Comparative Study Of Funicular Analysis And Finite Element Analysis For Masonry Dome

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Abstract- *Masonry Domes are common features for historical monuments covering large span. Funicular analysis and finite element analysis has been done in present study for axisymmetric hemispherical domes having different t / R ratio. Funicular analysis has been used to construct the thrust line of masonry dome. The stability of structure has been studied from the results of funicular analysis. Change in thrust line by considering different material properties like Density and Modulus of elasticity has been carried out. Optimization has been also done using varying thickness of domes at bottom and top. Finite element analysis has been done in ABAQUS / CAE 6.14 software.*

Keywords- Masonry Domes, Axisymmetric, Funicular analysis, Finite element analysis, Stability of dome, Thrust line approach, ABAQUS.

I. INTRODUCTION

The analysis of Masonry Dome can be done using two different approach. 1) Thrust line approach and 2) Finite element approach. Thrust line approach is graphical method to construct the thrust line of masonry dome. Funicular analysis is one of the thrust line approach. The stability of dome can be easily understood by results of thrust line approach. While finite element analysis is a numerical method for solving problems of engineering. To solve the problem we subdivides a large problem into smaller and simpler parts which is called finite elements.

1.1 Thrust Line Analysis

Funicular analysis is one of the thrust line approach. Funicular analysis is graphical analysis emerged from early factual investigations with hanging chain models. Half cross section of the arch is drawn at a large scale. Then, it is divided into smaller segments which are preferably of equal length. Their weights are calculated and thrust line has been drawn. Funicular method consists 1) Form diagram and 2) Force (Funicular) diagram. Funicular method is easy to understand compared to the finite element analysis. We can plot thrust line using funicular diagram which can be helpful to understand the stability of dome. Stress results cannot be obtained by funicular analysis.

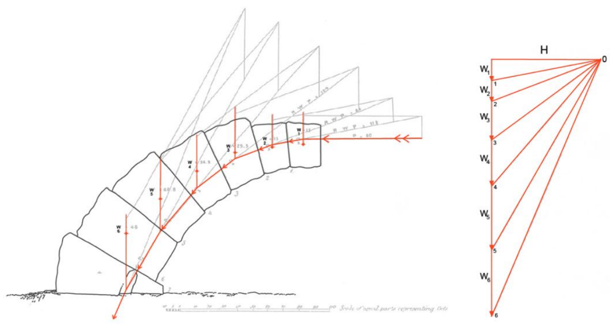

Fig -1: Form diagram and force diagram

Robert Hook's hanging chain model is explained by J.F.D. Dahmen and J.A. Ochsendorf [7] . The hanging chain model will give the thrust line of arch. The behaviour of chain is in total tension while hanging with two fixed ends. The arch will be in total compression so its thrust line will behave same as the hanging chain in total tension. The arch formed by hanging chain will give the thrust line of arch by inverting it.

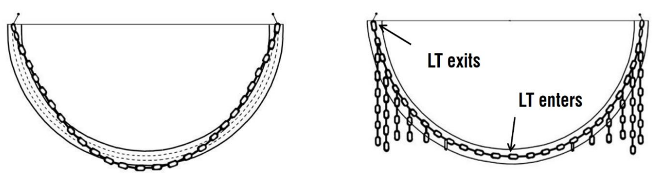

Fig -2: Hanging chain model

The stability of arch can be understood from the position of thrust line. If thrust line is passing through the cross section of arch then the arch is stable. We can understand from fig.-2 that if thrust line formed by hanging chain do not passes from the cross section of arch then we can also adjust thrust line by increasing the weight. In real construction practice, we can increase the weight by increasing the thickness of arch and can optimize the position of thrust line. We know that the cross section of dome is arch so from results of stability of arch, we can decide the stability of dome. In general domes are subjected to two types of forces, 1) Hoop force and 2) Meridional force as shown in fig-3.

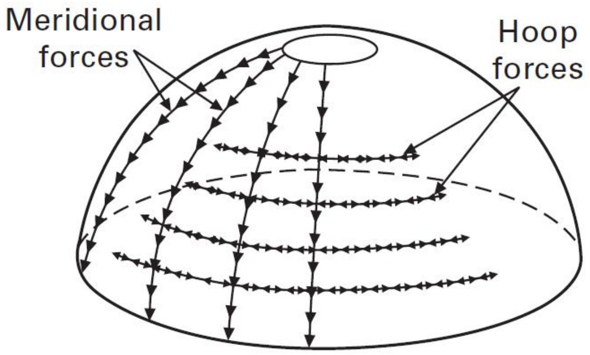

Fig -3: Forces acting on dome

1.2 Finite Element Analysis

Masonry finite element modelling can be divided into two categories:

1) Micro modelling
2) Macro modelling

In micro modelling we consider masonry unit and mortar as different elements. While in macro modelling masonry units and mortar are considered as a homogeneous continuum. In micro modelling, it is difficult to decide the thickness of mortar and pattern of masonry units. Thus macro modelling is used for finite element analysis of masonry. The finite element method focuses on a stress-based analysis. Finite element analysis can be used for domes or arches having complex geometry, boundary condition etc. while thrust line analysis can only be applied for simple geometry and boundary condition.

Dr. Mahesh N. Varma[1] proposed a new method FETLA for analysis of masonry dome. He was responsible for the structural design of world's now largest span masonry dome – '*Grand Pagoda'*. Finite element thrust line analysis (FETLA) is combination of finite element analysis and thrust line analysis. Using FETLA, we can generate the trust line from the stress results of finite element analysis.

Paolo Foraboschi[4] shows that resisting system of masonry dome is comprised of arches and dome splits into arches. The splitting of dome does not depends upon the mechanical properties of material. It depends upon the pattern of bricks or stone arranged. The brick or stone pattern dictates the thickness of the arches that the dome splits into and therefore it dictates the thickness of the resisting system.

According to Heyman[5] during limit analysis of the masonry dome we take some assumptions. The stresses are low enough for crushing strength of masonry to be considered as infinite and the construction will be assumed to be dry. Thus, the compressive strength of masonry unit is infinite and the tensile strength of masonry is zero.

II. FUNICULAR ANALYSIS

The funicular analysis of following models has been done by the method explained in the manual of Auroville Earth Institute[9] . The funicular diagram are drawn in AUTOCAD Software. The depth of arch has been assumed to be 1 m.

t / R Ratio	0.04	0.08	0.12	0.16	0.18	0.20
Radius	7 m					
Density	2 x 10^4 N/m^3			1.9 x 10^4 N/m^3		

1) t / R ratio = 0.04
Radius = 7 m
Thickness of arch = 0.280 m
Density = 2 x 10^4 N/m^3

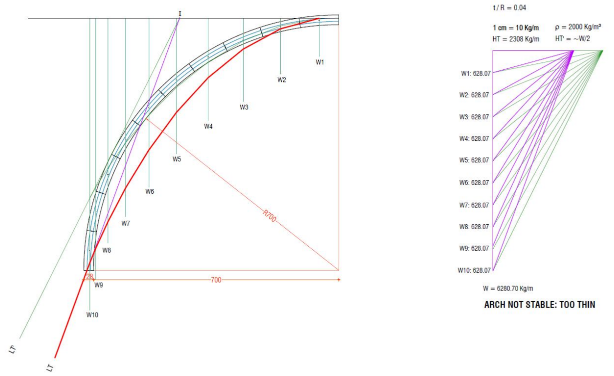

Fig -4: Funicular diagram for t/R=0.04

2) t / R ratio = 0.08
Radius = 7 m
Thickness of arch = 0.560 m
Density = 2 x 10^4 N/m^3

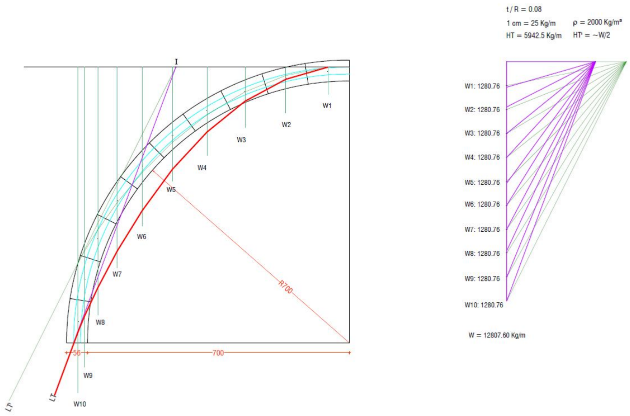

Fig -5: Funicular diagram for t/R=0.08

3) t / R ratio = 0.12
Radius = 7 m
Thickness of arch = 0.840 m
Density = 2 x 10^4 N/m^3

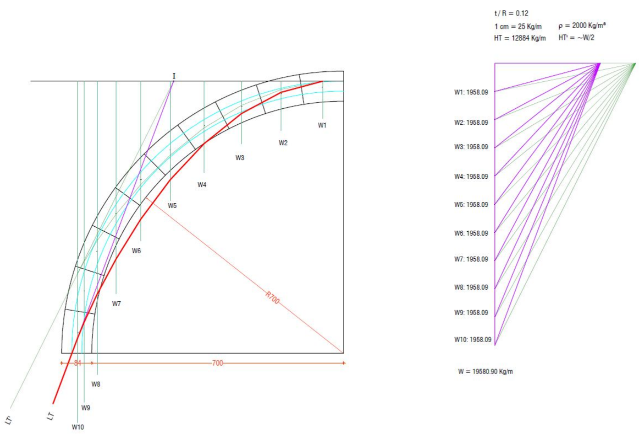

Fig -6: Funicular diagram for t/R=0.12

4) t / R ratio = 0.16
Radius = 7 m
Thickness of arch = 1.120 m
Density = 2 x 10^4 N/m^3

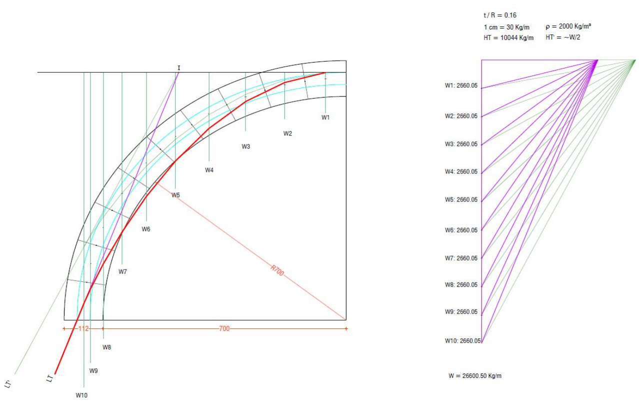

Fig -7: Funicular diagram for t/R=0.16

5) t / R ratio = 0.18
Radius = 7 m
Thickness of arch = 1.260 m
Density = 2 x 10^4 N/m^3

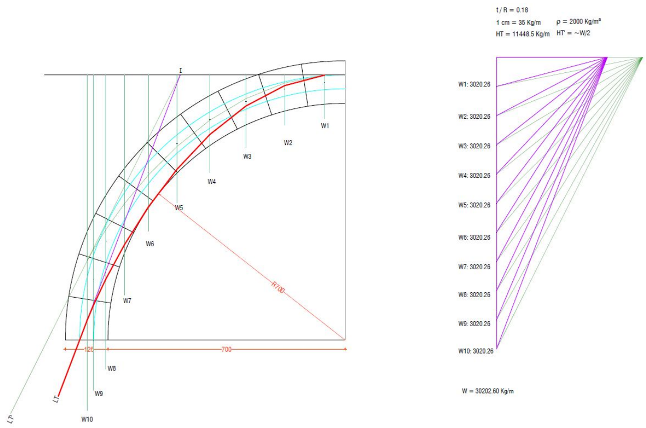

Fig -8: Funicular diagram for t/R=0.18

6) t / R ratio = 0.20
Radius = 7 m
Thickness of arch = 1.40 m
Density = 2 x 10^4 N/m^3

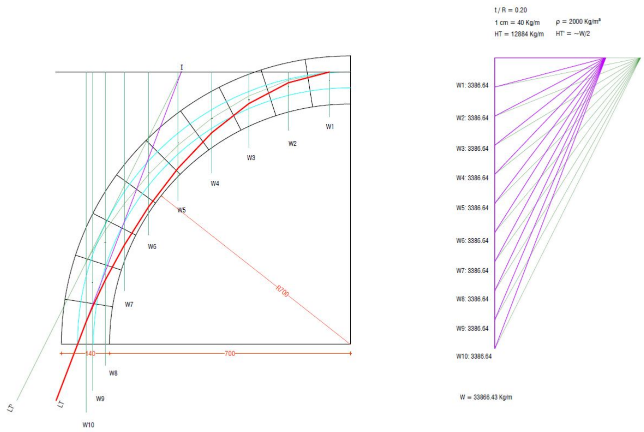

Fig -9: Funicular diagram for t/R=0.20

In all figures of funicular diagrams shown above, the red line indicates the thrust line of arch. We can clearly see in that for t/r = 0.04 to 0.16, the thrust line is not passing through cross section of arch thus these arches are not stable. As t/R ratio increases, the thrust line moves towards the intrados of arch. When t/R is 0.18, Thrust line enters in the cross section of the arch. Thus for t/R i.e. 0.18 and 0.20, the arches are stable as their thrust lines are passing through their cross section.

2.1 Change In Thrust Line With Different Density

To check the changes in thrust line due to change in density, we will take different density as 1.9 x 10^4 N/m^3 and draw funicular diagram for above models.

1) t / R ratio = 0.04
Radius = 7 m
Thickness of arch = 0.280 m
Density = 1.9 x 10^4 N/m^3

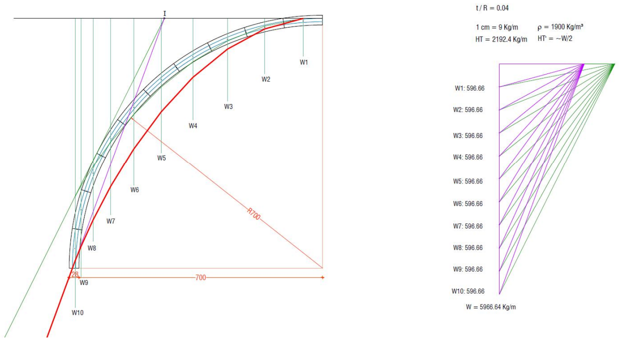

Fig -10: **Funicular diagram for t/R=0.04 with density 1.9 x 10^4 N/m^3**

2) t / R ratio = 0.16
Radius = 7 m
Thickness of arch = 1.120 m
Density = 1.9 x 10^4 N/m^3

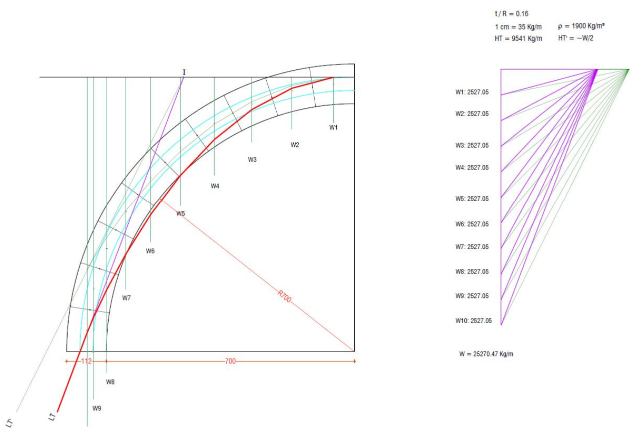

Fig -11: **Funicular diagram for t/R=0.16 with density 1.9 x 10^4 N/m^3**

Here we can see that thrust line do not changes its position for different density. The values of weight of segments and hoop tension (HT) varies with change in density. Thus, position of thrust line of arch do not depends on the density of material.

2.2 Optimization Of Arch

We can optimize the position of arches in funicular analysis. Let us take an example for optimization. The arch having 1.26 m thickness at bottom and 0.28 m thickness at top is considered. The thrust line of discussed arch will pass from section as shown below in fig.-12.

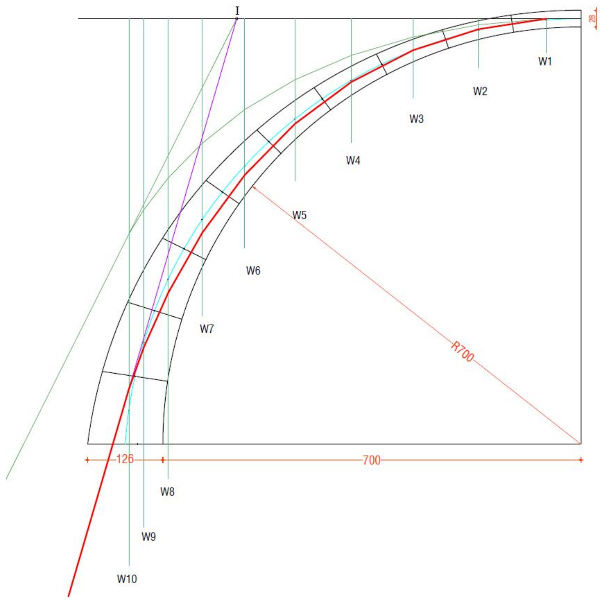

Fig -12: **Thrust line of arch having varying thickness**

Arch having uniform thickness of 0.28 m has thrust line outside of its cross section as shown in fig.-4. Arch having uniform thickness of 1.26 m has its thrust line inside the cross section but passing nearly from the intrados of arch. Thus with varying thickness of arch we can achieve thrust line passing nearly from the center line of arch which make arch more stable and safe.

III. FINITE ELEMENT ANALYSIS

Finite element analysis has been done for all the models discussed in funicular analysis with considering following material properties:

Modulus of Elasticity	2 x 10^9 N/m^2	2.1 x 10^9 N/m^2
Poisson's ratio	0.1	
Density	2 x 10^4 N/m^3	1.9 x 10^4 N/m^3

The finite element analysis has been done in ABAQUS / CAE 6.14 Software. The macro modelling approach has been adopted.
Available stress and strain results at intrados and extrados of dome are extracted.

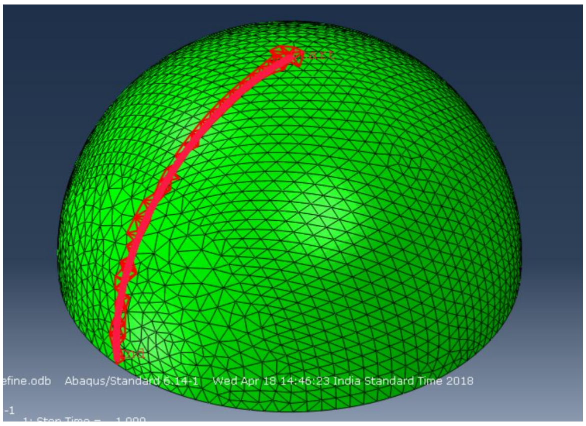

Fig -13: **Applied mesh to the dome and path to extract results**

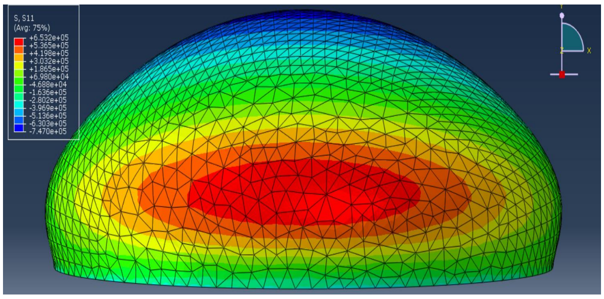

Fig -14: **Normal Stress S11 along X direction**

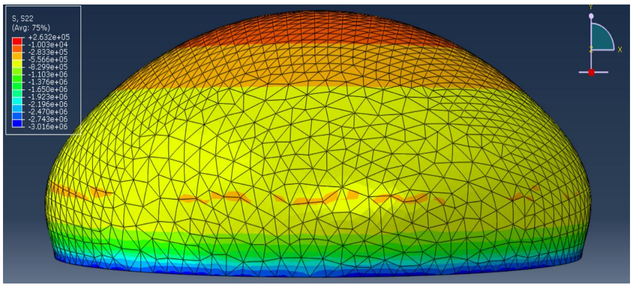

Fig -15: **Normal Stress S22 along Y direction**

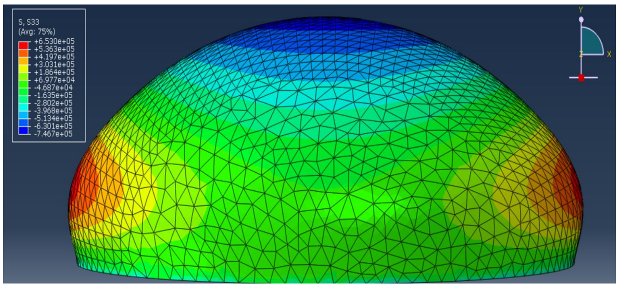

Fig -16: **Normal Stress S33 along Z direction**

As shown in figure above we can extract the normal stresses S11, S22 and S33 along X, Y and Z direction respectively. Similarly we can also extract the shear stresses S12, S23 and S13 along XY, YZ and XZ direction respectively. We can also extract the values of strain for each direction.

IV. STRESS RESULTS OF FEA

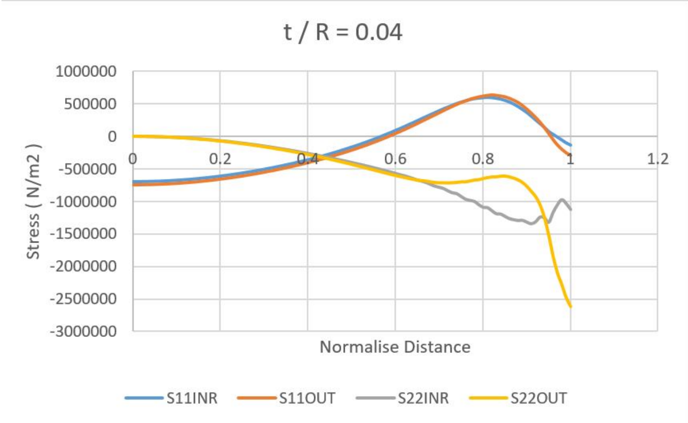

Fig -17: S11 and S22 for t / R = 0.04

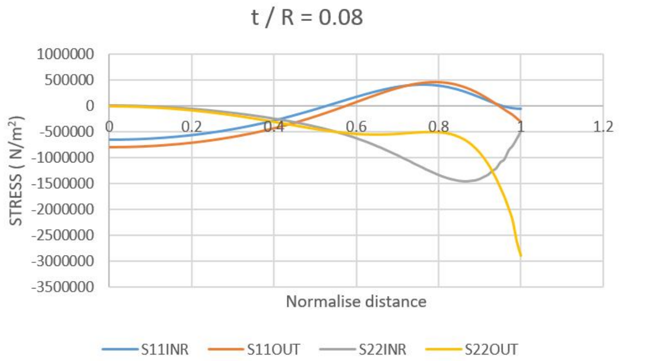

Fig -18: S11 and S22 for t / R = 0.08

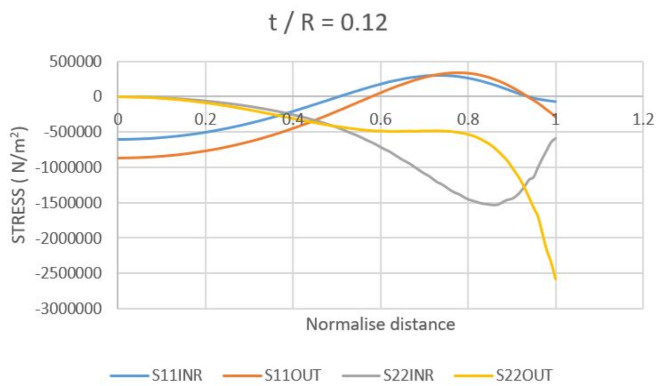

Fig -19: S11 and S22 for t / R = 0.12

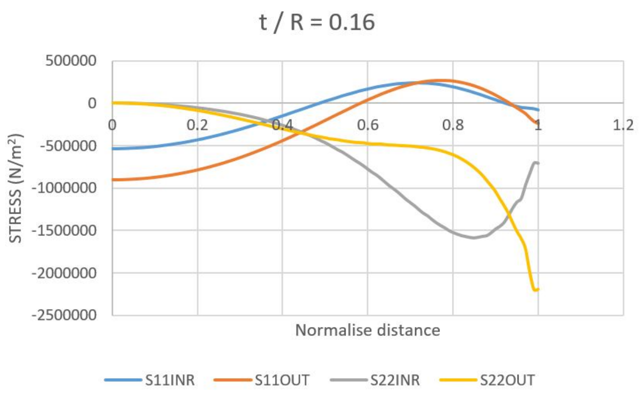

Fig -20: S11 and S22 for t / R = 0.16

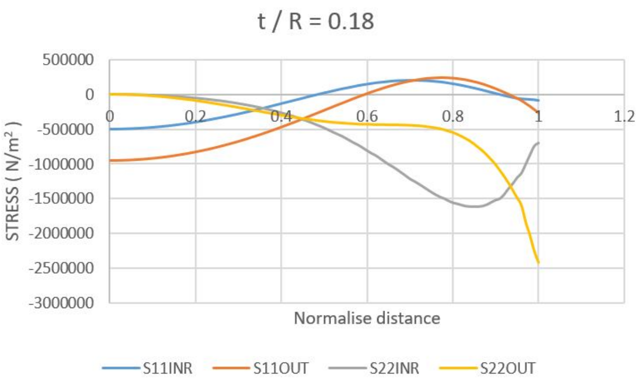

Fig -21: S11 and S22 for t / R = 0.18

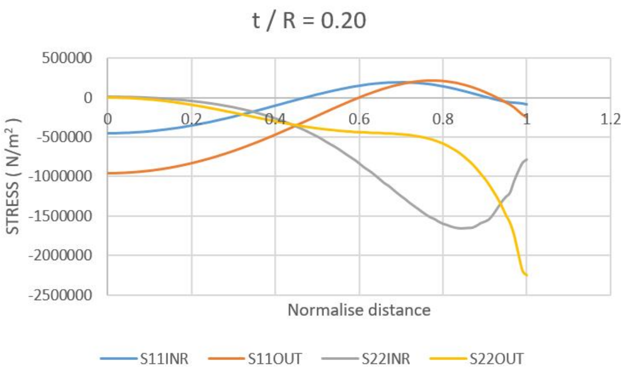

Fig -22: S11 and S22 for t / R = 0.20

V. CONCLUSION

From above stress results we can see that the values of S11 is negative in some portion and then it becomes positive and the values of S22 are negative in every case. Here, behavior of S11 is like bending stress which varies negative to positive for crown to bottom of dome respectively. And S22 behaves like direct stress which will be always in compression means negative from crown to bottom of dome. As thickness increases, the value of S11 at extrados becomes more negative.

The study of discussed dome has been done for different material properties but their stress behavior remains same. Thus stress behavior of dome and thrust line of dome do not depends upon the material properties. It depends upon the geometry of dome.

From funicular analysis, we can conclude that dome having uniform thickness and t/R ratio greater than or equal to 0.18 will be stable and if t/R ratio is less than 0.18 then dome will not be stable and safe. We can also optimize the position of thrust line by changing the thickness as shown in fig.-12.

We cannot directly estimate the position of thrust line from finite element analysis. It only gives as stress based results, but using FETLA proposed by Dr. Mahesh Varma[1] we can construct thrust line from stress results. According to FETLA the minimum t/R ratio for hemispherical dome to be stable is 4.32%.The detailed study of FETLA could be done in future work.

VI. ACKNOWLEDGEMENT

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