Some Performance Measures of M(T)/M/1 Queuing Model With Sinusoidal Arrival Rate

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***Abstract*-** *Analysis of queuing models basically end up with methods for some performance measures. Formulation of mathematical model and then obtained the expressions for average waiting time in the queue and in the system, average number of customers in the queue and in the system. All the performance measures are not studied. In this paper, we study average length of non-empty queue, variance of queue length and the probability of arrivals during the service time of any given customer arrive to the system with sinusoidal arrival rate function $\lambda(t)$ and served exponentially with the rate μ in M(t)/M/1 queuing system.*

Keywords: Queuing system, Exponentially, Probability, Sinusoidal function.

I. INTRODUCTION

Queuing theory is applicable to all most all the fields in our daily life. It is widely applicable to Banks, Railways, Supermarkets, Cinema halls, Playgrounds, Schools, Colleges, Telecommunications, Computer time sharing etc. Several authors studied performance measures of various queuing models. They studied the performance measures like average waiting time in the queue and in the system, average number of customers in the queue and in the system and traffic intensity in various queuing models. Some authors studied statistical inference in various queuing models. But some of the performance measures have not studied even though models are available. Clarke, 1957 obtained the maximum likelihood estimates (MLE) of the parameters in an M/M/1 queue model having a Poisson input and a negative exponential service time. Cox, 1965 has statistically analyzed the problems connected with congestion. Wolff, 1965 has discussed the maximum likelihood estimation and likelihood ratio tests for a class of ergodic queuing models which give rise to birth and death processes. Benes, 1957 has discussed a sufficient set of statistics for a simple telephone exchange model. Bhat and Rao, 1987 have statistically analyzed the queuing system1 in detail. Upendra Dave and Y. K. Shah, 1980 have discussed the maximum likelihood estimates for the parameters involved in a stationary M/M/2 queuing process with heterogeneous servers are obtained to make inferences about arrival and service rates. They considered the queue is to be in a state of equilibrium. Basawa and Prabhu, 1981 have discussed the asymptotic inference for single server queues. Basawa and Prabhu, 1988 have also proved the consistency and asymptotic normality of maximum likelihood estimators. Acharya,1999 has discussed with the rate of convergence of the distribution of the maximum likelihood estimators of the arrival and the service rates in a GI/G/1 queuing system. Sharma and Kumar, 1999 have studied the testing of hypothesis, maximum likelihood estimators, uniformly minimum variance unbiased estimators and Bayes estimators of various characteristics of the M/M/1 system. Borthakur and Choudhury, 2005 have studied the estimation and testing of hypothesis of the traffic intensity parameter of the classical single server Markovian M/M/1 queuing model. Ausín, Wiper, and Lillo, 2001 undertake Bayesian inference and prediction for M/G/1 queuing systems. They considered a semi parametric model based on mixtures of Erlang distributions to model the general inter-arrival time distribution. They proposed given arrival and service data, a Bayesian procedure based on birth-death Markov Chain Monte Carlo methods. They also gave an estimation of the system parameters and predictive distributions of measures such as the stationary system size and waiting time. Mukherjee, S.P. and Chowdhury, S., 2005 have studied the maximum likelihood and Bayes estimation in M/M/1 queue. In this paper, they obtained maximum likelihood estimation of traffic intensity in M/M/1 queuing model based on number of customers present at several sampled time points. Chowdhury, S. and Mukherjee, S.P., 2011 Estimation of waiting time distribution in M/M/1 queue. In this paper, they estimated waiting time distribution in the form of its right tail area, known as exceedance probability in M/M/1/FCFS/∞/∞ queue with the help of data consists of non zero waiting time of randomly chosen customer from n independent queues and number of queues with zero waiting time. Chowdhury, S. and Mukherjee, S.P., 2013 have worked out maximum likelihood estimator (MLE) as well as Bayes estimator of traffic intensity in an M/M/1/∞ queueing model in equilibrium based on number of customers present in the queue at successive departure epochs. They also derived estimates of some functions of traffic intensity which provide measures of effectiveness of the queue and a comprehensive simulation study starting with the transition probability matrix. The most recent work in this line seems to be that of Pant, A.P. and Ghimire, R.P., 2015 have studied varioue performance measures of M(t)/M/1 Queuing System with Sinusoidal Arrival Rate such as mean waiting time in the queue and in the system, mean number of customers in the queue and in the system.

In the present study, we obtain various performance measures such as average length of non-empty queue, variance of queue length and the probability of arrivals during the service time of any given customer arrive to the system with sinusoidal arrival rate function λ(t) and served exponentially with the rate μ in M(t)/M/1 queuing system. Thus, our study is the extension of M(t)/M/1 Queuing System with Sinusoidal Arrival Rate by Pant, A.P. and Ghimire, R.P., 2015.

II. MATHEMATICAL MODEL

Usual notations are

n = Number of customers in the system at time t.

λ(t) = Arrival rate function.

λ = Average arrival rate.

μ = Average service time.

P_n = Steady state probability of having n customers in the system.

$P_n(t)$ = Transient state probability that exactly n customers in the queuing system at time t, assuming the system started its operation at time zero.

L_s = Expected line length, i.e. expected number of customer in the system.

L_q = Expected queue length, i.e. expected number of customer in the queue.

W_s = Expected waiting time per customer in the system.

W_q = Expected waiting time per customer in the queue.

P = Traffic intensity.

(L/L > 0) = Expected length of non-empty queue.

K_r = The probability of r arrivals during the service time of any given customer.

Proposed by Pant, A.P. and Ghimire, R.P. solution of the transition differential equation for M(t)/M/1 queuing system with sinusoidal arrival rate function λ(t) and served exponentially with rate μ is

$$P_0 = \left[1 - \frac{\lambda(t)}{\mu}\right]$$

$$P_n = \left[\frac{\lambda(t)}{\mu}\right]^n \left[1 - \frac{\lambda(t)}{\mu}\right]$$

Performance measures of the system are

$$L_s = \frac{\lambda(t)}{\mu - \lambda(t)}$$

$$L_q = \frac{\lambda(t)}{\mu} \cdot \frac{\lambda(t)}{\mu - \lambda(t)}$$

$$W_s = \frac{1}{\mu - \lambda(t)}$$

$$W_q = \frac{\lambda(t)}{\mu} \cdot \frac{1}{\mu - \lambda(t)}$$

$$P = \frac{\lambda(t)}{\mu}$$

III. DERIVATION OF SOME OTHER PERFORMANCE MEASURES

To find the expected length of non-empty queue, (L/L > 0):

(L/L > 0) = L_s/ Probability (an arrival has to wait, L > 0)

$= L_s/(1 - P_0)$

$= \left(\frac{\lambda(t)}{\mu - \lambda(t)}\right) / \left(1 - \left[1 - \frac{\lambda(t)}{\mu}\right]\right)$

$= \frac{\mu}{\mu - \lambda(t)}$

$= \frac{1}{1-\rho}$

To find the variance of queue length:

$$\text{Variance (n)} = \sum_{n=0}^{\infty} n^2 P_n - \left[\sum_{n=0}^{\infty} n P_n\right]^2$$

$$= \sum_{n=1}^{\infty} n^2 P_n - [L_s]^2$$

$$= \sum_{n=1}^{\infty} \left[\frac{\lambda(t)}{\mu}\right]^n \left[1 - \frac{\lambda(t)}{\mu}\right] - \left[\frac{\lambda(t)}{\mu - \lambda(t)}\right]^2,$$

after simplification

$= \left[\frac{\lambda(t)}{\mu}\right] / \left[1 - \frac{\lambda(t)}{\mu}\right]^2$

$= \frac{\rho}{(1-\rho)^2}$

To find the probability of arrival of customer during the service time of any given customer, K_r:

Since the inter-arrival times are exponential and the probability distribution of sum of inter-arrival times follows gamma distribution and service times are exponential, the probability of r arrivals during the service time of any given customer is given by

$K_r = \int_0^\infty P_r(x)\, S(x)\, dx$

where, $P_r(t) = \frac{(\lambda(t))^r e^{-\lambda(t)x} x^{r-1}}{\Gamma r}$,

$S(x) = \mu e^{-\mu x}$

$= \int_0^\infty \frac{(\lambda(t))^r e^{-\lambda(t)x} x^{r-1}}{\Gamma r} \mu e^{-\mu x} dx$

$= \frac{(\lambda(t))^r \mu}{\Gamma r} \int_0^\infty e^{-(\lambda(t)+\mu)x} x^{r-1} dx$

$= \frac{(\lambda(t))^r \mu}{\Gamma r} \frac{\Gamma r}{(\lambda(t)+\mu)^r}$

$= \mu \left(\frac{\lambda(t)}{\lambda(t)+\mu}\right)^r$

$= \mu \left(\frac{\rho}{1+\rho}\right)^r$

IV. CONCLUSION

I have obtained various performance measures such as average length of non-empty queue, variance of queue length and the probability of arrivals during the service time of any given customer arrive to the system with sinusoidal arrival rate function λ(t) and served exponentially with the rate μ in M(t)/M/1 queuing system. I feel that these methods would be helpful in queuing model analysis and also carrying out various empirical analysis.

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