An Analytical Evaluation of Minimum Shear Reinforcement for RC Beams

Ravinder Kumar[1], Dr. B. P. Suneja[2]
[1]Dept of Civil Engineering
[2]HOD, Dept of Civil Engineering
[1, 2] Rajasthan Technical University, Kota, Rajasthan, 324010, India.

Abstract- *The present work is an analytical study to see the effect of grade of concrete (fck), shear span to depth ratio (a/d) and percentage longitudinal reinforcement (ρ) on minimum shear reinforcement in reinforced concrete beams. A comparative study of limits of minimum shear reinforcement as given in various international codes has been worked out. A rectangular section beam is analyzed on the basis of shear strength criterion. After that curve fitting and regression analysis is done to fit a power curve and new formula for determining the minimum shear reinforcement is generated.*

Keywords- Longitudinal reinforcement, Shear reinforcement, Shear span, Shear strength criterion.

Nomenclature and abbreviations

S_v = Stirrup spacing along the length of member

A_s = Area of shear reinforcement

a/d = Shear span to depth ratio

b_w = Width of beam.

d = Effective depth

D = Over all depth of beam

f_{ck} = Characteristic cube compressive strength of concrete.

f'_c = Cylindrical compressive strength

f_y = Characteristic yield strength of reinforcement.

f'_y = Characteristic yield strength of tensile steel

V_{cr} = Cracking shear

V_{cz} = Shear resistance of the uncracked portion of concrete.

V_d = Dowel force in tension reinforcement due to dowel action.

V_s = Shear resistance carried by the transverse reinforcement.

V_u = Ultimate shear resistance by cracked section

V_{uc} = Shear force resistance by uncracked concrete

ρ = percentage longitudinal reinforcement

V_u = Ultimate shear resistance by cracked section

V_c = Shear capacity of concrete

I. INTRODUCTION

The failure of beams is generally initiated with formation of cracks. The location of cracks depends on the direction of principal stresses which is combination of shear stresses and tensile stresses. Hence, in addition to flexural reinforcement, shear reinforcement also plays an important role in enhancing the strength and overall structural behavior of reinforced concrete members. Accordingly, the codes of every nation specify the requirement of shear reinforcement, its maximum limits and recommendation to adopt the minimum shear reinforcement. Different international codes of practice for reinforcement concrete recommend the minimum amount of shear reinforcement considering the various factors in different ways. Generally, minimum shear reinforcement is a function of characteristic compressive strength of concrete and yield strength of steel. The results, using these codes results show that the increase in grade of concrete increases the requirement of minimum shear reinforcement. In Indian code (IS 456:2000), percentage longitudinal reinforcement and characteristic compressive strength of concrete is used as a function of shear reinforcement design but that are not used as function of minimum shear reinforcement.

II. LITERATURE REVIEW

Minimum shear reinforcement is a function of many parameters such as grade of concrete, grade of steel, shear span to depth ratio and percentage longitudinal reinforcement but the Indian standard code IS 456 (2000) dose note consider all these parameters except yield strength of steel (fy) in determining minimum shear reinforcement. [1]**Guney Ozcebe, et. al. (1994)** gives an equation to determine minimum shear reinforcement for high strength concrete. [2]**Young-Soo Yoon, et. al. (1996)** presented evaluation of minimum shear reinforcement requirements in normal, medium and high strength reinforced concrete beams. A comparison and evaluation of minimum shear reinforcement provision of the 1989 ACI code and the 1994 CSA standard are made. [3]**Mark K. Johnson and Julio A. Ramirez (1989)** investigated adequacy of minimum shear reinforcement in concrete beams. [4]**S. Ahmad and A. Elahi (2003)** studied the effect of reinforcement ratio and shear span on shear strength of high strength concrete. [5]**Mohamed Zakaria et. al. (2009)** presented an experimental investigation to clarify shear cracking behavior of reinforced concrete beams. [6]**N. Subramanian (sep., 2010)** show the minimum and maximum limits on longitudinal and the Indian code provisions are compared with the ACI code, Eurocode, Canadian and New Zealand codes provisions, suitable modifications to the expressions are suggested for future editions of the Indian code. [7]**L. Sudheer Reddy et. al. (2010)** deals with review of available data base and predict shear strength of reinforced concrete beams without web reinforcement by different shear models. [8]**G. Appa Rao and S. S. Injaganeri (2013)** compared the minimum shear reinforcement provision of different nation codes. Also generate an equation of minimum shear reinforcement for high strength concrete. [9]**B. K. Kohlapur (2013)** an experimental investigation is carried out on HSC beams with constant size and effective length by varying the longitudinal reinforcement ratio and the web reinforcement ratio to understand the shear behavior of the beams with minimum web reinforcement as per IS CODE and ACI CODE and maximum web reinforcement.

III. THEORY

1.1 Comparative study of different nations codes for minimum shear reinforcement

Minimum shear reinforcement provisions given in different nations codes are different due to considering the different parameters. These provisions are generally similar in formulation but their coefficients are different. This is due to the different approaches and different definitions are used in different codes.

Table-1: Minimum Shear Reinforcement by various Codes of Practice

S N	CODE	EQUATION
1	ACI 318M-2008	$\frac{A_s}{b_w S_v} = \frac{0.9\sqrt{f_{ck}}}{16 f_y} \geq \frac{0.33}{f_y}$
2	CSA A23.3-2004	$\frac{A_s}{b_w S_v} = \frac{0.054\sqrt{f_{ck}}}{f_y}$
3	Eurocode 2-1992	$\frac{A_s}{b_w S_v} = \frac{0.08\sqrt{f_{ck}}}{f_y}$
4	NZS 3101: 2006, Part 1	$\frac{A_s}{b_w S_v} = \frac{0.9\sqrt{f_{ck}}}{16 f_y}$
5	IS 456: 2000	$\frac{A_s}{b_w S_v} = \frac{0.4}{0.87 f_y}$ When $\tau_v \geq 0.5\tau_c$
6	BS 8110: Part 1: 1997	$\frac{A_s}{b_w S_v} = \frac{0.4}{0.95 f_y}$
7	AASTHO LRFD 2000	$\frac{A_s}{b_w S_v} = \frac{\sqrt{f_{ck}}}{12 f_y}$

Value of minimum shear reinforcement index (MSRI or r^*f_y) calculated as per various codes are shown graphically in Figure 1. It has been observed that the minimum reinforcement calculated by different codes is nearly close to each other up to M-40 grade of concrete.

Table 2: Minimum Shear Reinforcement Index (r*fy or MSRI) for Different Grades of Concrete as per Various Codes

Code	Grade of Concrete								
	M20	M30	M40	M50	M60	M70	M80	M90	M100
ACI (2008)	0.33	0.33	0.356	0.398	0.436	0.471	0.503	0.534	0.563
AASTHO (2000)	0.373	0.456	0.527	0.589	0.645	0.697	0.745	0.791	0.833
CSA (2004)	0.224	0.296	0.342	0.382	0.418	0.452	0.483	0.512	0.54
IS:456 (2000)	0.46	0.46	0.46	0.46	0.46	0.46	0.46	0.46	0.46
BS:8110 (1997)	0.421	0.421	0.421	0.421	0.421	0.421	0.421	0.421	0.421
NZS (2006)	0.252	0.308	0.356	0.398	0.436	0.471	0.503	0.534	0.563
Eurocode (1992)	0.358	0.438	0.506	0.566	0.62	0.669	0.716	0.759	0.8

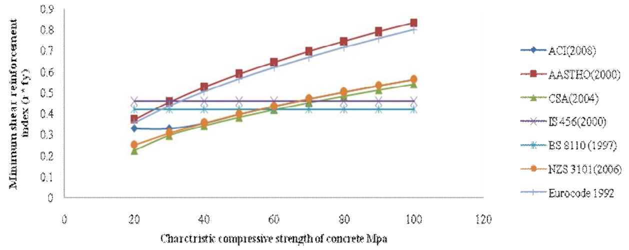

Figure 1: Variation of MSRI for Different Grade of Concrete for Different Nations Codes.

As the grade of concrete increases the effect of fck becomes dominant for calculation of minimum shear reinforcement. It realizes that the factor fck should be considered in determining the minimum shear reinforcement particularly for high strength concrete. In general the Indian standards compared to other Nation's code gives higher value of min. shear reinforcement for low grade of concrete and low values for higher grade of concrete

IV. NUMERICAL STUDY

The basic concept of determining minimum shear reinforcement is that when diagonal shear failure occurs then shear resistance of cracked section should be more than the cracking shear. Minimum shear reinforcement is corresponding to the limiting condition in which shear force resistance by cracked section is equal to the cracking shear. Shear resistance of beam is combination of shear force resistance provided by uncracked concrete, aggregate interlock, dowel force and resistance provided by reinforcement as stirrups.

$$V_u \geq V_{cr}$$

Under limiting condition,

$$V_u = V_{cr}$$

$$V_u = V_c + V_s$$

Cracking shear V_{cr} given by

$$V_{cr} = \left(v_u \frac{\sqrt[3]{\frac{a}{d}}}{2\rho^{\frac{1}{6}}} \right) bd$$

$$v_u = 0.5\sqrt{f'_c}$$

$$V_c = V_{uc} + V_a + V_d$$

$$V_c = \left(\frac{0.17\sqrt{f'_c}}{\Upsilon} \right) bd$$

$$V_s = A_{sv} f_y \left(\frac{d}{S_v} \right)$$

$$\left(\frac{0.17\sqrt{f'_c}}{\Upsilon} \right) bd + A_{sv} f_y \left(\frac{d}{S_v} \right) = 0.5\sqrt{f'_c} \left(\frac{\sqrt[3]{a/d}}{2\rho^{1/6}} \right) bd$$

$$\frac{A_{sv} f_y}{S_v} = 0.5\sqrt{f'_c} \left(\frac{\sqrt[3]{a/d}}{2\rho^{1/6}} \right) b - \left(\frac{0.17\sqrt{f'_c}}{\Upsilon} \right) b$$

$$\frac{A_{sv} f_y}{bS_v} = \sqrt{f'_c} \left(0.25 \frac{\sqrt[3]{\frac{a}{d}}}{\rho^{\frac{1}{6}}} - \frac{0.17}{\Upsilon} \right)$$

Here, ϒ is higher for light weight concrete.

Final equation of minimum shear reinforcement for different grade of concrete is as below:

$$\frac{A_{sv} f_y}{bS_v} = \sqrt{f'_c} \left(0.25 \frac{\sqrt[3]{a/d}}{\rho^{1/6}} - 0.22 \right)$$

4.1 Curve Fitting Analysis

In order to generate the formula to determine MSR that includes the various factors like f_{ck}, ρ, a/d etc., a curve fitting analysis is carried out using the various data obtained by theoretical analysis for MSR. For different value of a/d and ρ, a power curve in the form below is fitted for different value of a/d and ρ.

$$r * f_y = K (f'_c)^n$$

or

$$r * f_y = K (f_{ck})^n$$

Where 'K' and 'n' are constants

Taking logarithm of base ten of both sides, we get

$$log(r*f_y) = log[K(f_{ck})n]$$

$$log(r*f_y) = logK + log[(f_{ck})n]$$

$$log(r*f_y) = logK + nlog(f_{ck})$$

Putting the

$$r*f_y = Y \quad \text{and} \quad f_{ck} = X$$

Then

$$logY = logK + nlogX$$

For convenient putting

$$logY = y,\ logK = k \text{and} \quad logX = x$$

Hence equation is

$$y = k + nx \qquad (1)$$

The above equation represents a family of straight lines for different values of the arbitrary constants 'k' and 'n' so that equation is the line of best fit. The term best fit is interpreted in accordance with Legendre's Principle of Least Square which states that the sum of the square of the deviation of the actual values y from its estimated values should be minimum so it gives the best fit of line.

Now using method of least squares

$$\Sigma y = \Sigma k + \Sigma(nx)$$

$$\Sigma y = k\Sigma(1) + n\Sigma(x)$$

$$\Sigma y = Nk + n\Sigma(x) \qquad (2)$$

Here, N = Total numbers of observation points

Equation (1) is multiplying with 'x' both sides we get

$$xy = kx + nx^2$$

Taking summation of both sides we get

$$\Sigma(xy) = (\Sigma kx + \Sigma nx^2)$$

$$\Sigma(xy) = \Sigma(kx) + \Sigma(nx^2)$$

$$\Sigma(xy) = k\Sigma(x) + n\Sigma(x^2) \qquad (3)$$

Solving equation (2) and (3); We get

$$k = \frac{\Sigma y \Sigma x^2 - \Sigma x \Sigma xy}{N\Sigma x^2 - (\Sigma x)^2} \text{ and } n = \frac{N\Sigma xy - \Sigma x \Sigma y}{N\Sigma x^2 - (\Sigma x)^2}$$

Taking antilog of 'k' gives the value of K.

Values of 'n' are 0.5 for all values of a/d is 1 to 3 and ρ is 0.25% to 4%. Values of K are tabulated below for a/d 1 is to 3 and ρ is 0.25% to 4%. When a/d is 1 and ρ is greater than 2% then the reinforcement index is negative so value of coefficient K and n are used as same as when a/d is 1 and ρ is 2%.

General equation of minimum shear reinforcement is given below-

$$r * f_y = K\sqrt{f_{ck}}$$

Where value of coefficient K taken from Table 3.

Table 3: Values of Coefficient K for Different a/d and ρ

a/d	ρ=.5%	ρ=1%	ρ=1.5%	ρ=2%	ρ=2.5%	ρ=3%	ρ=3.5%	ρ=4%
1	0.05431	0.02716	0.01139	0.00238				
2	0.11739	0.08601	0.06532	0.05431	0.04498	0.03855	0.02902	0.02716
3	0.16435	0.12566	0.10386	0.08218	0.08122	0.07012	0.0657	0.05611

4.2 Correlation and Regression Analysis

Regression analysis is done to find a simple equation for minimum shear reinforcement for all value of Eighteen equation of minimum shear reinforcement for six different values of a/d ratio and ρ was generated. General form of minimum shear reinforcement as given below:

$$r = \frac{A_{sv}}{b_v S_v} = K\frac{\sqrt{f_{ck}}}{f_y}$$

Here K is constant which is different for different values of a/d ratio and ρ.

These equations show the change in a/d or ρ, change the value of coefficient K in the equation of minimum shear reinforcement, so the amount of minimum shear reinforcement also change. These values of coefficient K show that a relationship between K, a/d ratio and ρ. So a linear relationship establish between K, a/d ratio and ρ.

longitudinal reinforcement percentage at different a/d ratio.

So coefficient of correlation between coefficient K and ρ at a/d is 1 to 3 calculated.

- Calculation for Correlation Coefficient

Here 'x' used for 'ρ' and 'y' for the coefficient K for convenient.

Δx = Deviation from mean in x-series

Δy = Deviation from mean in y-series

x_m =Mean of x-series

y_m = Mean of y-series

Standard deviation of x-series, $\sigma_x = \sqrt{\frac{\Sigma(\Delta x)^2}{N}}$

Standard deviation of y-series, $\sigma_y = \sqrt{\frac{\Sigma(\Delta y)^2}{N}}$

Now coefficient of correlation, r is given by-

$$r = \frac{\sum(\Delta x * \Delta y)}{\sqrt{\sum(\Delta x)^2}\sqrt{\sum(\Delta y)^2}}$$

or

$$r = \frac{\sum(\Delta x * \Delta y)}{N\sigma_x\sigma_y}$$

N = Numbers of observations

All values of 'r' shows that there is negative correlation between coefficient K and ρ. Equation of line of regression for K is calculating in terms of ρ. General equation of line of regression is given below:

$$y - y_m = \frac{r\sigma_y}{\sigma_x}(x - x_m)$$

Three equations of minimum shear reinforcement generated by regression analysis for different a/d ratio given below:

$$r * f_y = (0.06042 - 0.0197\rho)\sqrt{f_{ck}} \qquad when \frac{a}{d} = 1$$

$$r * f_y = (0.13283 - 0.0311\rho)\sqrt{f_{ck}} \qquad when \frac{a}{d} = 2$$

$$r * f_y = (0.18119 - 0.0366\rho)\sqrt{f_{ck}} \qquad when \frac{a}{d} = 3$$

V. RESULT AND DISCUSSION

Table 4 (a): MSRI for Different grade of concrete and ρ for different a/d

ρ	Grade of Concrete								
	M20 concrete			M30 concrete			M40 concrete		
	a/d=1	a/d=2	a/d=3	a/d=1	a/d=2	a/d=3	a/d=1	a/d=2	a/d=3
0.25	0.38	0.71	0.94	0.47	0.87	1.15	0.54	1	1.33
0.5	0.24	0.53	0.74	0.3	0.65	0.9	0.34	0.76	1.04
1.0	0.12	0.38	0.56	0.15	0.47	0.69	0.17	0.54	0.8
1.5	0.05	0.3	0.47	0.07	0.36	0.57	0.08	0.42	0.66
2	0.01	0.24	0.4	0.01	0.3	0.5	0.02	0.34	0.57
3	-	0.17	0.32	-	0.21	0.39	-	0.24	0.45
4	-	0.12	0.26	-	0.15	0.32	-	0.17	0.37

Table 4(b): MSRI for Different grade of concrete and ρ for different a/d

ρ	M60 concrete			M80 concrete			M100 concrete		
	a/d=1	a/d=2	a/d=3	a/d=1	a/d=2	a/d=3	a/d=1	a/d=2	a/d=3
0.25	0.66	1.23	1.62	0.76	1.41	1.87	0.85	1.58	2.1
0.5	0.42	0.93	1.28	0.48	1.07	1.48	0.54	1.19	1.65
1.0	0.21	0.66	0.97	0.24	0.76	1.12	0.27	0.85	1.26
1.5	0.09	0.52	0.81	0.11	0.6	0.94	0.12	0.67	1.05
2	0.02	0.42	0.7	0.02	0.48	0.81	0.02	0.67	1.05
3	-	0.29	0.56	-	0.34	0.64	-	0.38	0.72
4	-	0.21	0.46	-	0.24	0.53	-	0.27	0.59

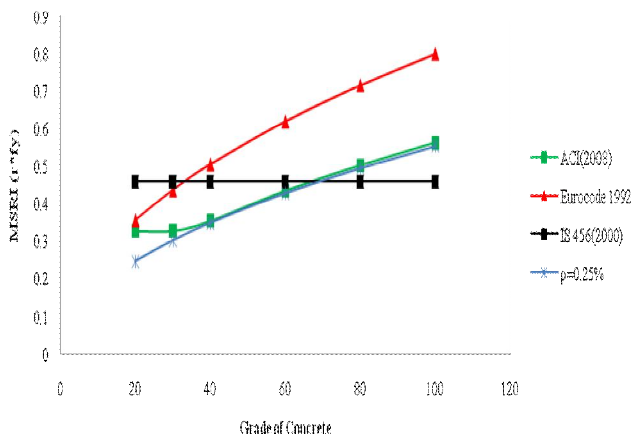

Figure 2: MSRI vs Grade of Concrete at a/d=1

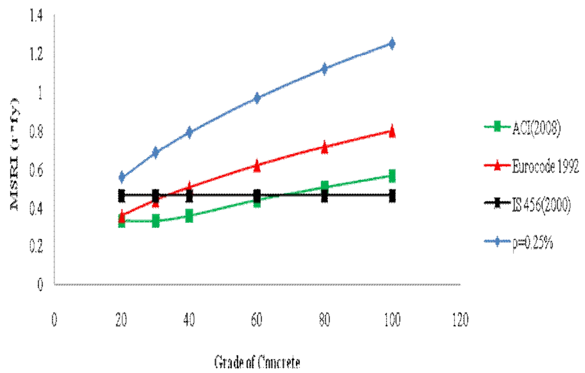

Figure 3: MSRI vs Grade of Concrete at a/d=2

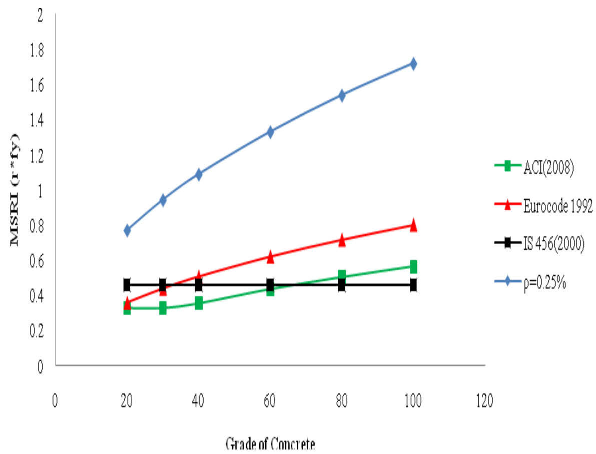

Figure 4: MSRI vs Grade of Concrete at a/d=3

IV. CONCLUSION

A_{svmin} is a function of grade of concrete, a/d ratio and percentage longitudinal reinforcement (ρ). Requirement of A_{svmini} increases with grade of concrete. This is true for all amounts of ρ and a/d ratio. This may be due to the in higher grade of concrete aggregate interface force is smaller due to smooth crack surface. Requirement of A_{svmini} decrease with increases in ρ because of indirect contribution of longitudinal reinforcement in resisting the shear of any section. This is true for all grade of concrete. Requirement of A_{svmini} increases with increase in a/d ratio for all grade of concrete and for all amount of ρ provided. This can be viewed as size effect of beam. Which effect the strength of reinforced beam due to arch action. Deep beams resist more load due to arch action, so beam require less amount of A_{svmin}. It has been revealed that most of codes gives the requirement of A_{svmin} which is closely corresponding to ρ = 1.5%, a/d= 2 and grade of concrete M-40 to M-50. Beyond these values of ρ, a/d ratio and grade of concrete the variation in A_{svmin} determined by different codes becomes large. It has also been observed that A_{svmin} also depends on grade of steel. For all grade of concrete and all a/d ratio A_{svmin} increase with increase in grade of steel.

As using higher grade of steel reduces an amount of tension steel required, hence the dowel force which contribute to shear strength also decrease.

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