Point To Point Stability Analysis

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Abstract- *Currently the demand for energy such as electricity is growing remarkably all over the world. Particularly in countries with growing economy, the provision of sustainable power is a primary concern. The economy is becoming highly dependent on various power demanding sectors despite the energy infrastructures is still in its juvenile state. The continuous demand of power urgently seek the operation of the power system to run at its most capacity. And the need for reliable, stable and quality power is rising due to the expansion of electric power sensitive industries like the manufacturing industries, information technology, communication, electronics as well as the rise in the service and commercial sector. In these circumstances, fulfilling the electric power demand is not the only condition expected but also it is the responsibility of the service providers and indeed power system engineers to provide a stable and quality power to consumers. These issue emphasizes the importance of understanding the power system stability so as to improve the provision of the power.*

This document tries to analyze power system stability based on a classical model suggested by E.W.Kimbark and S.B.Crary. Here, the analysis of a power system is achieved by representing synchronous generators to analyze the stability of a single generator connected to an infinite bus by representing mathematically power system components .

I. INTRODUCTION

"Power system stability is the ability of an electric power system, under a given initial operating condition, to regain a state of operating equilibrium after being subjected to a physical disturbance, with most of the system variables controlled so that practically the entire system remains unharmed" [1], [2]. The disturbances mentioned in the definition could be faults, load changes, generator outages, line outages, voltage collapse or some combination of these.

Power system engineering forms a vast and major portion of electrical engineering studies. It is mainly concerned with the production of electrical power and its transmission from the sending end to the receiving end as per consumer requirements, sustaining a minimum amount of losses. The power at the consumer end often changes due to the variation of load or due to disturbances induced within the length of transmission line. For this reason, the term power system stability is of utmost importance in this field. It is used to define the ability of the of the system to bring back its operation to steady state condition within a minimum possible time after having undergone any transience or disturbance in the line.

In the power plants, several synchronous generators with different voltage ratings are connected to the bus terminals having the same frequency and phase sequence as the generators, while the consumer ends are connected directly from those bus terminals. Therefore, for a stable operation, the bus with the generators has to be synchronized over the entre duration of the transmission. For this reason the power system stability is also referred to as synchronous stability and is defined as the ability of the system to return to synchronism after having undergone some disturbance due to many reasons such as switching on and off of loads or due to line transience.

Though the classical model is not convenient for current generators having with fast acting exciters, governors and power system stabilizers, it is nevertheless useful in understanding the basic phenomenon of stability.

The following assumptions are made for the sake of easily representation of synchronous generators mathematically:-

1. The exciter dynamics are not considered and hence the generator stator induced voltage is always constant.
2. The effect of damper and rotor windings present on the rotor of the synchronous generators is neglected. Damper windings (squirrel-cage winding) helps the synchronous motor to start on its own by providing starting torque.
3. The input mechanical power to the generator is assumed to be constant during the period of study. This leads to neglecting the dynamics of turbine and turbine speed governor.
4. Much emphasis is not given for the nature of the rotor construction. The saliency of the generator is neglected

For a simple understanding of the classical model the case of a generator connected to an infinite bus through a transformer and transmission line which is called single Machine Infinite Bus(SMIB) system is taken in to consideration. The single line diagram of SMIB system is shown in the figure below where the infinite bus is to represent rest of the system or grid, where the voltage magnitude and frequency are held constant. Only reactance of synchronous generator, transformer and transmission line are considered. Because resistance of synchronous generator stator, transformer and the transmission line are relatively negligible as compared to the corresponding reactance. The infinite bus can act *like infinite source or sink*. It can also be considered as a *generator with infinite inertia and fixed voltage*.

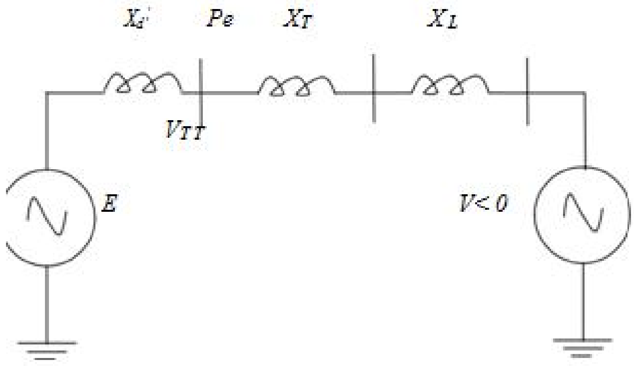

Fig.1: Single line diagram of the SMIB system

In Fig. above *E* represents the complex internal voltage of the synchronous generator behind the transient reactance X_d' . X_d' is the generator's transient reactance transient reactance. $V_{T\,T}$ is the terminal voltage of the synchronous generator. X_T and X_L represent the reactance of the transformer and the transmission line respectively. The complex infinite bus voltage is represented as $V<0$ .since the infinite bus voltage is taken as the reference ,the angle is taken as zero. The generator internal voltage angle δ is defined with respect to the infinite bus voltage angle. The input mechanical power is represented as P_m and the output electrical power is defined by Pe . H *and H∞* are the inertia constant of the generator and the grid equivalent generator connected at the infinite bus respectively. The electrical output of the synchronous generator are represented mathematically as follows:-

$$P_e = Real\left[e^{j\theta_T} * \left[\frac{\left(V_T e^{j\theta_T} - V_\infty e^{j\theta}\right)}{j(X_T + X_L)}\right]^*\right] \qquad \text{Eqn.1}$$

The maximum real power output of the synchronous generator that can be transferred to the infinite bus in this case is when sin δ=1 at an angle δ=90.

$$P_e = Real\left(V_T e^{j\theta} \times \left(\frac{\left(V_T e^{j\theta_T} - V_\infty e^{j0}\right)}{j(X_T + X_L)}\right)^*\right)$$

$$P_e = Real\left(E e^{j\delta} \times \left(\frac{\left(E_T e^{j\theta_T} - V_\infty e^{j0}\right)}{j(x_d' + X_T + X_L)}\right)^*\right)$$

$$= \frac{EV_\infty}{X_d' + X_T + X_L} sin\delta \qquad \text{Eqn.1.1.}$$

The maximum real power output of the synchronous generator that can be transferred to the infinite bus in this case is

$$P_{max} = \frac{EV_\infty}{x_d' + X_T + X_L'} \qquad \text{eqn.1.3}$$

The above discussion are all about the equivalent electrical representation of the synchronous machine. The synchronous machine also has a mechanical system which has to be modeled. The above discussion are all about the equivalent electrical representation of the synchronous machine. The synchronous machine also has a mechanical system which has to be modeled. The prime mover is responsible for the mechanical energy to the generator rotor and in turn the generator through magnetic coupling converts the mechanical energy into electrical energy. The dynamics representation of a rotational mechanical system can be shown as:-

$$J\frac{d^2\theta_m}{dt^2} = T_m - T_e \qquad \text{Eqn. 1.4}$$

- Where, *J* kg.m2 is the inertia constant of the rotating machine.
- The mechanical input torque due to the prime mover is represented as
- T_m N.m and the electrical torque, acting against the mechanical input torque, is represented by T_e N.m .

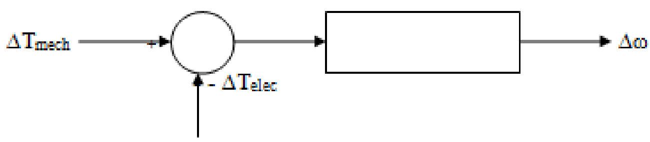

Fig.2. Relationship between mechanical and electrical power and speed change

The angle Θ_m is the mechanical angle of the rotor field axis with respect to the stator reference or fixed reference frame. Θ_m keeps varying with time as the rotor continuously rotating at synchronous speed in steady state. To make it constant in steady state Θ_m is measured with respect to a synchronously rotating reference instead of a stationary reference. Hence, we can write

$$\theta_m = \delta_m - \omega_{ms} t \qquad \text{Eqn. 1.5}$$

Where, $\delta\ m$ is the angle between the rotor field axis and the reference axis rotating synchronously at ω_{ms} rps . Differentiating the above equation with respect to time we get

$$\frac{d\theta_m}{dt} = \frac{d\delta_m}{dt} + \omega_m s$$

$$\frac{d^2\theta_m}{dt^2} = \frac{d^2\theta\delta_m}{dt^2} \qquad \text{Eqn.1.6}$$

But, the rate of change of the rotor mechanical angle Θm with respect to time is the speed of the rotor. Hence,

$$\omega_m = \frac{d\theta_m}{dt} \qquad \text{Eqn.1.7}$$

$$\frac{d\delta_m}{dt} = \omega_m - \omega_m s \qquad \text{Eqn.1.8}$$

Similarly, substituting Eqn.6 in to eqn. 4. we get

$$J\frac{d^2\delta_m}{dt^2} = T_m - T_e \qquad \text{Eqn.1.9}$$

If we multiply with ωm on both the side of Eqn.9 and noting that torque multiplied by speed gives power, Eqn.9 once again can be written as

$$J\omega_m \frac{d^2}{dt^2} = P_m - P_e \qquad \text{Eqn.1.10}$$

Now multiplying with the term 1/2 $\omega\ ms$ on both the sides and divide the entire equation with the base MVA (S_B), in order to express the equation in per unit form leads to

$$\frac{\frac{1}{2}J\omega_m\omega_{ms}}{S_B}\cdot\frac{d^2\delta_m}{dt^2} = \frac{1}{2}\omega_{ms}\left[\frac{P_m}{S_B} - \frac{P_e}{S_B}\right] \qquad \text{Eqn.1.11}$$

For a new parameter named machine inertia constant H is given by

$$H = \frac{\frac{1}{2}J\omega_m\omega_{ms}}{S_B}\cdot\frac{d^2\delta_m}{dt^2} = \frac{\frac{1}{2}J\omega_{ms}^2}{S_B}\cdot\frac{m_{ws}}{NVA}$$

Eqn.1.12

Suppose $\omega_m \approx \omega_{ms}$ as the variation of the speed, even during transients, from synchronous speed is quite less. This means $1/2\ J\omega^2 \approx 1/2\ J\omega_m\omega_{ms}$

$$H\frac{d^2\delta_m}{dt^2} = \omega_{ms}(P_m - P_e)_{\text{perunit}} \qquad \text{Eqn.1.13}$$

where δ_m and ω_{ms} are expressed in mechanical radians and mechanical radians per second. They can be converted in to electrical radians and electrical radians per second by considering the number of poles of the synchronous machine rotor. The electrical angle and electrical speed can be represented as

$$\delta = \frac{p}{2}\delta_m\ electric\ rad$$

$$\omega_s = \frac{p}{2}\omega_{ms} electric.rad \qquad \text{Eqn.14}$$

Substituting the expressions in Eqn.14 in to Eqn.13 and rearranging we get

$$\frac{d^2\delta}{dt^2} = \frac{\omega_s}{2H}(P_m - P_{max}sin\delta)\ perunit$$

$$\frac{d^2\delta}{dt^2} = \frac{\pi f_s}{H}(P_m - P_{max}sin\delta)\ \ per\ unit(at\ f = 50\ or\ 60\ Hz)$$

Eqn.1.15

Assuming all the parameters of Eqn.15 expressed in per units, can also be written as

$$\frac{d\delta}{dt} = (\omega - \omega_s)$$

$$\frac{d\omega}{dt} = \frac{\pi f_s}{H}(P_m - P_{max}sin\delta) \qquad \text{Eqn. 1.16}$$

II. SMALL-DISTURBANCE STABILITY ANALYSIS OF SMIB SYSTEM

The classical model of synchronous generator in a SMIB system was derived in the previous section. In this section we will try to understand how the stability of the generator in a SMIB system can be checked when subjected to small-disturbances. The swing equation, given in (1.16), can be written as

$$\frac{H}{Hf_s}\cdot\frac{d^2\delta}{dt^2} = P_m - P_{max}\sin\delta$$ Eqn. 1.17

In the steady state, that is when the speed of the generator rotor is constant at synchronous speed, the rate of change of rotor speed will become zero due to which the above equation can be written as

$$P_m = P_{max}\sin\delta$$ Eqn1.18

Since, the mechanical power input P_m and the maximum power output of the generator P_{max} are known for a given system topology and load, we can find the rotor angle from (1.18) as

$$\delta = sin^{-1}\left(\frac{P_m}{P_{max}}\right) \text{ or } \pi - \sin^{-1}\left(\frac{P_m}{P_{max}}\right)$$
Eqn.1.19

To understand which of the solutions (δ_0 , δ max) leads to a stable operation. Let us take first point A (corresponding rotor angle is δ_0) .If we perturb the rotor angle δ_0 by a small positive angle δ_0, so that the new operating point is at C, then electrical power output will also increase to P_{max} sin($\delta_{0,}$ Δ δ_0).Since, in steady state P_m=P $_{max}$ sin ($\delta_{0,}$ Δ δ_0), after perturbation P_m<P $_{max}$ sin($\delta_{0,}$ Δ δ_0), for a positive value of $\Delta\delta_0$

Now the output electrical power will become more than the input mechanical power and hence the rotor starts decelerating due to which the angle δ will be pulled back to the point A.

But since the rotor has certain inertia it cannot stop at the point A and decelerate further due to which the angle δ moves to the point say D. At point D $P_m > P_e$ that is input mechanical power will become more than the electrical power output and hence the rotor starts accelerating. Again the rotor angle δ starts increasing and will reach point A but due to inertia it cannot stop there and it will again move to point C.

This phenomenon repeats itself indefinitely if there is no damping to the rotor oscillations. This can be understood analogously from the motion of the pendulum in vacuum. If a pendulum is perturbed from its steady state position (the vertical position with the bob of the pendulum hung by a string attached to a fixed point) then it swings to one extreme point reverses it direction pass through its steady state position goes to the other extreme and reverse and pass through the extreme and this happens indefinitely as it is oscillating in vacuum and there is no air friction to stop the oscillations.

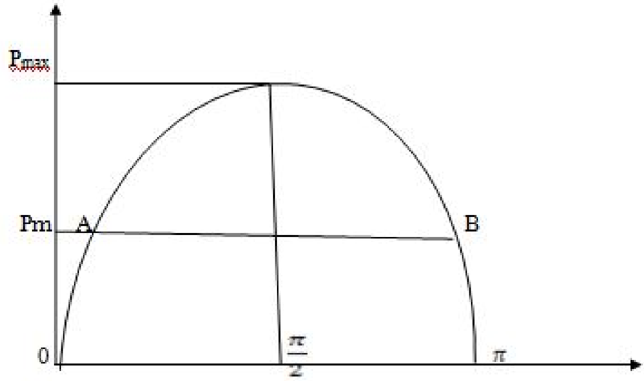

Fig.3 Swing curve

Now take the case of second operating point δ max that is point B in Fig.3 above .Again if we perturb by a positive angle δ_{max} then the electrical power output will be P_{max} sin (δ max+Δ δ max). In this case however, unlike the earlier case, and the rotor starts accelerating due to which the angle will increase further and this will lead to further decrease in the electrical power output. Hence, for a small positive perturbation in the rotor angle at the operating point B leads to continuous increase in the speed of the rotor there by leading to unstable operation of the generator. This discussion leads us to an important conclusion that out of the two operating points A and B, with rotor angles δ_0 and δ_{max} , operating point A is stable and operating point B is unstable for small disturbances.

Hence, point A is called stable equilibrium point and operating point B is called as unstable equilibrium point. Similarly δ_0 is a stable steady state rotor angle and δ_{max} is an unstable rotor angle. Though we have discussed about the implications of the two operating points A and B through their physical effect when subjected to disturbance, we can also prove that operating point A is stable as compared to operating point B mathematically.

III. CONCLUSION

From the swing equation given by Eqn.1.16 Where, ω=(p/2) ωm if P_m=P_{max} sinδ , then there will be no speed change and there will be no angle change. But, if $P_m \neq P_{max}$ sinδ due to disturbance in the system then either the speed increase or decrease with respect to time.

For the case of $P_m > P_{max}\sin\delta$, more input mechanical power than the electrical power output. In this case, as the energy has to be conserved the difference between the input and output powers will lead to increase in the kinetic energy of the rotor and speed increases. However, if $P_{max}\sin\delta > P_m$ then, the input power is less than the desired electrical power output. Again the balance power, to meet the load requirement, is drawn from the kinetic energy stored in the rotor due to which the rotor speed decreases.

This analysis again leads us to an important conclusion that out of the two operating points A and B, with rotor angles δ_0 and δ_{max} , operating point A is stable and operating point B is unstable for small disturbances. Hence, point A is called stable equilibrium point and operating point B is called as unstable equilibrium point. Similarly δ_0 is a stable steady state rotor angle and δ_{max} is an unstable rotor angle. Though we have discussed about the implications of the two operating points A and B through their physical effect when subjected to disturbance, we can also prove that operating point A is stable as compared to operating point B mathematically.

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