Effects of Arbitrarily Located Circular Cutouts on Vibration of Laminated Composite Plates

Himanshu S. Panda¹, Phatechand Naik², Pravat K. Parhi³

¹Associate Professor ³Professor, DEPT OF Civil Engineering ¹School of Architecture and Planning KIIT Deemed to be University, Bhubaneswar-24, Odisha, India ^{2, 3}College of Engineering and Technology, Bhubaneswar, Odisha-753001, India

Abstract- The present study deals with the effect of arbitrarily located circular cuts out on natural frequency of laminated composite plates. The free vibration analyses were done by finite element analysis software ANSYS15.0.A composite plate modeled using 8-noded shell element shell281, which implement first-order shear-deformation theory (FSDT) in analysis. Here various analyses with different boundary conditions, layer thickness, ply orientation, materials were solved. Validation of obtained result was done by comparing with those available previous researches. Present investigation showed good results for various cutouts except at center position.

Keywords- Free vibration, Laminated composite plate, Ply orientation, Stacking layers

I. INTRODUCTION

Presently, we are in era of composites after stone, wood, steel ages. Laminated composite plates are became major construction materials in mechanical, aerospace, automotive, marine, civil engineering structures because many advantage they offer like: high strength and stiffness for lower weight, fatigue strength. Orthotropic composite plates with internal holes are being widely used in different structural elements not only to access ports for mechanical and electrical systems, but also to reduce weight, to maintain availability of air, to transfer heat appropriately. The free vibration characteristics of composite curved panels are affected due to variability in shape, size and location of cut outs.

II. LITERATURE REVIEW

Recently many researchers had been analyzing free vibration study of laminated composite plates with cutouts. S.Dey et al (2016) studied on effect of cutout on stochastic natural frequency of composite curved panels [2]. Anand Venkatchari et al (2016) examined environmental effects on the free vibration of curvilinear fiber composite laminates with cutouts [3]. H.J Lin et al (1995) investigated the failure analysis of woven fabric composites with modeled in holes [4]. Sang Youl Lee (2010) examined dynamic stability of skew plates with cutouts by finine element approach based on HSDT [5]. K.S. Sairam and T. sreedhar Babu (2002) studied on free vibration of composite spherical shell cap with and without a cutout [6]. Mutra raja Sekhara reddy et al (2012) performed prediction of natural frequency of laminated composite plates using artificial neural networks technique [7]. N. fallah and M. Delzendeh (2018) studied on natural vibration analysis of laminated composite plates based in mesh less finite volume method [8]. Chai Gin Boay (1996), analyzed in free vibration of laminated composite plates with circular hole placed at center position [9]. A.L.Poore et al (2008) investigated in free vibration of laminated cylindrical shells with circular cutouts [10]. Sundararajan Natarajan et al (2014) studied hygrothermal effects on the free vibration and buckling for laminated composites with cut outs [11]. A.R.Abu Talib et al (2013) examined influence of cut out hole on multilayer kelar-29/epoxy composite laminated plates [12]. Sarmila sahoo (2014) investigated free vibration of laminated composite stiffened shallow spherical panels with cutouts and how it affects [13]. Liz G Nallim et al (2008) studied on natural frequency of symmetrically laminated elliptical and circular composite plates [14]. Vijay and K.Singh et al (2014) checked usefulness of non linear natural frequency analysis for single/doubly curved composite shallow shell panels [15]. Changshi Xu and Chuen Yuan chia (1995) studied effects of cutouts in non linear vibration and buckling behaviour of laminated shallow spherical shells [16]. Tiantang Yu et al (2016) examined the buckling and free vibration problems for laminated composite plates with complicated cutouts [17]. Sumit khare and N.D.Mittal (2017) studied three dimensional free vibration analyses of thick laminated composite circular plates with simply supported boundary conditions [18]. P.V.Joshi and N.K.Jain et al (2016) investigated effects of temperature on vibration and buckling analysis of partially cracked thin orthotropic rectangular plates [19]. H.S.Panda et al (2014) examined effects of moisture on the frequencies of vibration of woven fiber composite doubly curved panels with strip delaminating [20]. Bin Huang and Ji wang et al (2016) carried out research on vibration analysis of composite laminates with rectangular cutout by implementing

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independent coordinate coupling method [21]. Harsh Kumar Bhardwaj et al (2015) studie on free vibration analysis of laminated composite plates with triangular cutouts [22].

Previous researchers worked on effect of different cutouts like square, rectangle, elliptical, circular on free vibration of laminated composite plates. Free vibration characteristics of laminated composite plates are affected due to variability in shape, size and location of cutouts. In the present work, authors conducted analyses on effects of circular cutout at various locations and optimizes by implementing different stacking layers and ply orientation, which is not available in open literature.

III. FINITE ELEMENT MODELING

In the present work, the free vibration behavior of laminated composite plate has been investigated using finite element analysis software ANSYS. A composite plate model having length a, width b and total thickness h used and the geometry of two-dimensional laminated composite plates with positive set of coordinate axis are shown in Fig. 1.



Fig. 1 Geometry of laminated composite plate with positive set of coordinate axis

There are many types element available in software to model laminated composite materials. In our FE analysis, the linear layered structural shell element shell 281 is used. It is an eight nodded isoperimetric element with six degree of freedom at each node as shown in fig 2.



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Fig.2 shows the element in natural coordinate system

The shape functions for eight nodded shell element (j=8) are represented in natural $(\xi - \eta)$ coordinates and details of the element are given as follows:

The element geometry and displacement field are expressed by the shape functions $N_{\mathrm{i}}. \label{eq:nonlinear}$

$$x = \sum_{i=1}^{8} N_{i}x_{i}, \quad y = \sum_{i=1}^{8} N_{i}y_{i}, \quad u = \sum_{i=1}^{8} N_{i}u_{i},$$
$$v = \sum_{i=1}^{8} N_{i}v_{i}, \quad w = \sum_{i=1}^{8} N_{i}w_{i}, \quad \theta_{x} = \sum_{i=1}^{8} N_{i}\theta xi$$
$$\theta_{y} = \sum_{i=1}^{8} N_{i}\theta yi, \quad \theta_{z} = \sum_{i=1}^{8} N_{i}\theta zi$$
(1)

The shape functions N_i for different nodes as shown in fig.2 are defined as follows:

At corner nodes (i.e. for nodes 1, 2, 3, 4)

$$N_{i} = \frac{1}{4} (1 + \xi \xi_{i}) (1 + \eta \eta_{i}) (\xi \xi_{i} + \eta \eta_{i} - 1)$$
(2.a)

At middle nodes (i.e. for nodes 5, 7)

$$N_i = \frac{1}{2} (1 - \xi^2) (1 + \eta \eta_i)$$
(2.b)

At middle nodes (i.e. for nodes 6, 8)

$$N_i = \frac{1}{2} (1 + \xi \xi_i) (1 - \eta^2)$$
(2.c)

Where ξ and η are the local natural coordinates of ξ

the element and ξ_i and η_i are the respective values at node i. The derivatives of the shape functions N_i with respect to x and y is expressed in terms of their partial derivatives with respect to ξ and η by the relationships:

$$\frac{\partial N_i}{\partial x} = \frac{\partial N_i}{\partial \xi} \frac{\partial \xi}{\partial x} + \frac{\partial N_i}{\partial \eta} \frac{\partial \eta}{\partial x}$$
(3)

$$\frac{\partial N_i}{\partial y} = \frac{\partial N_i}{\partial \xi} \frac{\partial \xi}{\partial y} + \frac{\partial N_i}{\partial \eta} \frac{\partial \eta}{\partial y}$$
(4)

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$$\begin{cases}
\frac{\partial N_{i}}{\partial x} \\
\frac{\partial N_{i}}{\partial y}
\end{cases} = \begin{bmatrix}
\frac{\partial \xi}{\partial x} & \frac{\partial \eta}{\partial x} \\
\frac{\partial \xi}{\partial y} & \frac{\partial \eta}{\partial y}
\end{bmatrix}
\begin{cases}
\frac{\partial N_{i}}{\partial \xi} \\
\frac{\partial N_{i}}{\partial \eta}
\end{cases}$$
(5)

$$\begin{bmatrix} \mathbf{N}_{i,x} \\ \mathbf{N}_{i,y} \end{bmatrix} = \begin{bmatrix} \mathbf{J} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{N}_{i,\xi} \\ \mathbf{N}_{i,\eta} \end{bmatrix}$$
(6)

 $\begin{bmatrix} J \end{bmatrix} = \begin{bmatrix} x_{,\xi} & y_{,\xi} \\ x_{,\eta} & y_{,\eta} \end{bmatrix}$ (7)

[J] matrix is the Jacobian matrix.

The plates with initial stresses undergo small lateral deformations. It is well known that mid-plane kinematics of laminated composite has been considered as the first order shear deformation theory as implemented in ANSYS and conceded as follows:

$$u(x, y, z) = u^{0}(x, y) + z\theta_{x}(x, y)$$
$$v(x, y, z) = v^{0}(x, y) + z\theta_{y}(x, y)$$
$$w(x, y, z) = w^{0}(x, y)$$
(8)

Where, u, v, w are displacements in the X, Y, Z directions respectively for any point and the subscript (⁰) corresponds to the mid-plane values of the composite plate. ρ ρ

 θ_x , θ_y are the rotations of the cross section normal to the Y and X axes. The middle plane of the composite plate is considered as the reference plane of the panel.

Strains are obtained as derivatives of displacements as

$$\{\varepsilon\} = \{u_{,x} \ v_{,y} \ w_{,z} \ u_{,y} + v_{,x} \ v_{,z} + w_{,y} \ w_{,x} + u_{,z}\}_{\mathrm{T}}$$
(9)

Where,
$$\{\varepsilon\} = \{\varepsilon_x \ \varepsilon_y \ \varepsilon_z \ \gamma_{xy} \ \gamma_{yz} \ \gamma_{xz} \}_{T}$$
 is the normal and shear strain components of in plane and out of plane derivatives.

Assuming small deformations, the generalized linear in plane and out of plane strains of the laminate at a distance 'z' from the mid plane are expressed as

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$$\{ \varepsilon_{x}, \varepsilon_{y}, \varepsilon_{z}, \gamma_{xy}, \gamma_{yz}, \gamma_{zx} \}^{T} = \{ \varepsilon^{0}_{x}, \varepsilon^{0}_{y}, \varepsilon^{0}_{z}, \gamma^{0}_{xy}, \gamma^{0}_{yz}, \gamma^{0}_{zx} \}^{T} + z \{ k_{x}, k_{y}, k_{z}, k_{xy}, k_{yz}, k_{zx} \}^{T}$$

$$(10)$$

Where the deformation components are described as:

$$\begin{cases} \varepsilon^{0}{}_{x} \\ \varepsilon^{0}{}_{y} \\ \varepsilon^{0}{}_{z} \\ \gamma^{0}{}_{xy} \\ \gamma^{0}{}_{yz} \\ \gamma^{0}{}_{zx} \end{cases} = \begin{cases} \frac{\partial u^{0}}{\partial x} \\ \frac{\partial v^{0}}{\partial y} \\ \frac{\partial w^{0}}{\partial z} \\ \frac{\partial u^{0}}{\partial y} + \frac{\partial v^{0}}{\partial x} \\ \frac{\partial u^{0}}{\partial y} + \frac{\partial w}{\partial y} \\ \theta_{y} + \frac{\partial w}{\partial y} \\ \theta_{x} + \frac{\partial w}{\partial x} \end{cases}$$

And

$$\begin{cases} k_{x} \\ k_{y} \\ k_{z} \\ k_{z} \\ k_{zx} \\ k_{yz} \\ k_{zx} \end{cases} = \begin{cases} \frac{\partial \theta_{x}}{\partial x} \\ \frac{\partial \theta_{y}}{\partial y} \\ \frac{\partial \theta_{z}}{\partial z} \\ \frac{\partial \theta_{z}}{\partial z} \\ \frac{\partial \theta_{x}}{\partial y} + \frac{\partial \theta_{y}}{\partial x} \\ 0 \\ 0 \end{cases}$$

Where,
$$\varepsilon^0{}_x, \varepsilon^0{}_y, \varepsilon^0{}_z, \gamma^0{}_{xy}, \gamma^0{}_{yz}, and \gamma^0{}_{zx}$$
 are the

(11)

mid plane strains and k_x, k_y, k_{xy} are the curvatures of the laminate.

The strain vector expressed in terms of nodal displacement vector as

$$\{\varepsilon\} = [B]\{\delta\} \tag{12}$$

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Where, $\begin{bmatrix} B \end{bmatrix}$ indicates the strain displacement matrix containing interpolation functions and their derivatives and $\{\delta\}$ is the nodal displacement vector.

The generalized stress strain relation with respect to the reference plane is expressed as

$$\{\sigma\} = [D]\{\varepsilon\} \tag{13}$$

Where, $\{\sigma\}$ and $\{\varepsilon\}$ is the stress and strain vectors respectively and [D] is the matrix of stiffnesses. The element stiffness matrix [K] and mass matrix [M] can be easily defined with the help of virtual work method which, may be expressed as

$$\begin{bmatrix} K \end{bmatrix} = \int_{-1-1}^{1} \begin{bmatrix} B \end{bmatrix}^{T} \begin{bmatrix} D \end{bmatrix} \begin{bmatrix} B \end{bmatrix} J | d\xi d\eta$$
(14)
$$\begin{bmatrix} M \end{bmatrix} = \int_{-1-1}^{1} \begin{bmatrix} N \end{bmatrix}^{T} \begin{bmatrix} M \end{bmatrix} \begin{bmatrix} N \end{bmatrix} J | d\xi d\eta$$
(15)

Where, |J| is the determinant of the Jacobian matrix, [N] is the shape function matrix and [M] is the inertia matrix. The integration has been carried out using Gaussian quadrature method.

The free vibration analysis is used to determined natural frequencies by given equation:

$$\left\{\!\left[K\right]\!-\omega_{n}^{2}\left[M\right]\!\right\}\!\left\{\!\delta\right\}\!=\!0\tag{16}$$

IV. RESULTS AND DISCUSSIONS

The free vibration of square laminated composite plate analyzed using ANSYS.15. Natural frequencies of composite plates were computed for circular holes at various positions such as at center, left, bottom, near support end and corner as shown below in fig. 3-7.The size of hole taken as per relation with length by (a /D) ratio of 20. Plate without holes meshed using grid 30x30 and with hole meshed using smart sizing. The composite plates chosen are symmetric angle ply having stacking sequence $[(0/45/-45/90)_2]_s$ and all edges clamped (c-c-c-c) for all analysis except analysis in effect of boundary conditions. The dimension parameters are as follows:

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a=250 mm, b=250 mm, h=2.08 mm

The ply thickness considered as 0.13 mm. the carbon-epoxy composites are used and materials properties are:

$$E_{1} = 128GPa \qquad E_{2} = 11GPa \ G_{12} = 4.48GPa$$

$$G_{23} = 1.53GPa \ v_{12} = 0.25$$

$$\rho = 1500kg / m^{3}$$
(17)



Fig-3 shows hole at center



Fig-4 shows hole at left quarter position



Fig-5 shows hole at bottom quarter position



Fig-6 shows hole at corner

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Fig-7 shows hole near to support End.

*Result Validation-*To validate the applicability of software for free vibration analysis, finite element results are compared with the result by M. R. S Reddy et.al. [7]. the properties of the material are given in above equation 17. The plate parameters are as follows:

a=250 mm b=500 mm t=1.04 mm

The layer stacking sequence of the plate is $[0/45/45/90]_{s}$. The ply thickness considered as 0.13 mm. the plate is clamped at all edges. Numerical results which are obtained by software for the first five natural frequencies are presented in table 1. The present results show good agreement with the results of previous study.

Table 1: Natural frequency of rectangular [0/45/-45/90] s composite plates.

mode sequence	M. R. Reddy et.al.[7]	S	Present	
01	85.026		85.073	
02	133.87		133.98	
03	206.34		206.53	
04	215.51		215.80	
05	251.47		251.79	

A. Effect of boundary conditions:

The square composite plate analyzed with varying boundary conditions such as all sides clamped (C-C-C-C), all sides simply supported (S-S-S-S), cantilever- one side clamped and other sides free (C-F-F-F) and two opposite sides clamped and other two sides free (C-F-C-F) condition. Those results were plotted in table-2 and fig-8 as shown below. The clamped boundary condition shows maximum value of free vibration. For the all sides clamped condition natural frequencies value increase in 3.85%, 4.5%, 4.4%, 4.3% and 3.95% for positions- center, Left, bottom, corner and support respectively from without cutouts. For other boundary conditions frequencies value decrease in 9.7% to 12.7%, 1.54% to 2.94% and 11.22% to 13.22% for C-F-F-F, S-S-S-S and C-F-C-F condition respectively.

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Hole	Different boundary conditions				
positions	CCCC	CFFF	SSSS	CFCF	
Without	331.82	210.05	179.73	234.27	
Center	344.60	186.84	174.45	207.97	
Left	346.88	187.30	176.29	206.83	
Bottom	346.42	189.74	176.83	207.34	
Comer	346.09	188.82	176.83	207.34	
Near	344.92	183.44	176.96	203.30	
support					

Table 2



Fig-8 shows effect of different boundary conditions.

B. Effect of aspect ratios:

Natural frequencies obtained for laminated composite plates with all sides clamped (C-C-C-C) condition along with variation in aspect ratios from 0.5 to 3.The square composite plates give highest values of natural frequency as compared to rectangular plates. The results are plotted in graph as shown in fig-9 below. Frequency decreases with increase in aspect ratio. Natural frequencies value increase in 3.85% to 4.5%, 17.63% to 25% and 24.65% to 30.51% for aspect ratio 1, 2 and 3 respectively.



C. Effect of no. of laminates :

Series of analyses carried out for square composite plates with all sides clamped (C-C-C-C) along with variation in number of laminate layers- 16, 24 and 32 layers having length /thickness ratios 120, 80 and 60 respectively. Here

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natural frequencies value increase by increase in number of layers. The natural frequencies values are plotted for different length/thickness ratios as given in fig-10 below. Natural frequencies value increase in 3.85% to 4.5%, 2.63% to 3.2% and 1.88% to 2.45% for number of laminates 16, 24, 32 respectively.



Table-3: natural frequencies for various ply orientation

Ply orientation	with out	Center	Left	Bottom	Corner	Support
(0/90/90/0)2s	336.47	341.91	343.49	339.2	342.31	337.36
(90/0/90/0)2s	336.44	326.53	328.62	324.08	327.63	322.35
(0/0/90/90)2s	336.24	354.03	356.33	352.2	354.11	351.33
(90/90/0/0)2s	336.24	310.12	312.97	308.17	311.86	306.95
(0/90)8	335.15	334.77	336.77	335.6	338.84	338.84
(90/0)8	335.15	338.37	339.93	335.6	338.84	333.72
(0/45/-45/90)2s	331.82	344.6	346.88	346.42	346.09	344.92
(0/45/60/90)2s	325.11	339.97	336.04	338.38	336.05	333.37
(0/30/45/90)2s	325.61	347.25	344.64	344.39	345.91	343.16
(90/45/-45/0)2s	331.82	313.51	310.49	309.98	310.34	309.41
(0/90/45/90)2s	333.36	338.35	337.49	334.3	337.61	333.07
(0/30)8	325.17	333.57	343.21	341.69	342.36	345.15
(0/60)8	324.95	339.9	337.68	332.6	335.73	334.59
(0/45)8	321.65	336.59	338.86	337.4	338.37	338.39
(0/-45)8	321.65	327.88	334.57	334.2	331.8	335.17

D. Effect of ply orientations:

Natural frequencies obtained for square laminated composite plates with all sides clamped (C-C-C-C) condition along with variation in ply orientations. Laminates were positioned in various angles with horizontal axis of plate: cross ply and angle ply orientations with symmetric conditions. The natural frequencies values are given in table-3 and here observed highest values of frequency for (00/00/900/900) ply orientation. The natural frequencies value increase in 4.48% to 5.97% from without cutouts.



Fig-11 shows different in frequencies for three materials.

E. Effect of materials:

Natural frequencies are investigated for square laminated composite plates with different materials such as CFRP, Carbon/Epoxy and Glass/Epoxy composites used in above works. Material properties for CFRP referred from journal Reddy M.R.S et.al. [7], Glass/Epoxy composites referred from journal Shokrieh, M. M et.al. [23].The natural frequencies values are plotted in graph as shown in fig-11. Carbon/Epoxy composites show good result due to greater stiffness value.

V. CONCLUSIONS

In the present work, the ANSYS software is used to obtain the free vibration responses for laminated composites with circular hole at different positions such as at center, left, bottom, near support and corner. Results are obtained for first lowest frequency values under various conditions like different boundary conditions, aspect ratios, thickness ratios, ply orientations and materials. The following conclusions are drawn for laminated composite plates:

- The natural frequency value increases for clamped boundary condition and decreases for other CFCF, CFFF and simply supported boundary conditions from without cutouts respectively. Frequency increases for other than center position.
- The natural frequency of laminated plate decreases with increase in aspect ratios. Square composites show better results as compare to other and cutouts are suitable for any location except center.
- Similarly, the frequency values increase with increase in no of layers may be due to increase in stiffness as compared to mass.
- Symmetric cross laminate with ply orientation [0/0/90/90] gives maximum value of natural frequencies results.
- Carbon-epoxy composite material has good result as compared to other materials examined.

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