mid-sides

# **VI. CONCLUSIONS**

- The P-∆ effects found to decrease the storey displacement by earthquake using shear walls.
- Displacement with respect to earthquake load with  $P-\Delta$  effects are higher compared with earthquake load.
- As number of storey increases,  $P-\Delta$  effect becomes more important when shear wall provided.
- So, we should perform  $P-\Delta$  analysis for designing a 51 storey building with shear wall.
- The conclusion is valid for RCC building for seismic loading in all zones for India.

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