Half companion sequences of dio 3-tuples from $\frac{CC_n}{Gno_n}$

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Abstract-

In this paper we try to explicate sequence of dio 3-tuples $\{p_1, p_2, p_3\},\{p_2, p_3, p_4\},\{p_3, p_4, p_5\},.......$ involving half companion sequences under 3 cases with the properties $D[CS_n + (1-n)], D(-3)\ and\ D(2-n)$.

Keywords **-** Centered cubic number, Gnomonic number, Centered square number, dio 3-tuples.

I.INTRODUCTION

A sequence is an enumerated collection of object in which repetitions are not allowed. The number of elements describes the length of the sequence. A tuple is a finite ordered list of elements. An n-tuple is defined inductively using the construction of an ordered pair. The word tuple is originated as an abstraction of the sequence single, double, triple, quadruple, quintuple,………n-tuple ,where the prefixes are taken from Latin names of the numerals. A 3-tuple is a triple or a triplet.

[1-6] has been referred for various ideas on number theory. [7-13] has been studied for interesting ideas on diophantine triples. In [14, 15] special dio 3- tuples were constructed from a special number. Recently in [16], Gaussian triples were constructed with the property D (25).

In this paper we try to explicate sequence of dio 3-tuples $\{p_1, p_2, p_3\},\{p_2, p_3, p_4\},\{p_3, p_4, p_5\},.......$ involving half companion sequences under 3 cases with the properties $D[CS_n + (1-n)],\quad D(-3)\ \ and\ \ D(2-n)$.

Notations:

CC_n = centered cubic number of rank n

Gno_n = Gnomonic number of rank n

$CG_n = \frac{CC_n}{Gno_n}$

CS_n = centered square number of rank n

II.METHOD OF ANALYSIS

Case (i):

Let $p_1 = CG_{n-1} = n^2 - 3n + 2$

$p_2 = CG_n \quad = n^2 - n + 1$

We then have

$$p_1 p_2 + 2n^2 - 3n + 2 = \alpha^2 \quad (1)$$

Where $\alpha = n^2 - 2n + 2$ (2)

Let ‘ p_3 ’ be any positive integer such that $p_1 p_3 + 2n^2 - 3n + 2 = \beta_1^2$ (3)

$$p_2 p_3 + 2n^2 - 3n + 2 = \gamma_1^2 \quad (4)$$

Introduce the linear transformation

$$\beta_1 = x_1 + p_1 y_1 \qquad \gamma_1 = x_1 + p_2 y_1 \quad (5)$$

Using some algebra from (3), (4) and (5) with taking $y_1 = 1$ we’ve

$$x_1 = n^2 - 2n + 2 \quad (6)$$

Using (6) in (5) we get $\beta_1 = 2n^2 - 5n + 4$

Therefore from (3) we’ve $p_3 = 4n^2 - 8n + 7 = CG_{2n-2} + 2n$

Hence $\{p_1, p_2, p_3\}$ is a dio 3-tuple.

ie., $\{CG_{n-1}, CG_n, CG_{2n-2} + 2n\}$ is a dio 3-tuple with the property $D[CS_n + (1-n)]$.

Now let ' p_4 ' be any non-zero integer such that

$$p_2 p_4 + 2n^2 - 3n + 2 = \beta_2^2 \quad (7)$$

$$p_3 p_4 + 2n^2 - 3n + 2 = \gamma_2^2 \quad (8)$$

Introduce linear transformation

$$\beta_2 = x_2 + p_2 y_2 \quad , \gamma_2 = x_2 + p_3 y_2 \quad (9)$$

Using some algebra from (7), (8) and (9) with taking $y_2 = 1$ we have

$$x_2 = 2n^2 - 3n + 3 \quad (10)$$

Using (10) in (9) we get $\beta_2 = 3n^2 - 4n + 4$

Substituting β_2 in (7) we have

$$p_4 = 9n^2 - 15n + 14 = CG_{3n-2} + 7 \quad (11)$$

Therefore $\{p_2, p_3, p_4\}$ is a dio 3-tuple

ie., $\{CG_n, CG_{2n-2} + 2n, CG_{3n-2} + 7\}$ is a dio 3-tuple with the property $D[CS_n + (1-n)]$.

Now let ‘p_5’be any positive non-zero integer

We then have

$$p_3 p_5 + 2n^2 - 3n + 2 = \beta_3^2 \quad (12)$$

$$p_4 p_5 + 2n^2 - 3n + 2 = \gamma_3^2 \quad (13)$$

Introduce linear transformation

$$\beta_3 = x_3 + p_3 y_3 \quad , \gamma_3 = x_3 + p_4 y_3 \quad (14)$$

Using some algebra from (12), (13) and (14) with taking $y_3 = 1$, we’ve

$$x_3 = 6n^2 - 11n + 10 \quad (15)$$

Using (15) in (14) we get $\beta_3 = 10n^2 - 19n + 17$

Substituting β_3 in (12) we've

$$p_5 = 25n^2 - 45n + 41 = CG_{5n-4} + 20$$

Hence $\{p_3, p_4, p_5\}$ is a dio 3-tuple.

ie., $\{CG_{2n-2} + 2n, CG_{3n-2} + 7, CG_{5n-4} + 20\}$ is a dio 3-tuple with the property $D[CS_n + (1-n)]$.

Some numerical examples satisfying the property are listed in the following table

TABLE 1

n	(p_1, p_2, p_3)	(p_2, p_3, p_4)	(p_3, p_4, p_5)	D(n)
3	(2,7,19)	(7,19,50)	(19,50,131)	11
4	(6,13,39)	(13,39,98)	(39,98,261)	22
5	(12,21,67)	(21,67,164)	(67,164,441)	37
6	(20,31,103)	(31,103,248)	(103,248,671)	56
7	(30,43,147)	(43,147,350)	(147,350,951)	79

Hence $\{p_1, p_2, p_3\}, \{p_2, p_3, p_4\}, \{p_3, p_4, p_5\}, ...$ is a sequence of dio 3-tuples involving half companion sequences with the property $D[CS_n + (1-n)]$

Case (ii):

Let $p_1 = CG_{n-2} = n^2 - 5n + 7$

$p_2 = CG_n = n^2 - n + 1$

Proceeding as in case (i) we've

$$p_3 = 4n^2 - 12n + 12 = CG_{2n-3} + 2n - 1$$

$$p_4 = 9n^2 - 21n + 19 = CG_{3n-4} + 6n - 2$$

$$p_5 = 25n^2 - 65n + 61 = CG_{5n-7} + 10n + 4$$

$\therefore \{p_1, p_2, p_3\}, \{p_2, p_3, p_4\}, \{p_3, p_4, p_5\}, ...$ is a sequence of dio 3-tuples involving half companion sequences with the property $D(-3)$.

Some numerical examples satisfying the property are listed in the following table.

TABLE 2

n	(p_1, p_2, p_3)	(p_2, p_3, p_4)	(p_3, p_4, p_5)	D(n)
1	(3,1,4)	(1,4,7)	(4,7,21)	-3
2	(1,3,4)	(3,4,13)	(4,13,31)	-3
3	(1,7,12)	(7,12,37)	(12,37,91)	-3
4	(3,13,28)	(13,28,79)	(28,79,201)	-3
5	(7,21,52)	(21,52,139)	(52,139,361)	-3

Case (iii):

Let $p_1 = CG_{n-2} = n^2 - 5n + 7$

$p_2 = CG_{n-1} = n^2 - 3n + 2$

Proceeding as in earlier cases we've

$$p_3 = 4n^2 - 16n + 17 = CG_{2n-3} - 2n + 4$$

$$p_4 = 9n^2 - 33n + 31 \quad = CG_{3n-5}$$

$$p_5 = 25n^2 - 95n + 94 = CG_{5n-9} + 3$$

Hence $\{p_1, p_2, p_3\}, \{p_2, p_3, p_4\}, \{p_3, p_4, p_5\}, ...$ is a sequence of dio 3-tuples involving half companion sequences with the property $D(2-n)$

Some numerical examples satisfying the property are listed in the following table

TABLE 3

n	(p_1, p_2, p_3)	(p_2, p_3, p_4)	(p_3, p_4, p_5)	D(n)
3	(1,2,5)	(2,5,13)	(5,13,34)	-1
4	(3,6,17)	(6,17,43)	(17,43,114)	-2
5	(7,12,37)	(12,67,91)	(37,91,244)	-3
6	(13,20,65)	(20,65,157)	(65,157,424)	-4
7	(21,30,101)	(30,101,241)	(101,241,654)	-5

III. CONCLUSION

In this paper we have explicated sequence of dio 3-tuples $\{p_1, p_2, p_3\}, \{p_2, p_3, p_4\}, \{p_3, p_4, p_5\}, ...$ involving half companion sequences under 3 cases. One may also try to explicate similar dio 3-tuples from other numbers with suitable property.

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