# Fuzzy Strongly (Gsp)*- Super Closed Sets In Fuzzy Topological Space

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***Abstract****- In this paper we have introduced a new class offuzzy setscalled fuzzystrongly (gsp)*-super closed sets, properties of this set are investigated and we introduce new fuzzy spaces namely, fuzzy $T_s$(gsp)*-space, fuzzy g$T_s$(gsp)*-space, fuzzy g*$T_s$(gsp)*- space and fuzzy g**$T_s$(gsp)*-space.*

***Keywords***- fuzzy Strongly (gsp)*- super closed sets, fuzzy Strongly (gsp)*- super continuous maps, fuzzy $T_s$(gsp)*-space, fuzzyg$T_s$(gsp)*-space, fuzzy g*$T_s$(gsp)*-space and fuzzy g**$T_s$(gsp)*-space.

## I. PRELIMINARIES

Throughout this paper (X,τ) and (Y,σ) represent fuzzy topological spaces. For a fuzzy subset A of a space (X,τ),cl(A) and int(A) denote the fuzzy closure and the fuzzy interior of A respectively.

**Definition 1.1:**A Subset A of fuzzy topological space (X,τ) is called;

1. Fuzzysemi super open set if A ≤cl(int(A)) and a fuzzy semi- super closed set if int(cl(A)) ≤ A.
2. Fuzzy semipre- super open set if A ≤cl(int(cl(A)) and a fuzzy semi-pre super closed set if int(cl(int(A))) <A
3. Fuzzy regular - super open set if int(cl(A))=A and a fuzzy regular - super closed.

**Definition 1.2:**A Subset A of fuzzy topological space (X,τ) is called;

1. Fuzzy generalized super closed set (resp. fuzzy g- super closed) if cl(A) ≤U whenever A≤ U and U is fuzzy super open in (X,τ)
2. Fuzzy g*- super closed set if cl(A) ≤ U whenever A ≤ U and U is fuzzy g super open in (X,τ)
3. Fuzzy g**-super closed set if cl(A) ≤ U whenever A ≤ U and U is fuzzy g* super open in (X,τ)
4. Fuzzy wg -super closed set if cl(int(A)) ≤ U whenever A ≤ U and U is fuzzy super open in (X,τ)
5. Fuzzy regular generalized super closed set (resp. fuzzyrg- super closed) if cl(A) ≤U whenever A ≤U and U is fuzzy regular super open in (X,τ)
6. Fuzzy sg**-super closed set if scl(A) ≤U whenever A ≤U and U is fuzzy g** super open in (X,τ)
7. Fuzzy sg*-super closed set if scl(A) ≤U whenever A ≤U and U is fuzzy g* super open in (X,τ)
8. Fuzzy generalized semi- super closed set (briefly fuzzy gs- super closed) if scl(A) ≤U whenever A≤U and U is fuzzy super open in (X,τ)
9. Fuzzy gsp - super closed set if spcl(A) ≤U whenever A ≤U and U is fuzzy super open in (X,τ)
10. Fuzzy (gsp)*-super closed set if cl(A) ≤U whenever A ≤U and U fuzzy gsp is super open in (X,τ)

**Definition 1.3:** A function f :(X,τ ) → (Y,σ) is called ;

1. Fuzzy g –super continuous if $f^{-1}$(V) is a fuzzy g- super closed set of (X,τ) for every fuzzy super closed set V of (Y,σ)
2. Fuzzy g*–super continuous if $f^{-1}$(V) is a fuzzy g*- super closed set of (X,τ) for every fuzzy super closed set V of (Y,σ)
3. Fuzzy g**–super continuous if $f^{-1}$(V) is a fuzzy g**- super closed set of (X,τ) for every fuzzy super closed set V of (Y,σ)
4. Fuzzyrg-super continuous if $f^{-1}$ (V) is a fuzzy rg - super closed set of (X,τ) for every fuzzy super closed set V of (Y,σ)
5. Fuzzywg–super continuous if $f^{-1}$(V) is a fuzzy wg - super closed set of (X,τ) for every fuzzy super closed set V of (Y,σ)
6. Fuzzy(gsp)*–super continuous if $f^{-1}$(V) is a fuzzy (gsp)* - super closed set of (X,τ) for every fuzzy super closed set V of (Y,σ)

**Definition 1.4:**A fuzzy topological space (X,τ ) is said to be;

1. Fuzzy$T_{1/2}$* space if every fuzzy g*- super closed set in it is fuzzy super closed.
2. Fuzzy$T_d$ space if every fuzzy gs - super closed set in it is fuzzy g- super closed.

## II. FUZZYSTRONGLY (GSP)* - SUPER CLOSED SETS IN FUZZY TOPOLOGICAL SPACE

**Definition 2.1:**A subset A of a fuzzy Topological space (X,τ ) is said to be a fuzzy strongly (gsp)*- super closed set if cl(int(A)) ≤ U whenever A ≤ U and U is fuzzy gsp- super open.

**Lemma2.1:** Everyfuzzy super closed set is fuzzy strongly (gsp)*- super closed.

**Proof:**Let A be a fuzzy super closed.Then cl(A) = A. Let us prove that A is fuzzy strongly (gsp)* - super closed. Let A≤ Uand U be fuzzygsp- super open. Then cl(A) ≤ U .Since A is fuzzy super closed . cl(int(A)) ≤ cl(A) <U. Then cl(int(A)) <U whenever A ≤ U and U is fuzzygsp-super open.so A is fuzzy strongly (gsp)*-super closed. The converse of the above proposition need not be true in general as seen in the following example.

**Lemma 2.2:** Everyfuzzy g- super closed set is fuzzy strongly (gsp)*- super closed.

**Proof:**Let A be fuzzy g- super closed. Then cl(A) ≤U Whenever A ≤U and U is fuzzy super open in (X, τ ).To prove A is fuzzy strongly (gsp)* - super closed.Then A ≤U and U be fuzzy (gsp) super open.We have cl(A)≤U Whenever A ≤U and U is fuzzy super open in (X, τ ).Since every fuzzy super open set is (fuzzy gsp)–super open.We have cl(A) ≤U Whenever A ≤ U and U is fuzzy (gsp)- super open in (X, τ ).But cl(int(A)) ≤cl(A) ≤U whenever A ≤U and U is (fuzzy gsp)- super open in (X, τ ) then cl(int(A)) ≤U whenever A ≤U and U is fuzzy (gsp) - super open in (X, τ ) so A is fuzzy strongly (gsp)*- super closed.The converse of the above proposition need not be true in general as seen in the following example.

**Lemma 2.3:** Everyfuzzy g*- super closed set is fuzzy strongly (gsp)* - super closed.

The converse of the above proposition need not be true in general as seen in the following example.

**Lemma 2.4:** Every fuzzyrg– super closed set is fuzzy strongly (gsp)* - super closed.

**Proof:**Let A be fuzzy rg- super closed set. Then cl(A) ≤U Whenever A ≤U and U is fuzzy regular- super open in (X, τ)**.**To prove A is fuzzy strongly (gsp)* - super closed.Let A ≤U and U be fuzzy (gsp) super open. Since every fuzzy regular- super open set is fuzzy (gsp)- super open.We have cl(A) ≤U Whenever A ≤U and U is fuzzy (gsp)- super open in (X, τ )**.**But cl(int(A)) ≤cl(A) ≤U whenever A ≤U and U is fuzzy (gsp)- super open in (X, τ )so cl(int(A)) ≤U whenever A<U and U is fuzzy (gsp)- super open in (X, τ ). So A is fuzzy strongly (gsp)*- super closed.

**Remark 2.1:** fuzzy strongly (gsp)* - super closed is independent of fuzzy semi- super closed.

**Lemma 2.5:** Everyfuzzy (gsp)* - super closed set is fuzzy strongly (gsp)* - super closed set

**Proof:**Let A be fuzzy (gsp)*-super closed set .Then cl(A) ≤U Whenever A ≤U and U is fuzzy gsp- super open in (X,τ ).To prove A is fuzzy strongly (gsp)*-super closed.Let A ≤U and U be fuzzy (gsp) super open. Since every fuzzy (gsp)*-super open set is fuzzy (gsp)- super open.We have cl(A) ≤U Whenever A ≤U and U is fuzzy (gsp)- super open in (X,τ ).But cl(int(A)) ≤cl(A) ≤U whenever A ≤U and U is fuzzy (gsp)- super open in (X, τ )≤cl(int(A)) ≤U whenever A ≤ U and U is fuzzy (gsp)- super open in (X, τ ) so A is fuzzy strongly (gsp)* - super closed. The converse of the above proposition need not be true in general as seen in the following example.

**Theorem 2.14:**Every fuzzy g** - super closed set is fuzzy strongly (gsp)* - super closed.

The converse of the above proposition need not be true in general.

**Remark 2.2:** fuzzy strongly (gsp)* - super closed is independent of fuzzy sg**- super closed.

**Remark 2.3:** Stronglyfuzzy (gsp)* - super closed is independent of fuzzy sg* - super closed.

**Lemma 2.4:** Everyfuzzy wg- super closed set is fuzzy strongly (gsp)* - super closed.

**Proof:**Let A be fuzzy wg- super closed**.**Then cl(int(A)) ≤U Whenever A ≤U and U is fuzzy super open in (X, τ )**.**To proveA is fuzzy strongly (gsp)* - super closed.Let A ≤U and U be fuzzy (gsp) super open. Since every fuzzy wg- super open set is fuzzy (gsp)-super open**.**We have cl(int(A)) ≤U whenever A ≤U and U is fuzzy (gsp)- super open in (X, τ )then cl(int(A)) ≤Uwhenever A ≤U and U is fuzzy (gsp)- super open in (X, τ )**.**A is fuzzy strongly (gsp)* - super closed.

**Lemma 2.5:**If A and B isfuzzy strongly (gsp)* - super closed sets, then AUB are also fuzzy strongly (gsp)* - super closed.

**Proof:** Let A and B be fuzzy strongly (gsp)* - super closed. Let AUB ≤ U and U be fuzzy (gsp)- super open. Then A ≤U andB ≤U where U is fuzzy (gsp)- super opencl(int(A)) ≤U, whenever A ≤U and U is fuzzy (gsp)- super open and cl(int(B))≤U, whenever B ≤U and U is fuzzy (gsp)- super openSince A and B are fuzzy strongly (gsp)* -super closed**.** cl(int(A) Uint(B)) = cl(int(A)) U cl(int(B)) ≤U whenever AUB ≤U and U is fuzzy (gsp)- super open.Therefore AUB isalso fuzzy strongly (gsp)* - super closed.

**Lemma 2.5:**If A is a fuzzy strongly (gsp)* - super closed set of (X,τ ) such that A ≤B ≤cl(int(A)) , then Bis also strongly fuzzy (gsp)* - super closed set of (X,τ )

**Proof:** Let U be a fuzzy (gsp) – super open set in (X,τ ) such that B ≤U Then A ≤U, Since A is strongly (gsp)*- super closed ,cl(int(A)) ≤U**.** Now cl(int(B)) ≤cl(int(A)), since B ≤cl(int(A)).Therefore cl(int(B)) ≤cl(int(A)) ≤U cl(int(B)) ≤U whenever B ≤U and U is (gsp)- super open.=> B is strongly (gsp)* - super closed.

## III. FUZZY STRONGLY (GSP)* - SUPER CONTINUOUS MAPPING

We recall the followings;

**Definition 3.1:**A function f:(X,τ ) → (Y,σ) is called fuzzy Strongly (gsp)* - super continuous if $f^{-1}(V)$ is a fuzzy strongly(gsp)* - super closed set in (X,τ ) for every fuzzy super closed set V of (Y,σ).

**Theorem 3.1:**Every fuzzy super continuous map is fuzzy strongly (gsp)* -continuous

**Proof:**Let f:(X,τ )→(Y,σ) be a fuzzy super continuous map let us prove that f is fuzzy strongly (gsp)* - continuous and let F be a fuzzy super closed set in (Y,σ).Since f is fuzzy super continuous $f^{-1}(F)$ is super closed in (X,τ ) then $f^{-1}(F)$ is fuzzy strongly (gsp)* - super closed so f is fuzzy strongly (gsp)* - super continuous the converse of the above Theorem is not true

**Theorem 3.2:** Everyfuzzy g- super continuous map is fuzzy strongly (gsp)* -continuous

**Proof:**Let f :(X,τ ) → (Y,σ) be a fuzzy g-continuous**.** Let F be a fuzzy super closed set in (Y,σ). Since f is fuzzy g- super continuous $f^{-1}(F)$ is fuzzy g- super closed in (X,τ ) .then ,$f^{-1}(F)$ is fuzzy strongly (gsp)* - super closedso f is fuzzy strongly (gsp)* -super continuous fuzzy super closedg**- super closed fuzzy g- super closed fuzzywg- super closed fuzzysg**- super closed fuzzyg*- super closedfuzzy strongly (gsp)*- super closedfuzzy (gsp)*- super closedfuzzy Semi-super closedsg*- super closedsg- super closed.The converse of the above Theorem is not true

**Theorem 3.3:** Every g* - super continuous map is strongly (gsp)* -continuous.

**Proof:**Let f :(X,τ ) → (Y,σ) be a g* -continuous**.** Let F be a super closed set in (Y,σ)**.** Since f is g* - super continuous $f^{-1}(F)$ is g* - super closed in (X,τ ) **.**Then $f^{-1}(F)$ is strongly (gsp)* - super closedso f is strongly (gsp)*- super continuous The converse of the theorem is not true

**Theorem 3.4:** Every fuzzy g** - super continuous map is fuzzy strongly (gsp)* -continuous.

**Proof:** Let f :(X,τ ) → (Y,σ) be a fuzzy g** -continuous**.** Let F be a fuzzy super closed set in (Y,σ).Since f is fuzzy g** -super continuous $g^{-1}(F)$ is g** - super closed in (X,τ ).then $f^{-1}(F)$ is fuzzy strongly (gsp)* - super closed so f is fuzzy strongly (gsp)*-super continuous The converse of the above Theorem is not true.

**Theorem 3.5:**Every fuzzy rg- super continuous map is fuzzy strongly (gsp)* -continuous.

**Proof:** Let f :(X,τ )→(Y,σ) be a fuzzy rg- super continuous **.** Let F be a fuzzy super closed set in (Y,σ) **.** Since f is fuzzy rg super continuous $f^{-1}(F)$ is fuzzyrg- super closed in (X,τ ).By Then ,$f^{-1}(F)$ is fuzzy strongly (gsp)* - super closedso f is fuzzy strongly (gsp)* -continuous.

**Theorem 3.6:** Everyfuzzy (gsp)*-super continuous map is fuzzy strongly (gsp)* - super continuous.

**Proof:** Let f :(X,τ ) → (Y,σ) be a fuzzy (gsp)* -super continuous **.** Let F be a fuzzy super closed set in (Y,σ) **.** Since f is fuzzy (gsp)*-super continuous $f^{-1}(F)$ is (gsp)*- super closed in (X,τ ).then $f^{-1}(F)$ is fuzzy strongly (gsp)*-super closed**so**fis fuzzy strongly (gsp)* -continuous. The converse of the above Theorem is not true.

**Theorem 3.7:** Everyfuzzy wg- super continuous map is fuzzy strongly (gsp)* -continuous.

**Proof:**Let f :(X,τ )→(Y,σ) be a fuzzy wg- super continuous **.** Let F be a fuzzy super closed set in (Y,σ) **.** Since f is fuzzy wg-super continuous $f^{-1}(F)$ is fuzzy wg- super closed in (X,τ ) then , $f^{-1}(F)$ is fuzzy strongly (gsp)* - super closedf is fuzzy strongly (gsp)* - super continuous

## IV. APPLICATIONS OF FUZZY STRONGLY (GSP)* - SUPER CLOSED SETS

As application of fuzzy strongly (gsp)*- super closed sets, new spaces, namely fuzzy $T_s$(gsp)* space , g$T_s$(gsp)* space , fuzzyg*$T_s$(gsp)* , fuzzy g**$T_s$(gsp)*space are introduced. We introduced the following definitions.

**Definition4.1:** Afuzzy space (X, τ) is called a fuzzy $T_s$(gsp)* - space if every fuzzy strongly (gsp)* - super closed set is super closed.

**Definition 4.2:** Afuzzy space (X, τ) is called a fuzzy g$T_s$(gsp)* - space if every fuzzy strongly (gsp)* - super closed set is fuzzy g super closed.

**Definition 4.3:** A fuzzy space (X, τ) is called a fuzzy g*$T_s$(gsp)* -space if every fuzzy strongly (gsp)* - super closed set is g*- super closed.

**Definition4.4:**A fuzzy space (X, τ) is called a fuzzy g**$T_s$(gsp)* -space if every fuzzy strongly (gsp)* - super closed set is fuzzy g**- super closed.

**Theorem 4.1:**Every fuzzy $T_s$(gsp)* -space is fuzzy $T_{1/2}$* - space.

**Proof:** Let (X, τ ) be a fuzzy $T_s$(gsp)* - space. Let us prove that (X, τ ) is a fuzzy $T_{1/2}$* - space**.** Let A be a fuzzy g* - super closedset. Since every fuzzy g* - super closed set is fuzzy strongly (gsp)* - super closed, A is fuzzy strongly (gsp)* - super closed. Since (X, τ) is a fuzzy $T_s$(gsp)* - space , A is fuzzy super closed.(X,τ) is a fuzzy $T_{1/2}$* - space.

**Theorem4.2:**Every fuzzy $T_s$(gsp)* -space is fuzzy gTs(gsp)*- space.

**Proof:** Let A be a fuzzy strongly (gsp)*- super closed set**.** Then A is fuzzy super closed. Since the fuzzy space is $T_s$(gsp)* - space.And every super closed set is fuzzy g- super closed**.**Hence A is fuzzy g- super closed**.** (X,τ) is a fuzzy g$T_s$(gsp)*- space. The converse is not true.

**Theorem 4.3:** Everyfuzzy g*$T_s$(gsp)* -space is fuzzy gTs(gsp)* -space.

**Proof:**Let A be a fuzzy strongly (gsp)* - super closed.Then A is fuzzy g* - super closed, since the fuzzy space is a fuzzy g*$T_s$(gsp)* -space since. Every fuzzy g* - super closed set is fuzzy g- super closed .Hence A is fuzzy g- super closed then (X,τ) is a fuzzy g$T_s$(gsp)* -space. The converse is not true.

**Theorem 4.4:** Every g*$T_s$(gsp)* -space is g**$T_s$(gsp)* -space.

**Proof:**Let A be a strongly (gsp)*- super closed set**.** Then A is g* - super closed, since the space is a g*$T_s$(gsp)* .SinceEvery g**- super closed set is g*- super closed**.**Hence A is g**- super closed. (X,τ) is a g**$T_s$(gsp)* -space.

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